Survivable Embedding of Logical Topologies in WDM Ring Networks

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Abstract

We consider the design of survivable logical topologies over physical WDM ring networks. The logical topology consists of the same set of nodes as the physical topology, and the links of the logical topology are lightpaths in the physical topology. The logical topology is said to be *survivable* if the failure of any single physical link does not disconnect the logical topology. In this paper, we consider the following problem. Given a logical topology with lightpath end-nodes, route the lightpaths to make the logical topology survivable if possible. Otherwise, determine and embed the minimum number of additional lightpaths to achieve survivability.

Key words: WDM, Survivable, Logical Topology, Physical Topology, Logical Topology Embedding, Ring Network

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1 Introduction

Optical networks employing Wavelength Division Multiplexing (WDM) and wavelength-routing are capable of providing lightpaths to higher service layers. Lightpaths are optical circuit-switched paths that have transmission rates of a few Gb/s. By the use of WDM, multiple lightpaths may traverse the same optical fiber link, each one using a different wavelength.

⋆ This work was supported in part by the DARPA under grant N66001-00-18949 (co-funded by NSA), by the DISA under NSA-LUCITE contract, and by the NSF under grant ANI-9973098.

Survivability is a very important requirement for high-speed optical networks. There has been a large amount of work that focuses on pre-allocating backup capacity so that any failed lightpaths may be restored rapidly as soon as normal operation is disrupted in the event of link break. The proposed techniques are classified as either link protection or path protection, depending on whether the rerouting of lightpaths is done around the failed link, or on an end-to-end basis. Protection at the optical layer is considered to be fast, partly because of the proximity of the optical layer to the physical layer at which the failure is first detected, and partly because of the coarse granularity at which restoration is done (at the lightpath or fiber level).

When an electronic service layer is embedded over a WDM optical network, then it may be the case that the electronic layer incorporates its own survivability functions, thereby making the optical layer recovery redundant and, in the worst case, perhaps conflicting. Furthermore, when a physical link fails, it may not be necessary for all the affected lightpath traffic to be restored. Thus, there is a case to be made for recovery to be done solely at the electronic layer. If the electronic layer were the IP layer, then the only requirement for the layer to be survivable is that it be connected.

Motivated by the above, we consider in this paper the embedding of an electronic layer on a physical WDM network such that the electronic layer network is connected when a single link fails. The connectivity at the electronic layer is represented by the *logical topology*. The logical topology is a topology which has as its nodes the set of electronic nodes. The edges of the logical topology correspond to the set of lightpaths that are established over the physical topology.

As mentioned above, multiple lightpaths may be routed over the same physical link, and therefore, it is possible for a single physical link failure to break more than one edge in the logical topology. Since survivability at the logical topology depends on the availability of multiple routes between nodes at the logical layer, it is clear that there must be some amount of coordination between the two layers if survivability has to be achieved at the logical layer. In this paper, we focus on the design of logical topologies that are *survivable*. We define a logical topology to be survivable if the failure of any single physical link does not disconnect the logical topology. Survivable logical topology design not only involves the determination of the logical edges but also the embedding of those edges on the physical topology, i.e., the routing of the lightpaths.

There has been some recent research in the survivable design of survivable logical topologies. In [1], the problem of embedding lightpaths such that the minimum number of source-destination pairs are disconnected upon a physical link failure at the logical layer was considered, and some optimization heuristics were presented. In [2] and [3], a similar problem was considered and some conditions for the survivability of a logical topology were presented. In those three papers, the physical topology was assumed to be an arbitrary mesh. In this paper, we consider a physical ring network with links oriented in both the clockwise and counterclockwise directions. Ring networks are important because the prevalent topology for SONET is the ring. As these networks are upgraded to WDM, it is likely that the topology will be maintained for some time before growing into a mesh network. Secondly, the simplicity of the topology enables us to take a deeper look into the complexity of the problem.

In the next section, we formally state the problem we attempt to solve in this paper. Some insight into the complexity of the problem is presented in Section 3. We present a heuristic algorithm based on shortest path routing in Section 4 and obtain some numerical results. Concluding remarks in Section 5 complete the paper.

2 Problem Formulation

Consider a logical topology shown in Figure 1(a) corresponding to the connection request set $R = \{(0,1),(0,2),(0,4),(1,3),(1,5),(2,4),(2,5),(3,5)\}$ to be embedded over a WDM ring network with six nodes. Figure 1(b-c) show the physical ring topology and two different lightpath embeddings. The embedding shown in (b) is survivable, i.e., the logical topology remains connected when any single physical link fails, whereas the embedding in (c) is not because, when ring link (0,1) fails, the logical topology becomes disconnected.

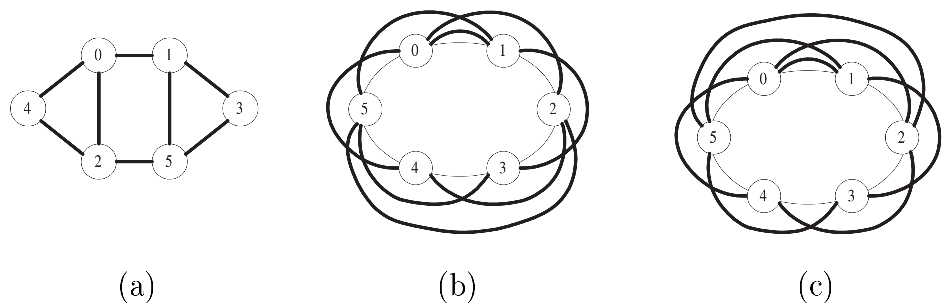

Fig. 1. (a) A logical topology, (b) a survivable embedding, and (c) a non-survivable embedding.

Consider another example shown in Figure 2, in which the logical topology in (a) has an edge-cut of size two $\{e_1, e_2\}$ (these two logical edges correspond to the two connection requests (a,b) and (c,d) assuming nodes a, c, b, and d are located in the ring in this sequence). Any route assignment of lightpaths corresponding to logical links (a,b) and (c,d) always share a physical link, and the logical topology becomes disconnected when the shared physical link

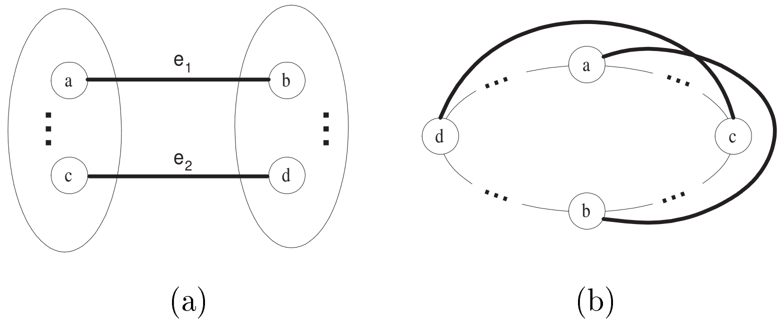

Fig. 2. (a) A logical topology with an edge-cut of size 2, and (b) a non-survivable embedding of the cut-edge.

fails. The above two examples lead us to the formulation of the following optimization problem.

- *Survivable Logical Topology Design Problem (SLTDP):* Given a physical WDM ring network with n nodes (where the node set is denoted by $V = \{0, \cdots, n-1\}$) and a set of connection requests $R = \{(i,j) \mid i,j \in V\}$ (where for any two connections $(i,j),(i',j') \in R$, $(i,j) \neq (i',j')$), find a route for each lightpath $(i,j) \in R$ such that the logical topology remains connected, if possible, after the failure of any single physical link. Otherwise, determine and embed the minimum number of additional lightpaths to make the logical topology survivable.

Note that when the given logical topology is not connected or has a cut-edge, there is no route assignment that can make the logical topology survivable without having additional lightpaths. Some logical topologies that have edge-cuts of size two, however, have route assignments that make the logical topology survivable, while some do not as shown in Figure 2. In this paper, we choose to characterize the survivability of a logical topology based on its edge-connectivity. We first observe that if the logical topology is completely connected, then it can always be made survivable by establishing the single-hop lightpaths $\{(i,(i+1)\%n) \mid 0 \leq i \leq n-1\}$. The rest of the lightpaths may be embedded in any arbitrary manner. An interesting question then is to determine in polynomial time whether there exists an embedding of lightpaths when the given logical topology is k-edge connected for an arbitrary k. In our earlier paper [4], we showed that if the logical topology G is 3-edge connected or 4-edge connected, there exist examples of G that for which survivable embeddings do not exist. Those examples are reproduced in the next section, where we also consider a special routing, namely the shortest path routing, and present some results on the problem complexity.

3 Problem Complexity

In this section, we first address the question: given an arbitrary integer k, is there any k-edge connected logical topology for which no lightpath embedding can make the logical topology survivable? We then consider a special routing algorithm, namely shortest path routing and show that if the minimum degree of the logical topology is at least $\lceil 2n/3 \rceil$ (i.e., each node is connected to at least $\lceil 2n/3 \rceil$ other nodes), shortest path routing of the lightpaths always makes the logical topology survivable.

3.1 k-Edge Connected Logical Topologies

As discussed earlier, no lightpath embedding can achieve survivability if the logical topology is 1-edge connected, and some 2-edge-connected logical topologies cannot achieve survivability. In the following, we show examples of 3-edge and 4-edge connected logical topologies that cannot achieve survivability.

Figure 3 shows a 4-edge connected logical topology and a ring network with the corresponding nodes. First note that the lightpaths corresponding to logical edges (a_1, a_2) and (a_3, a_4) must share at least one physical link regardless of their route assignments. The ring network in Figure 3 shows the case when lightpaths are established from a_1 to a_2 and from a_3 to a_4 both in the counter-clockwise direction, and they share physical links between a_1 and a_4. Now, consider the lightpath assignments for logical links (e_1, e_2) and (e_3, e_4). Four possible assignments are shown in Figure 4, in which any two lightpaths corresponding to (e_1, e_2) and (e_3, e_4) share at least one physical link, and the shared physical links lie between a_1 and a_4. Therefore, the failure of any physical link shared by four lightpaths corresponding to (a_1, a_2), (a_3, a_4), (e_1, e_2), and (e_3, e_4) will break all these logical links, hence making the logical topology disconnected. (Note that the edge-cut C_1 corresponds to the four logical links.) Similar arguments can be applied to show that the logical topology will become disconnected for any of the remaining three possible route assignments for (a_1, a_2) and (a_3, a_4) by deleting C_1, C_2, and C_3 when the shared physical link fails.

Figure 5 shows a 3-edge-connected logical topology and a ring network with the corresponding nodes. One can verify by a similar argument used in the case for the 4-edge connected logical topology that no lightpath embedding can make the shown logical topology survivable.

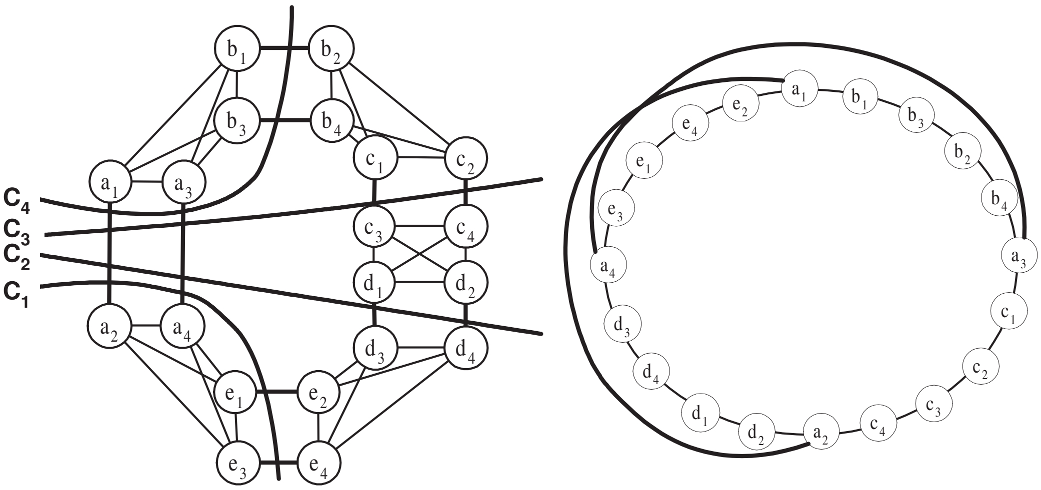

Fig. 3. A logical topology (left), and the nodes on the physical ring (right).

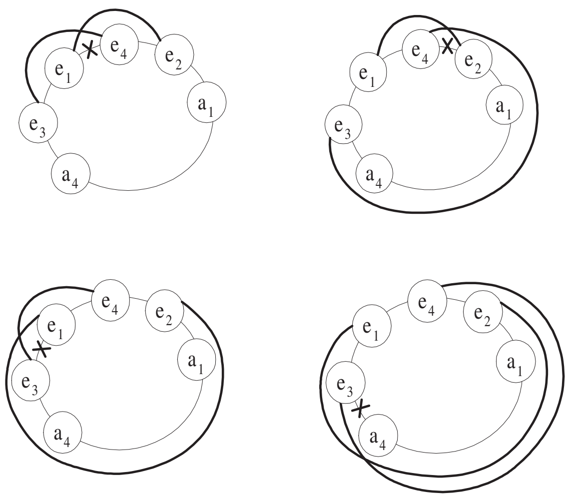

Fig. 4. All possible embeddings of logical links (e_1, e_2) and (e_3, e_4).

3.2 Shortest Path Routing

Theorem 3.1 *Given an arbitrary set of connection requests R where every node has to be connected to at least $\lceil 2n/3 \rceil$ other nodes, the logical topology is survivable with shortest path routing of lightpaths.*

Proof: In the following, we assume that n is a multiple of six and prove the theorem. When n is not a multiple of six, similar arguments can be applied to prove the theorem, and we omit the details in this paper.

Let $V = \{0, \cdots, n-1\}$ be the set of nodes in the ring network, and assume the lightpaths are routed along the shortest path on the ring. As shown in Figure 6,

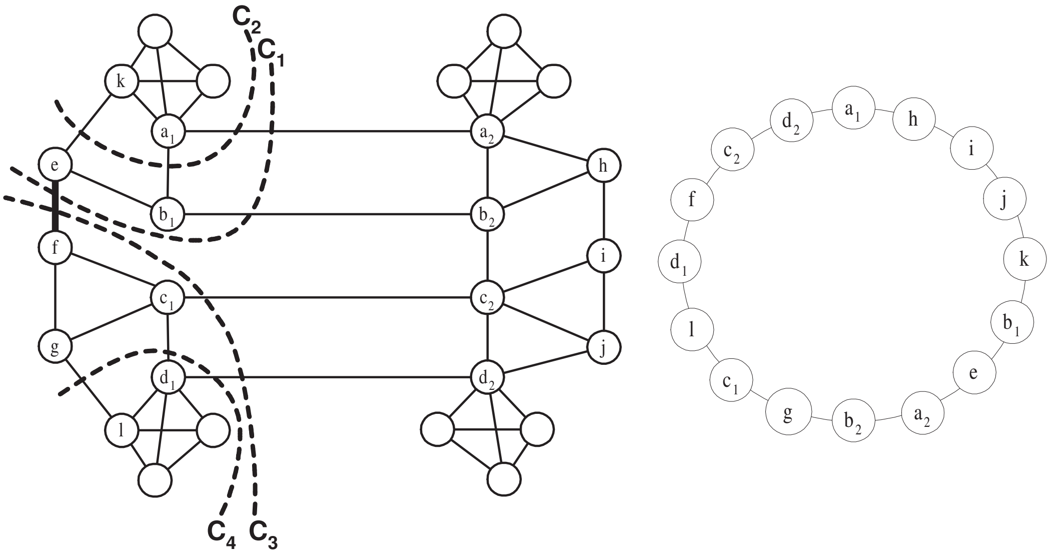

Fig. 5. A non-survivable 3-edge connected logical topology (left), and the nodes on the physical ring (right).

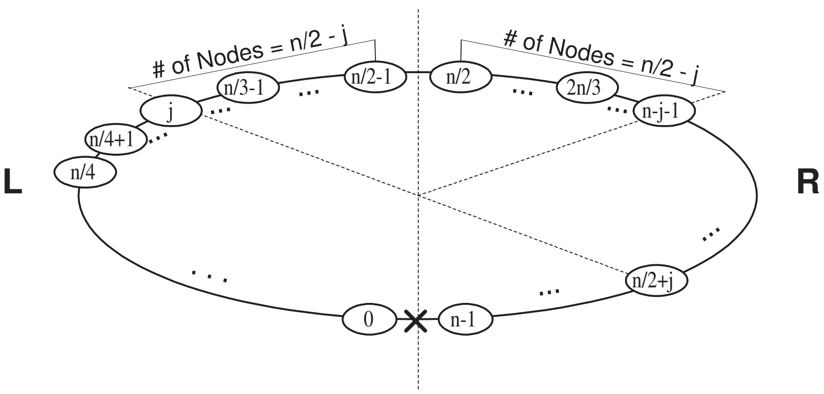

Fig. 6. Figure for proof of *Theorem 3.1*.

suppose $(0, n-1)$ is the failed physical link. Define $L = \{0, 1, \cdots, \frac{n}{2} - 1\}$ and $R = \{\frac{n}{2}, \frac{n}{2} + 1, \cdots, n-1\}$. Let s_i be the number of lightpaths (i.e., logical links) terminated at node i that do not use link $(0, n-1)$. We then observe the following:

$$s_i \geq \begin{cases} \frac{n}{6} + i & \text{if } i \in L \\ \frac{n}{6} - i + n - 1 & \text{if } i \in R. \end{cases}$$

Suppose the logical topology becomes disconnected after the failure of link $(0, n-1)$, and let C denote the smallest component (i.e., a component with the minimum number of nodes) connected via logical links after the failure of physical link $(0, n-1)$. Clearly, $|C| \leq n/2$. Note that $L \cap C \neq \emptyset$ or $R \cap C \neq \emptyset$, and assume that, without loss of generality, $L \cap C \neq \emptyset$. Let t denote the largest index in $L \cap C$. Let t' denote the smallest index in $R \cap C$ if $R \cap C \neq \emptyset$, and t' is not defined if $R \cap C = \emptyset$. Assume without loss of generality that the distance from node 0 to t is no less than the distance from node $n-1$ to t' (i.e., $t \geq n - t' - 1$). In the following, we consider four cases for the value of t and show the contradiction to the existence of C in each case.

Case 1: $t \leq \frac{n}{4} - 1$.
In this case, the distance in the clockwise direction from t to t' is larger than $n/2$, and $C \subset L$ (i.e., $R \cap C = \emptyset$.) Since t is the largest index in $L \cap C$, it implies that $|C| \leq t+1$, hence, node t can only be connected to at most t other nodes in C. However, by the definition of s_t, node t must be connected to at least $\frac{n}{6} + t$ nodes all in C, a contradiction.

Case 2: $t \geq \frac{n}{3}$.
By the definition of s_t, node t must be connected to at least $\frac{n}{6} + t$ (i.e., at least $\frac{n}{2}$) nodes in C. However, $|C| \leq n/2$, a contradiction.

Case 3: $\frac{n}{4} \leq t \leq \frac{n}{3} - 2$.
For any node $i \in C \cap R$, we then have $n - t - 1 \leq i \leq \frac{n}{2} + t$. (See Figure 6 for clarification when $j = t$.) This then implies that $|C \cap R| \leq 2t - \frac{n}{2} + 2$, which is $|C \cap R| \leq \frac{n}{6} - 2$. By the definition of s_t, node t must be connected to at least $t + \frac{n}{6}$ nodes in C, where node t can be connected to at most $\frac{n}{6} - 2$ nodes in $C \cap R$. Therefore, node t must be connected to at least $t+2$ nodes in $C \cap L$, and this is impossible since $|C \cap L| \leq t+1$.

Case 4: $t = \frac{n}{3} - 1$.
Since $n - t - 1 \leq i \leq \frac{n}{2} + t$ for any node $i \in C \cap R$, $|C \cap R| \leq \frac{n}{6}$. Again, by the definition of s_t, node t must be connected to at least $t + \frac{n}{6}$ nodes in C. Therefore, node t (i.e., node $\frac{n}{3} - 1$) must be connected to all of the $\frac{n}{6}$ nodes in $C \cap R$ and all of nodes in $\{0, 1, \cdots, \frac{n}{3} - 2\}$. Consequently, we have $C = \{0, 1, \cdots, \frac{n}{3} - 1\} \cup \{\frac{2n}{3}, \frac{2n}{3} + 1, \cdots, \frac{5n}{6} - 1\}$ (i.e., $|C| = \frac{n}{2}$). Clearly, any node in C can only be connected to nodes in C, and consider node $j = \frac{2n}{3}$ which is in $C \cap R$. Again, by the definition of s_j, node j must be connected to at least $\frac{n}{2} - 1$ nodes implying that node j must be connected to all nodes (except j itself) in C. However, this is impossible since node 0, for example, cannot be connected to node j without using link $(0, n-1)$ since the shortest route must be applied.

This completes the proof of the theorem. ∎

The result in Theorem 3.1 shows that the shortest path routing guarantees the logical topology's survivability if its minimum degree (i.e., the minimum number of nodes that each node is connected to) is at least $\lceil 2n/3 \rceil$. On the other hand, if the minimum degree is less than $\lfloor n/2 \rfloor$, the shortest path routing does not always guarantee survivability as can be seen in the following example.

Let $V = \{0, \cdots, n-1\}$ be the set of nodes in the ring network with n being an even number. Define $V_e = \{2p \mid 0 \leq p \leq n/2-1\}$ and $V_o = \{2p+1 \mid 0 \leq p \leq n/2-1\}$. Consider a connection request set $R = R_e \cup R_o \cup \{(0, n-1)\}$, where $R_e = \{(i,j) \mid i,j \in V_e\}$ and $R_o = \{(i,j) \mid i,j \in V_o\}$, and assume that each lightpath is established using the shortest path routing. From this example, we note the following: (i) the logical topology corresponding to R is connected, (ii) all nodes in V_o (and V_e, respectively) are connected to each other; hence, the minimum degree is at least $n/2-1$, and (iii) when physical link $(0, n-1)$ fails, the nodes in V_e are completely disconnected from the nodes in V_o. The discussion for this example is illustrated in Figure 7.

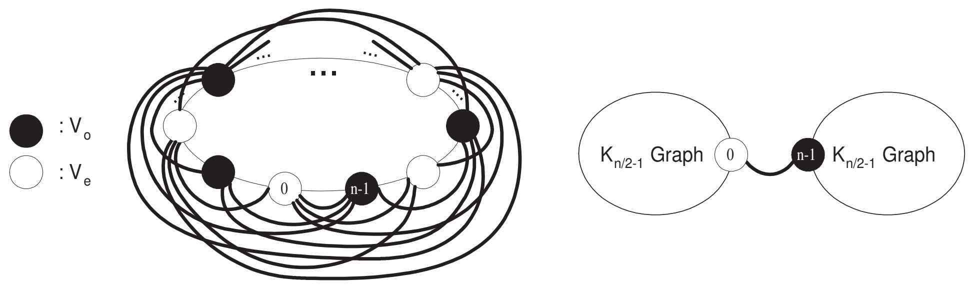

Fig. 7. An example of a simple logical topology with minimum degree $n/2-1$, which is not survivable under shortest path lightpath routing.

From this example and the result of Theorem 3.1, we observe that the shortest path routing can guarantee the logical topology's survivability only when each node has rich connectivity. But the survivability is not guaranteed when the minimum degree is less than or equal to $n/2-1$ and when the shortest path routing is applied. In fact, the result in this example can be extended to a $(n/2-1)$-edge connected logical topology by adding $n/2-2$ more lightpaths between any two nodes $(i,\ j)$ for $i \in V_o$ and $j \in V_e$ using physical link $(0, n-1)$ if it is the shortest path between i and j. Clearly, this is always possible for sufficiently large n. Note that the failure of physical link $(0, n-1)$ would still disconnect the logical topology. In the next section, we will look into the shortest path routing more closely and show some interesting numerical results.

4 Numerical Results

In this section, we present some numerical results obtained by a heuristic algorithm developed based on shortest path routing. Our heuristic algorithm is outlined in the following.

Given a set of connection requests R and a ring network with n nodes, each lightpath is established using shortest path routing. We then compute the number of connected components of the logical topology after the failure of each physical link. If the logical topology remains connected after the failure of any physical link, then we do not need to add any lightpath. Otherwise, let $(m, m+1)$ be the link whose failure disconnects the logical topology and creates the largest number of components. Let $C_1, \cdots, C_{k_m}$ be the k_m components that result from the failure of $(m, m+1)$. We then choose arbitrary nodes $u \in C_{k_m}$ and $v \in C_{k_m-1}$. If no lightpath exists between u and v, we establish a lightpath between u and v without using link $(m, m+1)$. (Note that this newly established lightpath may not be the shortest path between the two nodes.) If a lightpath between u and v already exists, we choose another node w such that not both lightpaths between u and w and between v and w exist, and establish one or two lightpath(s) to set up lightpaths both between u and w and between v and w, both without using link $(m, m+1)$. Upon completion of this process, the number of components of this new logical topology after the failure of $(m, m+1)$ is now decreased at least by one. We recompute the number of components by considering the failure of each link, and repeat the above procedure until the resulting logical topology becomes survivable.

We report simulation results for three different ring sizes: $n = 100$, 200 and 300. For each network size, the connection request set R is created using the uniform distribution of probability p that a logical link exists between pairs of nodes. For each p and n, two sets of 1000 logical topologies are generated. The first set is obtained by generating 1000 random logical topologies (i.e., 1000 random connection request sets) where some logical topologies may not be connected or may be only 1-edge connected. The second set is obtained by generating 1000 random logical topologies which are all 2-edge connected. For each p, n, and each set of 1000 logical topologies, our heuristic algorithm is applied, and the average number of additional logical links are computed. The results are shown in Figures 8-10.

Under the same link probability, an arbitrary logical topology requires a larger average number of additional lightpaths than 2-edge connected logical topology because an arbitrary logical topology itself can initially be divided into more than one connected components.

We also notice that, as the number of nodes in a logical topology increases, the additional number of lightpaths required to achieve survivability decreases under the same link probability. For large networks, very few additional lightpaths are required under shortest path routing even with small link probabilities.

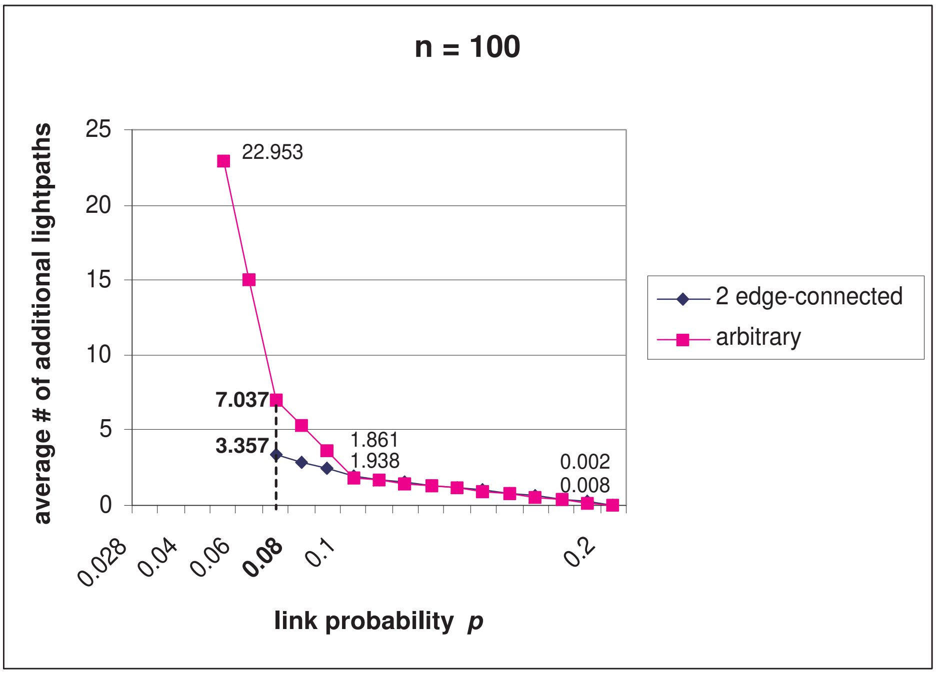

Fig. 8.

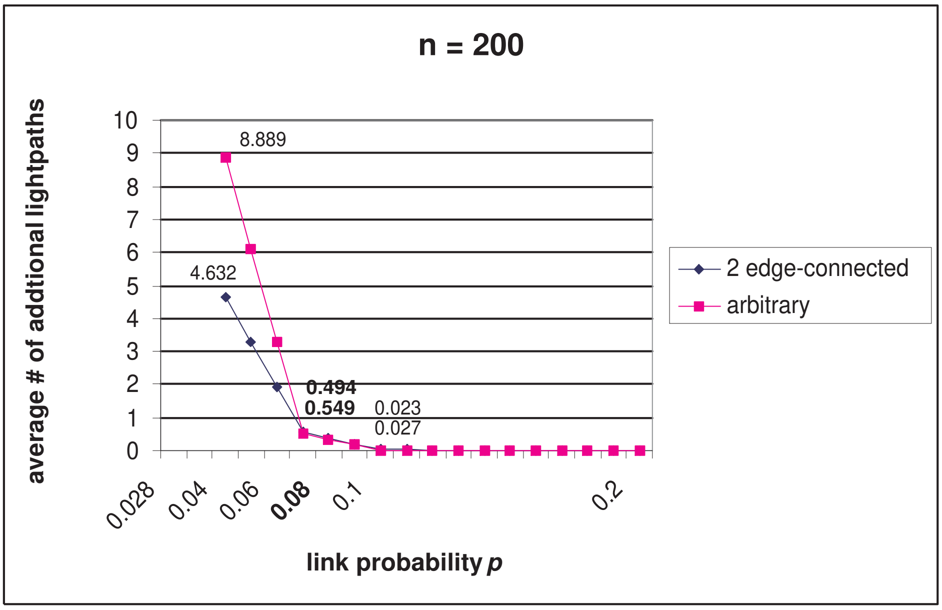

Fig. 9.

5 Concluding Remarks

In this paper, we addressed the issue of embedding survivable logical topologies in WDM optical rings. Specifically, we considered the problem of maintaining

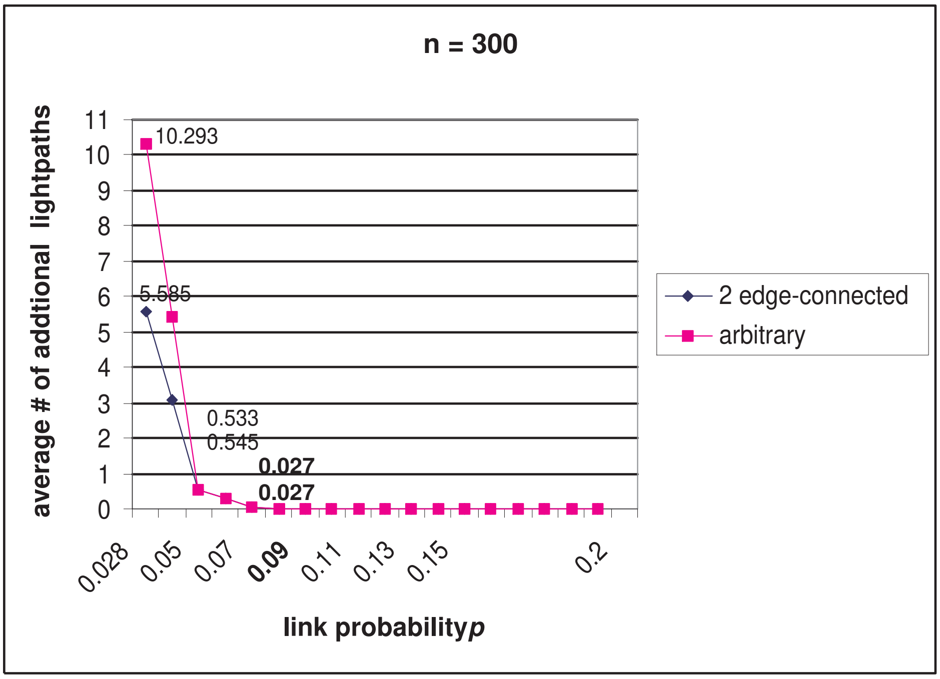

Fig. 10.

the connectivity of a logical topology even when a single physical link fails, using minimum extra lightpaths.

We first presented some examples that show the complexity of the problem. In particular, we showed that for 2-edge, 3-edge, and 4-edge-connected logical topologies, there exist logical topologies which are not survivable under a single link failure, no matter how the topology is embedded over the physical topology, i.e., no matter how the lightpaths are routed. We then showed that if shortest-path routing is used to embed a logical topology, then the logical topology may not be survivable even if it is $(n/2-1)$-edge connected, where n is the number of nodes in the ring. Correspondingly, we showed that if the logical topology is rich enough so that the minimum degree of any node is at least $\lceil 2n/3 \rceil$, then shortest-path routing of the lightpaths guarantees the survivability of the logical topology. Finally, we presented a simple heuristic to solve the survivable logical topology embedding problem based on shortest-path routing and presented some numerical results. These results indicate that the heuristic is almost optimal.

Future work may concentrate on sharpening the bounds presented in this paper. The complexity of the problem in other physical topologies is another possible topic for future investigation.

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