Section 7.3: Separable Equations

Practice HW from Stewart Textbook (not to hand in)
p. 519 # 1-21 odd

In Section 7.2, we looked at graphical and numerical techniques for examining the solutions of differential equations. For differential equations in special forms, there are special approaches for obtaining the exact (analytical) solution. We look at one of these methods in this section.

Finding Solutions Analytically – Separation of Variables

Suppose we are given the initial value problem.

$$y' = \frac{dy}{dx} = F(x, y), \; y(x_0) = y_0$$

and suppose we can write $\frac{dy}{dx}$ as product of a function of *x* and a function of *y*.

$$\frac{dy}{dx} = h(y)g(x)$$

We can then find a solution, $y(x)$, by "separating" the variables.

<u>Steps – Separation of Variables</u>
1. Get all terms involving *y* on one side of the equation and all terms involving *x* on the other side.
2. Integrate both sides.
3. Solve for the solution $y(x)$ (if possible).

<u>Some Useful Facts</u>
1. $\ln k = \log_e k$
2. $e^{\ln u} = u$
3. $|\, y \,|$ means $y = \begin{cases} y & \text{if} \quad y > 0 \\ -y & \text{if} \quad y \le 0 \end{cases}$

Example 1: Solve the differential equation $y' = x^4 y$.

Solution:

$\blacksquare$

Example 2: Solve the differential equation $\frac{dy}{dt} = 2ty^2 + 3t^2y^2$, $y(0) = -1$.

Solution:

$\blacksquare$

Example 3: Solve the differential equation $y' = \dfrac{y\cos t}{y^4 + 2y}$, $y(0) = -1$.

Solution:

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Example 4: Solve the differential equation $\frac{dy}{dx} = \frac{t}{t^2 y - y}$, $y(0) = 2$.

Solution:

$\blacksquare$