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ITEC452
Distributed Computing

Lecture 9
Program Correctness

Program correctness

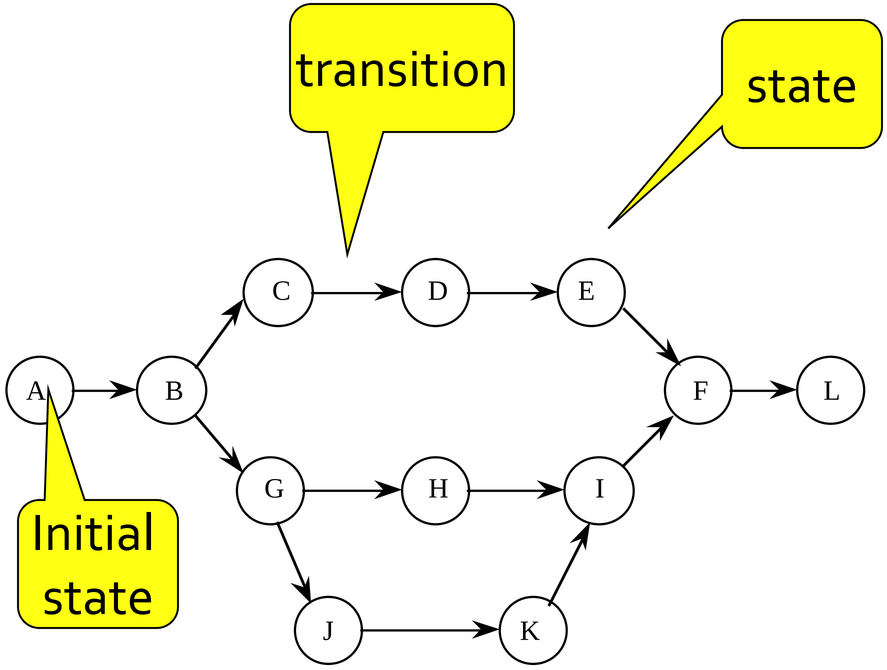
The State-transition model

The set of global states =

$s_0 \times s_1 \times \ldots \times s_m$

$\{s_k$ is the set of local states of process k$\}$

$S0 \xrightarrow[action]{} S1 \xrightarrow[action]{} S2 \xrightarrow[action]{}$



Each transition is caused by an action of an eligible process.

We reason using interleaving semantics

Correctness criteria

- Safety properties
  - Bad things never happen
- Liveness properties
  - Good things eventually happen

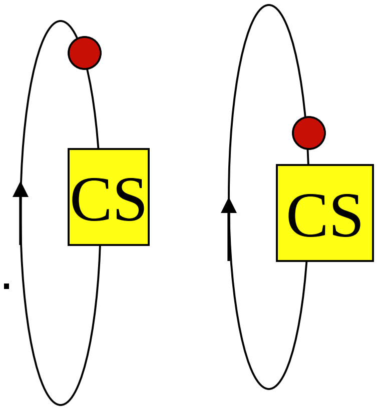
Example 1: Mutual Exclusion

Process 0
do true →
Entry protocol
Critical section
Exit protocol
od

Process 1
do true →
Entry protocol
Critical section
Exit protocol
od

Safety properties
(1) There is no deadlock
(2) At most one process enters the critical section.

Liveness property
A process trying to enter the CS must **eventually succeed**.
(This is also called the ***progress property***)



Testing vs. Proof

Testing: Apply inputs and observe if the outputs satisfy the specifications. Fool proof testing can be painfully slow, even for small systems. Most testing are partial.

Proof: Has a mathematical foundation, and a complete guarantee. Sometimes not scalable.

Correctness proofs

- Since **testing is not a feasible** way of demonstrating the correctness of program in a distributed system, we will use **some form of mathematical reasoning** as follows:
  - Assertional reasoning of proving safety properties
  - Use of well-founded sets of proving liveness properties
  - Programming logic
  - Predicate transformers

Review of Propositional Logic

- <u>Example</u>: Prove that $P \Rightarrow P \vee Q$

- Pure propositional logic is sometimes not adequate for proving the properties of a program, since propositions can not be related to program variables or program state. Yet, an extension of propositional logic, called ***predicate logic***, will be used for proving the properties.

Review of Predicate Logic

- Predicate logic is an extension of propositional logic

  cf. A proposition is a statement that is either true or false.

- A predicate specifies the property of an object or a relationship among objects. A predicate is associated with a set, whose properties are often represented using the universal quantifier ____ (for all) and the existential quantifier ____(there exists).

<quantifier><bound variable(s)>:<range>::<property>

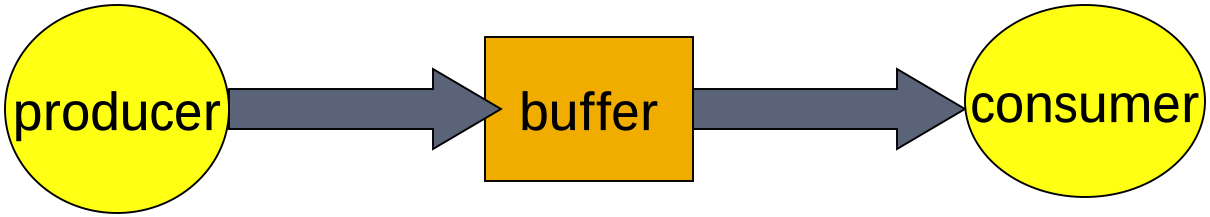
(ex) $\exists\, j : j \in N(i) :: c[j] = c[i] + 1 \textbf{ mod } 3$

Examples of Safety invariant
Well-known synchronization problems

Invariant means: a logical condition which should always be true.

1. ***The mutual exclusion problem***. $N_{CS} \le 1$,
   where N_{CS} is the Total number of processes in CS at any time

2. ***Producer-consumer problem***. $0 \le N_P - N_C \le$ buffer capacity
   (N_P = no. of items produced, N_C = no. of items consumed)



Exercise

What can be a safety invariant for the readers and writers problem?

- Only one write can write to the file at a time.
- When a writer write to the file, no process can read.
- Many processes can read at the same time.

Let N_W denote the number of writer processes updating the file and N_R denote the number of reader processes reading the file.

➔ $((N_W = 1) \wedge (N_R = 0)) \vee ((N_W = 0) \wedge (N_R \geq 0))$

Assertional reasoning of proving safety properties (1)

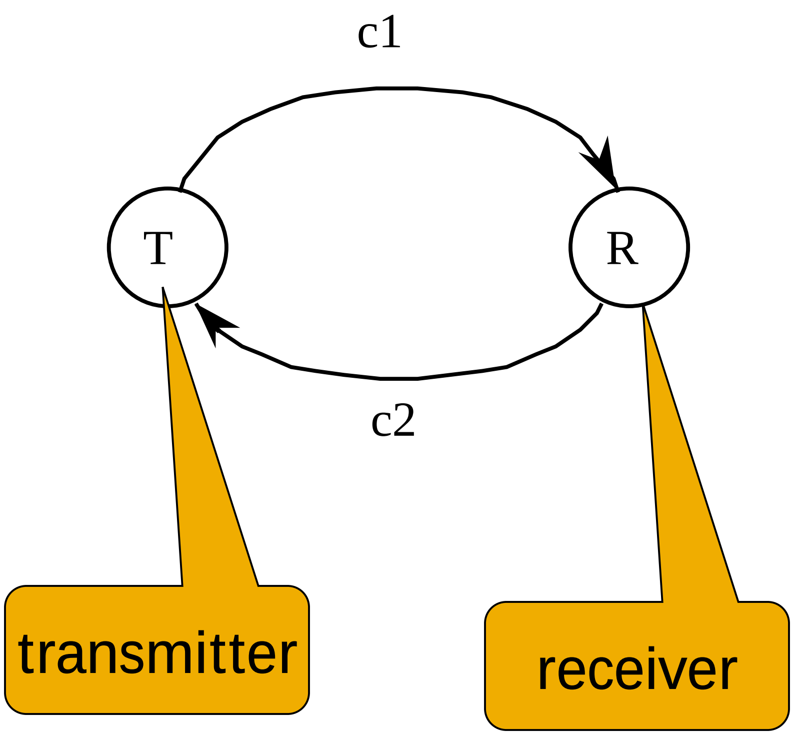
define c1, c2 : channel; {**init** c1 = Φ, c2 = Φ}
r, t : integer; {**init** r = 5, t = 5}

{program for **T**}
1 **do** t > 0 → send msg along c1; t := t -1
2 □ ¬empty (c2) → rcv msg from c2; t := t + 1
od

{program for **R**}
3 **do** ¬empty (c1) → rcv msg from c1; r := r+1
4 □ r > 0 → send msg along c2; r := r-1
od

We want to prove the safety property **P:**
$P \equiv n1 + n2 \leq 10$

n1= # of messages in c1
n2= # of messages in c2



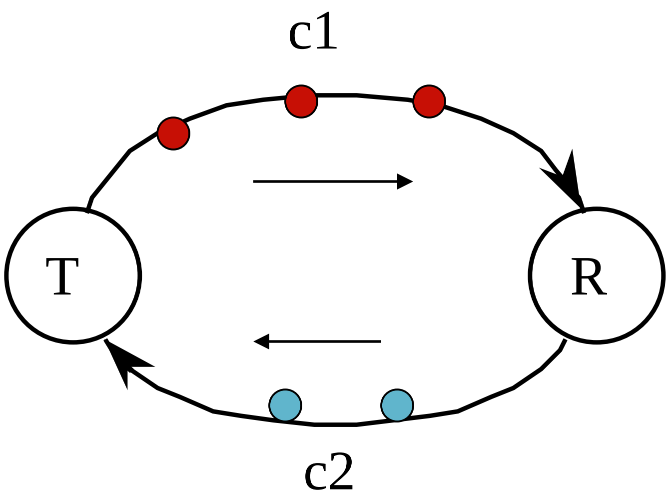
Assertional reasoning of proving safety properties (2)

n1, n2 = # of msg in **c1**and **c2** respectively.
We will establish the following invariant:

I ≡ $(t \geq 0) \wedge (r \geq 0) \wedge (n1 + t + n2 + r = 10)$
(**I** implies **P**). Check if **I** holds after **every action**.

{program for **T**}
1 **do** $t > 0 \rightarrow$ send msg along c1; t := t -1
2 □ ¬empty (c2) $\rightarrow$ rcv msg from c2; t := t+1
od

{program for **R**}
3 **do** ¬empty (c1) $\rightarrow$ rcv msg from c1; r := r+1
4 □ $r > 0$ $\rightarrow$ send msg along c2; r := r-1
od



Use the method of induction

Liveness properties

- Eventuality is tricky. There is no need to guarantee ***when*** the desired thing will happen, as long as it happens.

Type of Liveness Properties

Progress Properties

- If the process want to enter its critical section, it will eventually do.
- No deadlock?

Reachability Properties

: The question is that S_t is reachable from S_o?

- The message will eventually reach the receiver.
- The faulty process will be eventually be diagnosed

Fairness Properties

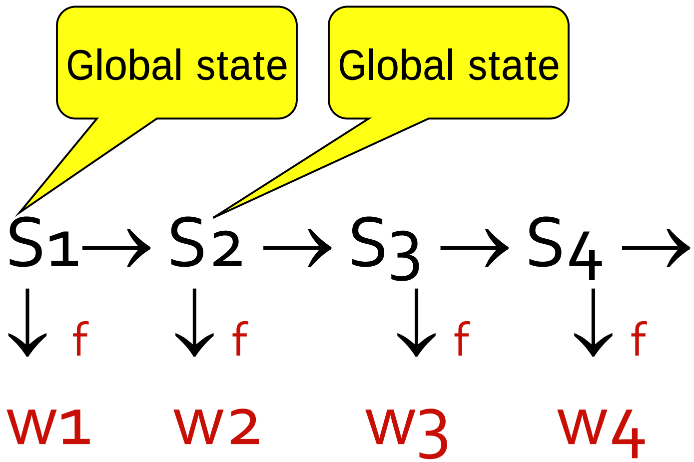
: The question is if an action will eventually be scheduled.

Termination Properties

- The program will eventually terminate.

Proving liveness

Use of well-founded sets of proving liveness properties



- **w1, w2, w3, w4 ∈ WF**
- **WF is a well-founded set whose elements can be ordered by]**

f is called a measure function

If there is no infinite chain like

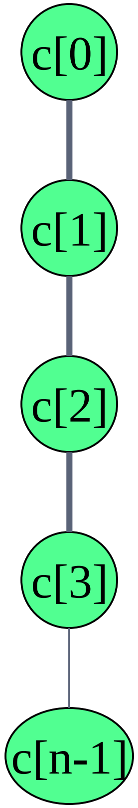
w1] w2] w3] w4 ..., *i.e.*

If an action changes the system state from s1 to s2

$f(s_i)$] $f(s_{i+1})$] $f(s_{i+2})$ ...

then the computation will definitely terminate!

Proof of liveness: an example



Clock phase synchronization

System of n clocks ticking at the same rate.

Each clock is 3-valued, i,e it ticks as 0, 1, 2, 0, 1, 2...

A failure may arbitrarily alter the clock phases.

The clocks need to return to the same phase.

Proof of liveness: an example

$c[k] \in \{0,1,2\}$

Clock phase synchronization
{Program for each clock}
(c[k] = phase of clock k, initially arbitrary)

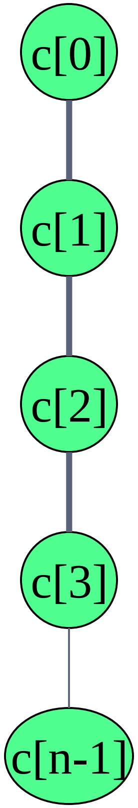
do $\exists$ j: j $\in$ N(i) :: c[j] = c[i] +1 **mod** 3 $\rightarrow$

  c[i] := c[i] + 2 **mod** 3

□ $\forall$ j: j $\in$ N(i) :: c[j] $\neq$ c[i] +1 **mod** 3 $\rightarrow$

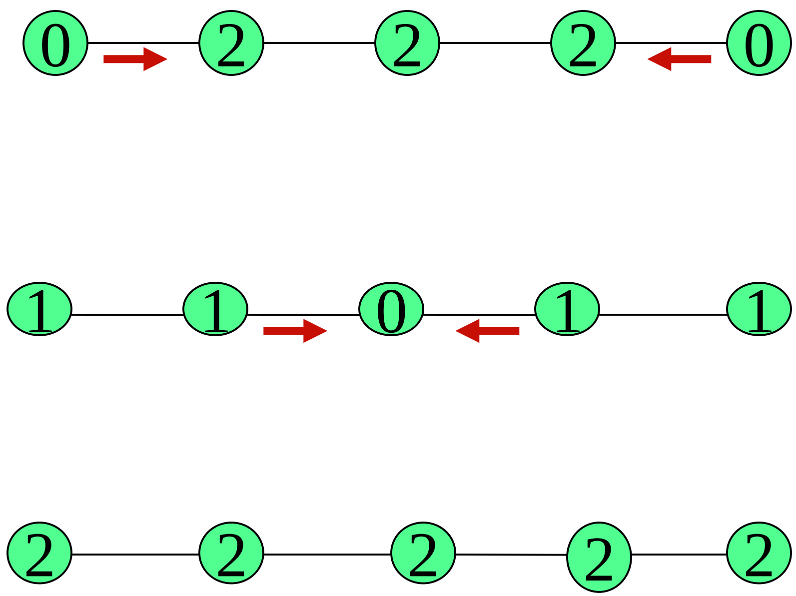
  c[i] := c[i] + 1 **mod** 3

od



Show that eventually all clocks will return to the same phase (convergence), and continue to be in the same phase (closure)

Proof of convergence



Understand the game of arrows

Let **D** = d[0] + d[1] + d[2] + … + d[n-1]

d[i] = 0 if no arrow points towards clock **i**;
= **i** + 1 if a ← pointing towards clock **i**;
= n – **i** if a → pointing towards clock **i**;
= 1 if both ← and → point towards clock **i**.

By definition, $D \geq 0$.

Also, D decreases after every step in the system. So the number of arrows must reduce to 0.

D= 0 means all the clocks are synchronized.