Section 6.3: Arc Length

Practice HW from Stewart Textbook (not to hand in)
p. 465 # 1-13 odd

Parametric Equations

Used to describe the *x* and *y* coordinates of a quantity in terms of a parameter *t*.

Parametric equations have the form

$$x = f(t), \quad y = g(t)$$

Orientation of a Curve: the direction the curve traces out as we plot (*x*, *y*) coordinates for increasing values of *t*.

Example 1: Sketch the curve represented by the equations $x = t^2 - 2t$ and $y = t + 1$.

Solution:

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Note: We can find the rectangular equations relating *x* and *y* by eliminating the parameter *t*.

Example 2: Find the rectangular equation for the parametric equations $x = t^2 - 2t$ and $y = t + 1$.

Solution:

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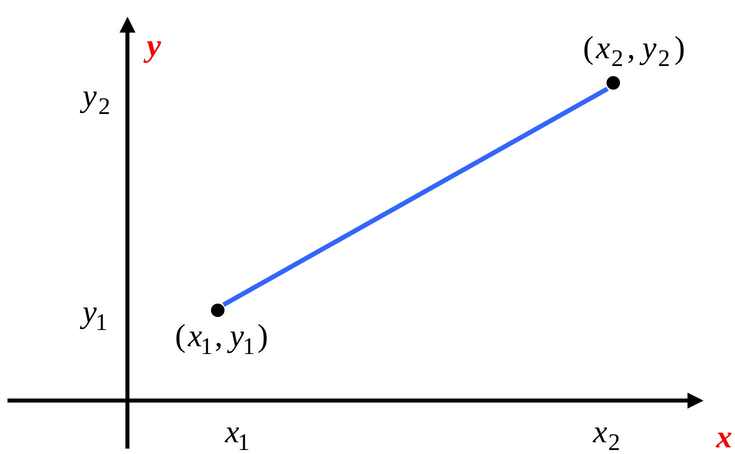
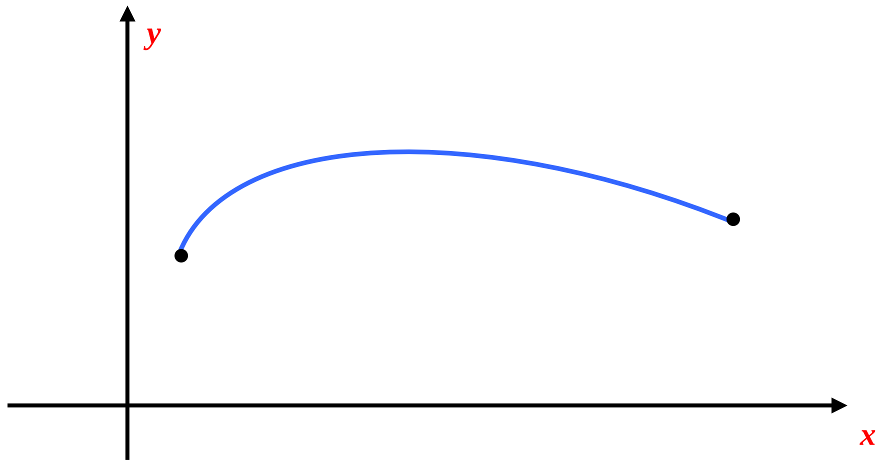
Arc Length

If we are given two points (x_1, y_1) and (x_2, y_2) on a graph.



$$\text{Distance of Line Connecting } (x_1, y_1) \text{ and } (x_2, y_2) = d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Suppose we now want to determine the length L of the segment (or arc) defined over some interval $[a, b]$.



<u>3 Cases to Define Arc Length</u>

1. If the curve is defined parametrically, $x = f(t)$, $y = g(t)$, where $a \le t \le b$, then

$$L = \int_{t=a}^{t=b} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

2. If y is a function of x, $y = f(x)$ for $a \le x \le b$, then

$$L = \int_{x=a}^{x=b} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

3. If x is a function of y, $x = f(y)$ for $a \le y \le b$, then

$$L = \int_{y=a}^{y=b} \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

Example 3: Find the arc length of the curve given by

$$x = e^t + e^{-t}, \ \ y = 5 - 2t, \ \ 0 \le t \le 3$$

Solution:

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Example 4: Find the arc length of the curve given by

$$x = \ln(1 - y^2), \quad 0 \le y \le \frac{1}{2}$$

Solution:

$\blacksquare$