

Section 5.3: Evaluating Definite Integrals

Practice HW from Stewart Textbook (not to hand in)
p. 374 # 1-27 odd, 31-43 odd

Definite Integral

The definite integral is an integral of the form

$$\int_a^b f(x) dx .$$

This integral is read as the integral from a to b of $f(x) dx$. The numbers a and b are said to be the limits of integration. For our problems, $a < b$.

Definite Integrals are evaluated using The Fundamental Theorem of Calculus.

Fundamental Theorem of Calculus

Let $f(x)$ be a continuous function for $a \leq x \leq b$ and $F(x)$ be an antiderivative of $f(x)$. Then

$$\int_a^b f(x) dx = [F(x)]_a^b = F(b) - F(a)$$

Example 1: Evaluate $\int_1^2 2x dx$.

Solution:



Example 2: Evaluate $\int_1^3 (3x^2 + x - 2) dx$.

Solution:



Additional Integration Formulas

1. $\int e^x dx =$

$$\int e^{kx} dx =$$

2. $\int a^x dx =$

3. $\int \frac{1}{\sqrt{1-x^2}} dx =$

4. $\int \frac{1}{1+x^2} dx =$

Example 3: Evaluate $\int_0^{\frac{1}{2}} (e^{2x} - \frac{1}{\sqrt{1-x^2}}) dx$.

Solution:

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Example 4: Evaluate $\int_1^3 \frac{4+u^2}{u^3} du$.

Solution:

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Example 5: Evaluate $\int_1^3 (x^3 - 1)^2 dx$.

Solution:



Recall that if $f(x) \geq 0$ for $a \leq x \leq b$. Then

$$\text{Definite Integral: } \int_a^b f(x) dx = \begin{array}{l} \text{Area Between } f(x) \text{ and the} \\ x \text{ axis for } a \leq x \leq b \end{array}$$

Example 6: Find the area under the graph of $y = x^2 + 1$ on $[0, 2]$.

Solution:



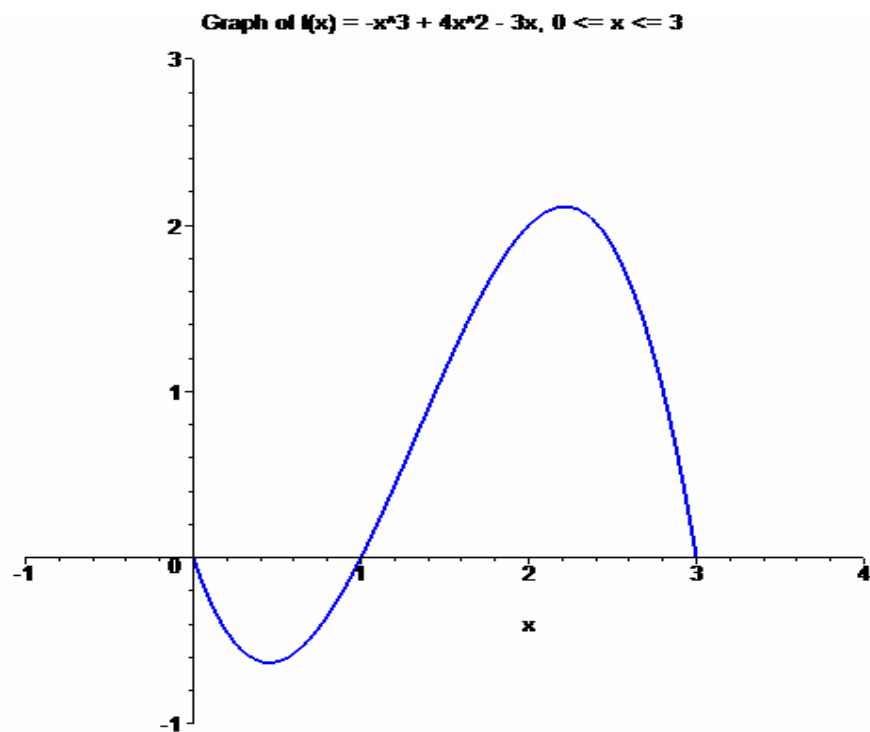
Example 7: Evaluate $\int_{-1}^2 e^{-x^2} dx$.

Solution:



Note: For a function $f(x)$ that is both positive and negative over an interval, the total area is the area enclosed by the negative part of the curve minus the negative part of the curve.

Example 8: Consider the function $f(x) = -x^3 + 4x^2 - 3x$ over the interval $0 \leq x \leq 3$. The graph of the function over this interval is given by



Find the total area enclosed between the function and the x axis.

Solution:

