Section 7.1: Modeling with Differential Equations

Practice HW from Stewart Textbook (not to hand in)
p. 503 # 1-7 odd

Differential Equations

Differential Equations are equations that contain an unknown function and one or more of its derivatives. Many mathematical models used to describe real-world problems rely on the use of differential equations (see examples on pp. 501-503).

Most of the differential equations we will study in this chapter involve the first order derivative and are of the form

$$y' = F(x, y)$$

Our goal will be to find a function $y = f(x)$ that satisfies this equation. The following two examples illustrate how this can be done for a basic differential equation and introduce some basic terminology used when describing differential equations.

Example 1: Find the general solution of the differential equation $y' = 3x^2$

Solution:

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The general solution (or family of solutions) has the form $y = f(x) + C$, where C is an arbitrary constant. When a particular value concerning the solution (known as an *initial condition*) of the form $y(x_0) = y_0$ (read as $y = y_0$ when $x = x_0$) is known, a *particular solution*, where a particular value of C is determined, can be found. The next example illustrates this.

Example 2: Find the particular solution of the differential equation $y' = 3x^2$, $y(0) = 1$.

Solution:

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To check whether a given function is a solution of a differential equation, we find the necessary derivatives in the given equation and substitute. If the same quantity can be found on both sides of the equation, then the function is a solution.

Example 3: Determine if the following functions are solutions to the differential equation $y'' - 2y' + 8y = 0$.

a. $y = e^{x}$

b. $y = 2e^{4x}$

Solution:

∎

Example 4: Verify that $y = \sin x \cos x - \cos x$ is a solution of the initial value problem $y' + (\tan x)y = \cos^2 x,\ y(0) = -1,$ on the interval $-\dfrac{\pi}{2} \le x \le \dfrac{\pi}{2}$.

Solution:

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Example 5: For what value of r does the function $y = e^{rx}$ satisfy the differential equation $y' + y = 0$?

Solution: For the function $y = e^{rx}$ to be a solution, we must, after computing the necessary derivative, obtain the same quantities on both sides of the equation after substitution. For $y' + y = 0$, we must compute, using the chain rule applied to the exponential function of base *e*, $y' = re^{rx}$. Hence, we obtain

$$y' + y = 0$$

$$re^{rx} + e^{rx} = 0 \qquad \text{(Substitute for } y' \text{ and } y\text{)}$$

$$e^{rx}(r+1) = 0 \qquad \text{(Factor } e^{rx}\text{)}$$

$$\frac{e^{rx}(r+1)}{e^{rx}} = \frac{0}{e^{rx}} \qquad \text{(Divide both sides by } e^{rx}\text{, which is allowable since } e^{rx} \neq 0 \text{ for all } x\text{)}$$

$$r - 1 = 0 \qquad \text{(Simplify)}$$

$$r = -1 \qquad \text{(Solve)}$$

Thus, $\boxed{r = -1}$ for $y = e^{rx}$ to be a solution.

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