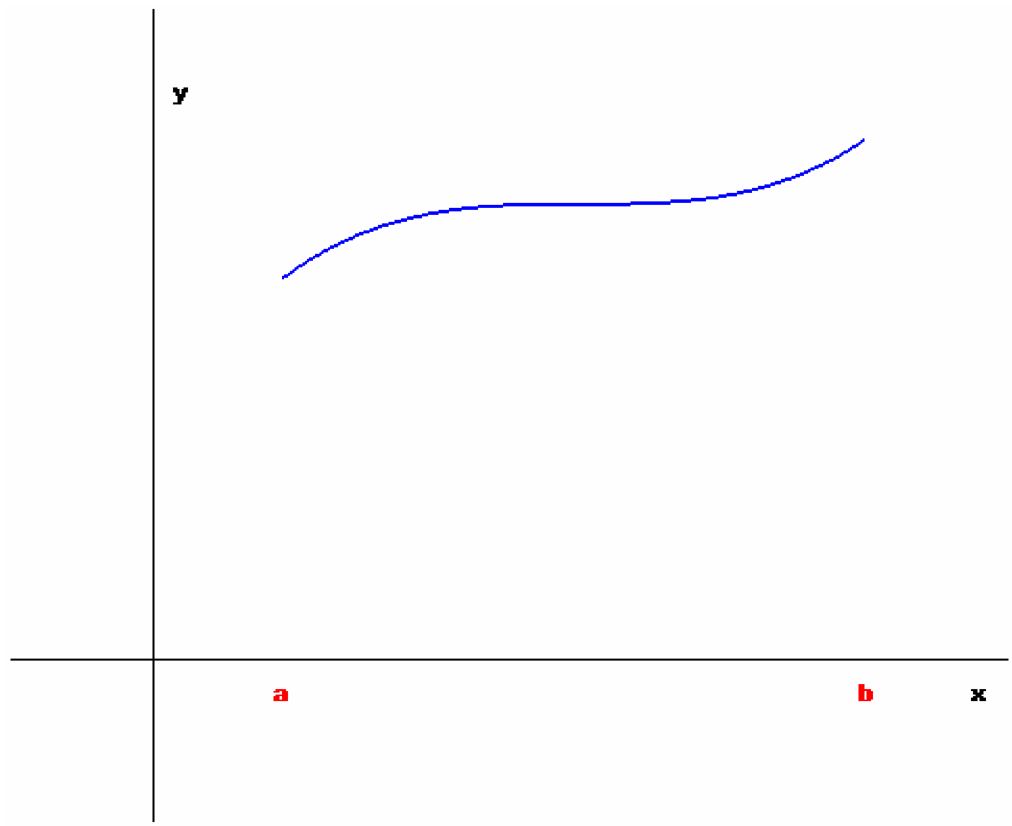
Section 6.1: More About Areas

Practice HW from Stewart Textbook (not to hand in)
p. 446 # 1-15 odd

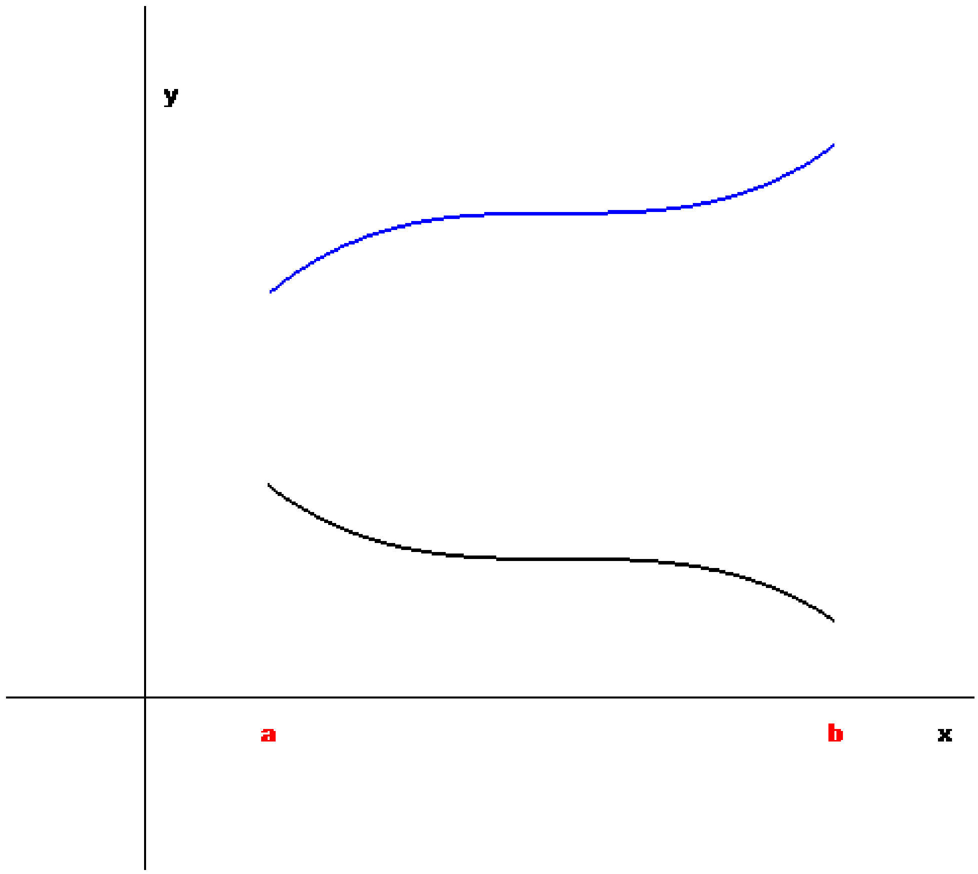
Areas Between Curves

Suppose we are given a continuous function $y = f(x)$ where $f(x) \geq 0$ over the interval $[a, b]$.



Recall: $$\begin{array}{c}\text{Area under } f \\ \text{on } [a,b]\end{array} = \int_a^b f(x)\,dx$$

Suppose we are given two functions f and g and we want to find the area bounded between the functions from $x = a$ and $x = b$.



Area between f and g = Area under f – Area under g

$$= \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

$$= \int_a^b (f(x) - g(x))\,dx$$

Area Between Two Curves

Given f and g are continuous functions on $[a, b]$ and $g(x) \le f(x)$ for all x in $[a, b]$. Then

$$\begin{array}{c}\text{Area between } f \text{ and } g \\ \text{on the interval} \\ [a,b]\end{array} = A = \int_a^b (f(x) - g(x))\,dx$$

Example 1: Determine the area bounded between the graphs of $f(x) = x^2 + 2$ and $g(x) = 1$ between x = -1 and x = 2.

Solution:

$\blacksquare$

Note: When asked to find the area bound by two graphs, to find the interval we many times have to find the points of intersection.

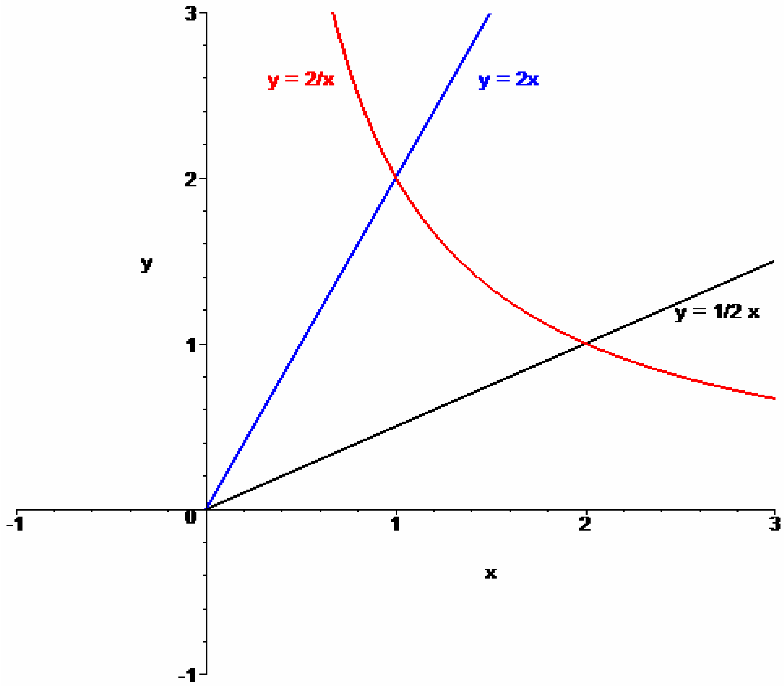
Example 2: Sketch the graph and find the area enclosed by the graphs of $y = -x^2 + 4x + 2$ and $y = x + 2$.

Solution:

Note: The upper and lower curves can switch over an interval. In this case, we must divide the interval and switch the upper and lower curves when integrating to find the area.

Example 3: Sketch the graph and find the area enclosed by the graphs of $y = \frac{2}{x}$, $y = 2x$, and $y = \frac{1}{2}x$ where $x > 0$.

Solution: The following represents the graphs of the functions $y = \frac{2}{x}$, $y = 2x$, and $y = \frac{1}{2}x$ for $x > 0$.



As the graphs indicate, from $x = 0$ to $x = 1$, $y = 2x$ is the upper curve and $y = \frac{1}{2}x$ is the lower curve. From $x = 1$ to $x = 2$, the upper curve becomes $y = \frac{2}{x}$ and the lower curve is $y = \frac{1}{2}x$. Hence,

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$$\text{Total Area} = \int_0^1 (2x - \frac{1}{2}x)\,dx \;\; + \;\; \int_1^2 (\frac{2}{x} - \frac{1}{2}x)\,dx$$

$$= \int_0^1 (\frac{4}{2}x - \frac{1}{2}x)\,dx \;\; + \;\; \int_1^2 (\frac{2}{x} - \frac{1}{2}x)\,dx$$

$$= \int_0^1 \frac{3}{2}x \;\, dx \;\; + \;\; \int_1^2 (\frac{2}{x} - \frac{1}{2}x)\,dx$$

$$= \left.\left(\frac{3}{2}\frac{x^2}{2}\right)\right|_0^1 \;\; + \;\; \left.\left(2\ln|x| - \frac{1}{2}\frac{x^2}{2}\right)\right|_1^2$$

$$= \left.\left(\frac{3}{4}x^2\right)\right|_0^1 \;\; + \;\; \left.\left(2\ln|x| - \frac{1}{4}x^2\right)\right|_1^2$$

$$= \frac{3}{4}(1)^2 - \frac{3}{4}(0)^2 + (2\ln|2| - \frac{1}{4}(2)^2) - (2\ln|1| - \frac{1}{4}(1)^2)$$

$$= \frac{3}{4} - 0 + 2\ln 2 - \frac{4}{4} - 2(0) + \frac{1}{4}$$

$$= \frac{3}{4} + 2\ln 2 - 1 - 0 + \frac{1}{4}$$

$$= \frac{4}{4} + 2\ln 2 - 1$$

$$= 1 + 2\ln 2 - 1 = \boxed{2\ln 2}$$

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Horizontal Area Representation

Involves integrating with respect to *y* to find the area when *x* is written as a function of *y*.

Example 4: Find the area enclosed by $x = y^2 + 1$ and $x = y + 3$.

Solution: