Section 5.5: The Substitution Rule

Integration by Substitution

Example 1: Integrate $\int 2x(x^2 + 5)^6 \, dx$

Solution:

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Fact: Integration by substitution undoes the chain rule of differentiation. Its goal is the make a variable transformation to transform the integral into a form where we can integrate it using the basic integration formulas.

To recognize substitution, look for a term (to set $u = $ to) whose derivative is similar to another term in the integral.

Example 2: Integrate $\int \dfrac{(\ln x)^3}{x}\,dx$

Solution:

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Example 3: Integrate $\int x^3 \sqrt{x^4 + 2}\,dx$

Solution:

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Example 4: Integrate $\int \dfrac{1}{(5x-2)^4}\, dx$

Solution:

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Example 5: Integrate $\int \sin 3x\ dx$

Solution:

∎

Example 6: Integrate $\int \tan^3 x \, \sec^2 x \, dx$

Solution:

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Example 7: Integrate $\int x\sqrt{x+1} \, dx$

Solution:

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Definite Integrals Using Substitution

<u>Two Methods</u>
1. Integrate the function by substitution, and write the answer back in terms of the original variable. Then evaluate the limits of integration.
2. Integrate the function by substitution and also write the limits of integration in terms of substitution variable (normally u), evaluating the integral in terms of the substitution variable.

Example 8: Integrate $\displaystyle\int_0^1 (2x+1)^5 \, dx$

Solution: