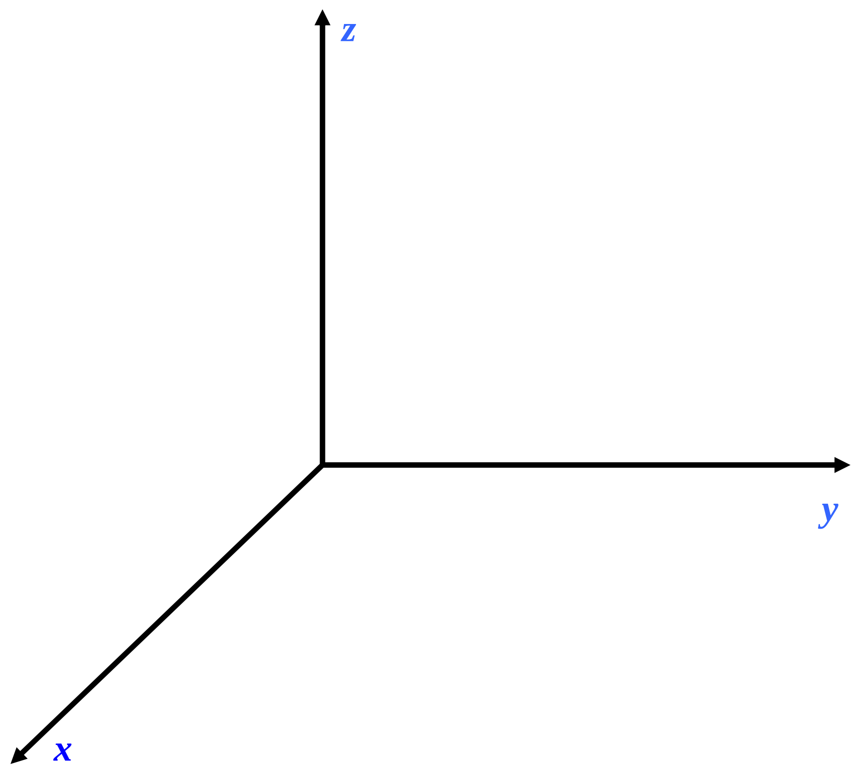
Section 9.1: Three Dimensional Coordinate Systems

Practice HW from Stewart Textbook (not to hand in)
p. 641 # 1, 2, 3, 7, 10, 11, 13, 14, 15b, 16

3-D Coordinate Axes

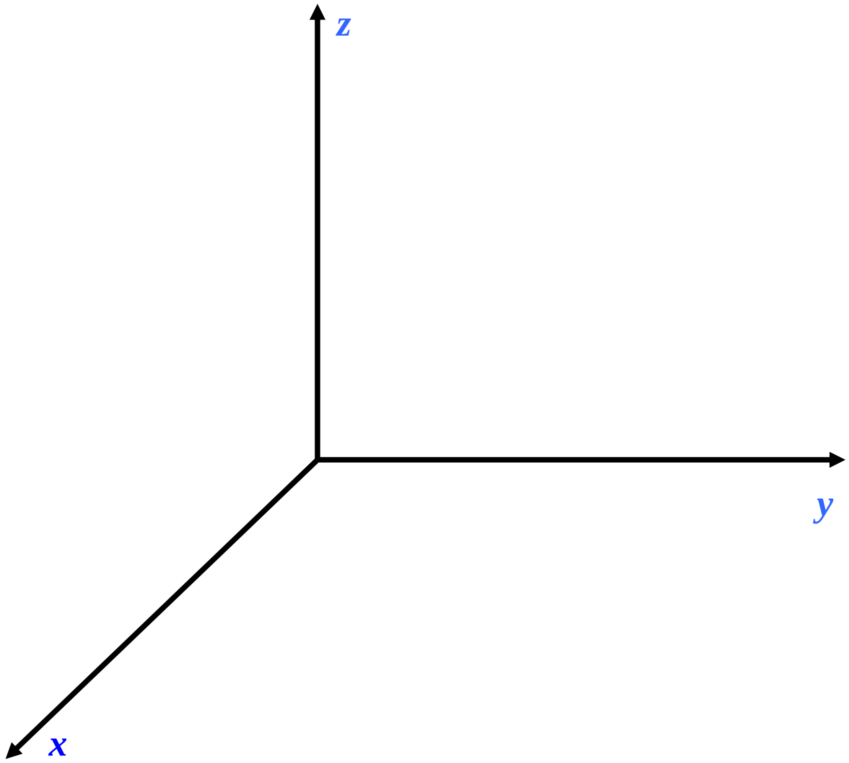
In this chapter, we want to consider the 3 dimensional coordinate axes.



Points are located using ordered triples (x, y, z).

Example 1: Plot the points (1, 1, 1), (2, 5, 0), (-2, 3, 4), (1, 1, -4), and (2, -5, 3).

Solution:



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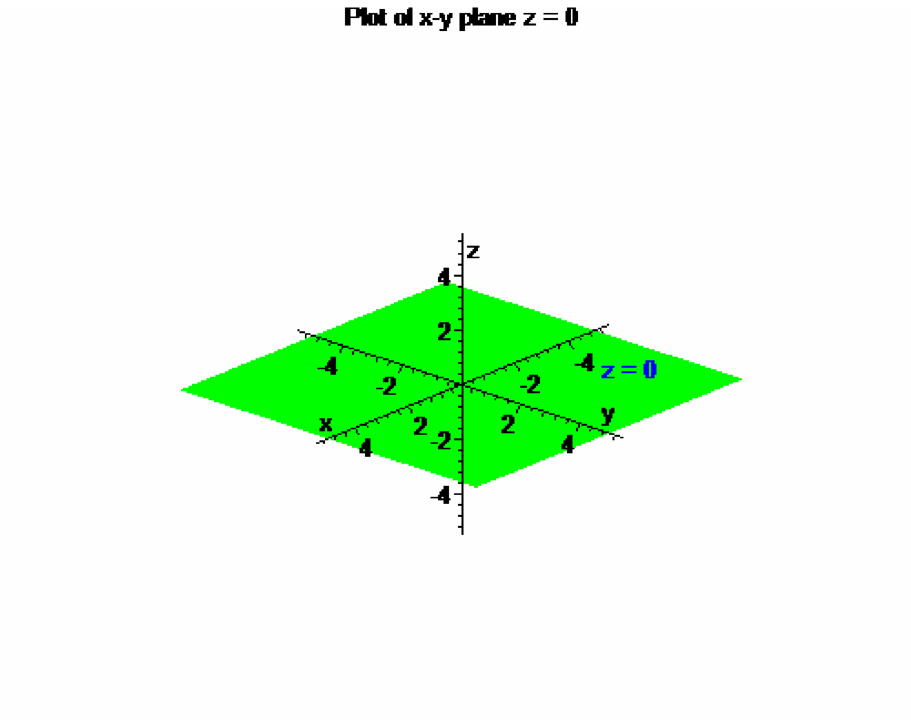
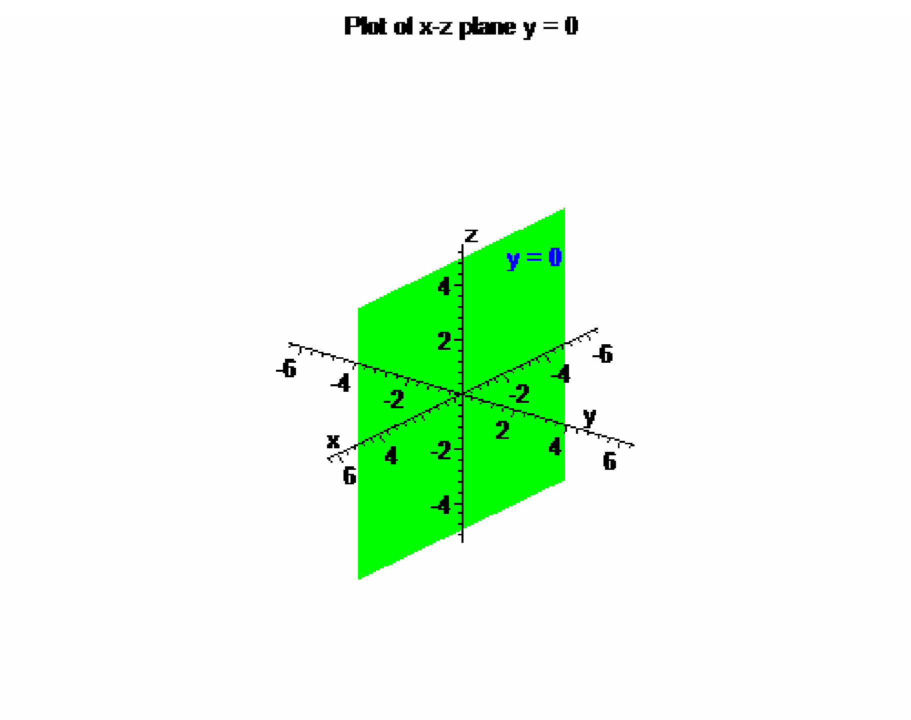
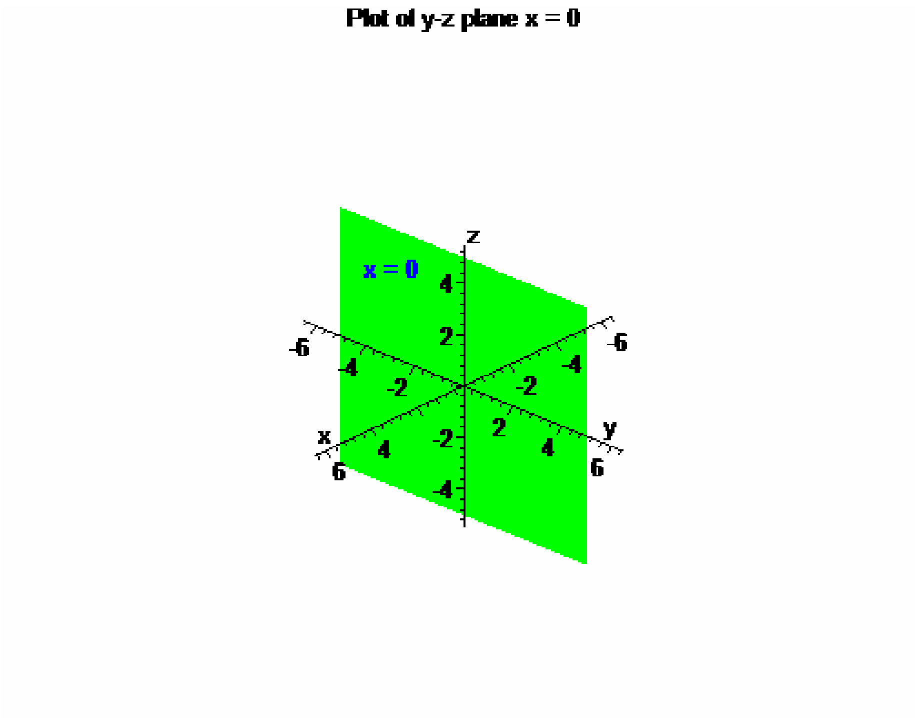
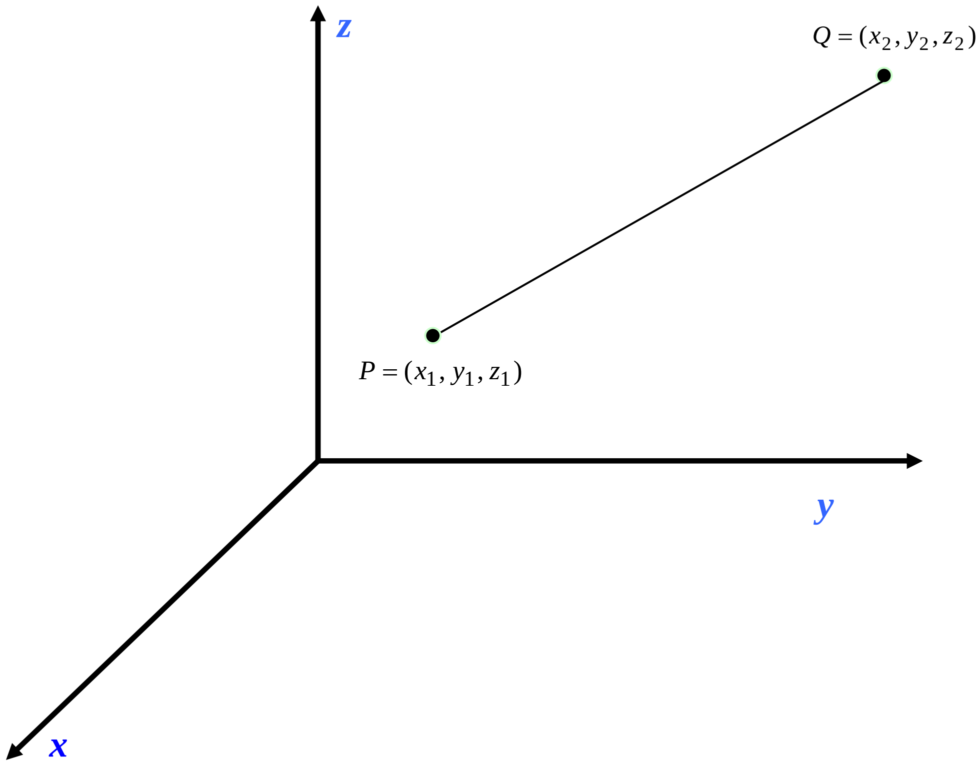
Note: The 3-D coordinate axes divides the coordinate system into 3 distinct planes – the *x*-*y* plane $z = 0$, the *y*-*z* plane $x = 0$, and the *x*-*z* plane $y = 0$. (see next page)


Plot of x-y plane z = 0
z
4
2
-2
-4
-4
-2
x
2
4
-4
-2
2
4
y
z = 0


Plot of x-z plane y = 0
z
y = 0
4
2
-2
-4
-6
-4
-2
x
4
6
-6
-4
-2
2
4
6
y


Plot of y-z plane x = 0
z
x = 0
x
y

Suppose we are now given the two points $P = (x_1, y_1, z_1)$ and $Q = (x_2, y_2, z_2)$ in 3D space.



Then

$$\text{Distance between the points } (x_1, y_1, z_1) \text{ and } (x_2, y_2, z_2) = d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$\text{Midpoint between the points } (x_1, y_1, z_1) \text{ and } (x_2, y_2, z_2) = M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$$

Example 1: Find the distance and midpoint between the points (2, 1, 4) and (6, 5, 2).

Solution:

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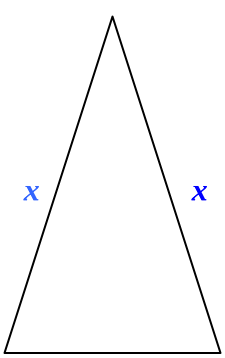
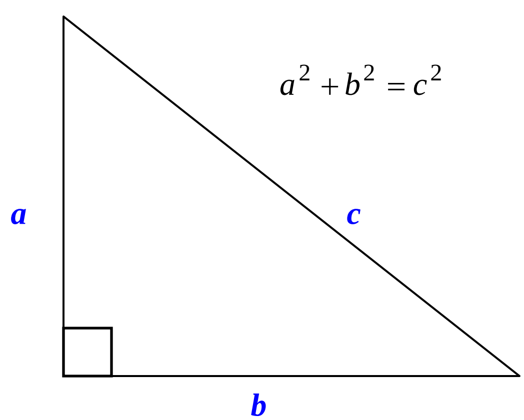
Recall: An isosceles triangle is a triangle where the lengths of two of its sides are equal. A right triangle is a triangle with a 90 degree angle where the sum of squares of the shorter sides equals the square of the hypotenuse.



Icosceles Triangle



Right Triangle

Example 2: Find the lengths of the sides of the triangle PQR if P = (1, -3, -2), Q = (5, -1, 2) and R = (-1, 1, 2). Deterrmine if the resulting triangle is an isosceles or a right triangle.

Solution: We will use the distance formula to find the length of the triangles sides PQ, QR, and RP. We obtain the following results:

Length of side PQ = $\sqrt{(5-1)^2+(-1--3)^2+(2--2)^2} = \sqrt{4^2+2^2+4^2} = \sqrt{36} = 6.$

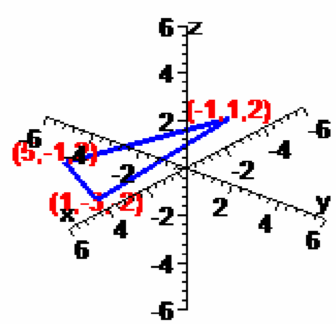
Length of side QR = $\sqrt{(-1-5)^2+(1--1)^2+(2-2)^2} = \sqrt{(-6)^2+2^2+0^2} = \sqrt{40} = \sqrt{4\cdot 10} = 2\sqrt{10}$

Length of side RP = $\sqrt{(1--1)^2+(-3-1)^2+(-2-2)^2} = \sqrt{2^2+(-4)^2+(-4)^2} = \sqrt{36} = 6\,.$

Since sides PQ and RP are equal, the triangle is isosceles. It is not a right triangle since the sum of squares of the shorter sides PQ and RP does not equal the square of the longest side QR. That is,

$$(PQ)^2+(QR)^2 = 6^2+6^2 = 36+36 = 72 \neq (2\cdot\sqrt{10})^2 = 4\cdot 10 = 40 = (QR)^2\,.$$

The following picture graphs the isosceles triangle in 3D space.



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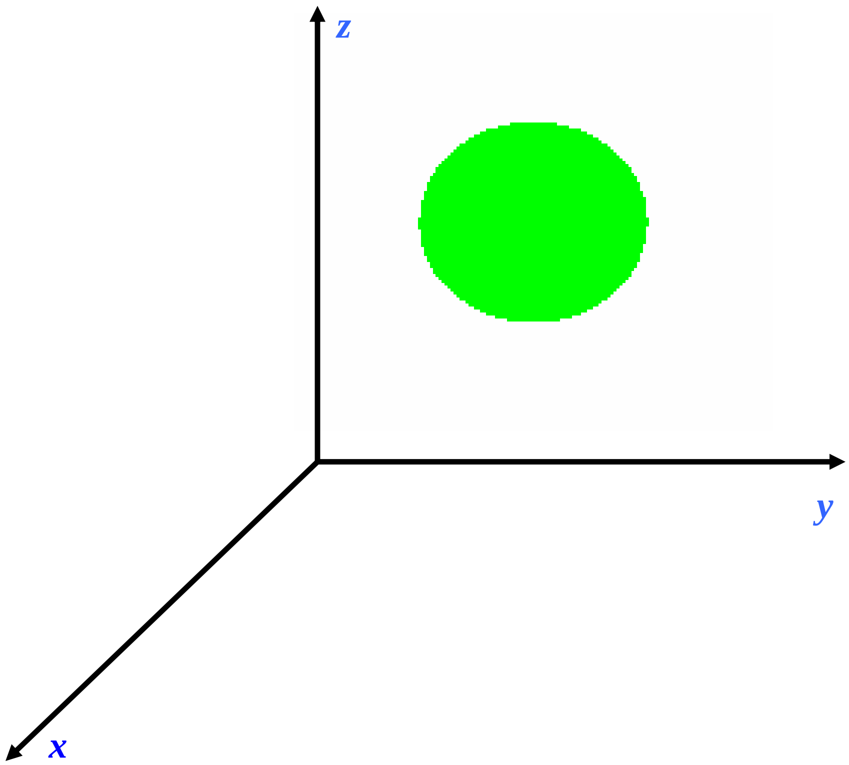
Standard Equation of a Sphere



Standard Equation of a Sphere

The standard equation of a sphere with radius r and center (x_0, y_0, z_0) is given by

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$$

In particular, if the center of the sphere is at the origin, that is, if $(x_0, y_0, z_0) = (0,0,0)$, then the equation becomes

$$x^2 + y^2 + z^2 = r^2$$

Fact: When the standard equation of a sphere is expanded and simplify, we obtain the general equation of a sphere

Standard Equation of a Sphere

$$Ax^2 + Ay^2 + Az^2 + Bx + Cy + Dz + E = 0$$

Example 3: Find the standard and general equation of a sphere that passes through the point (2, 1, 4) and has center (4, 3, 3)

Solution:

Note: To convert the general form of the sphere equation to standard form, we must complete the square.

Example 4: Find the center and radius of the sphere

$$4x^2 + 4y^2 + 4z^2 - 4x - 32y + 8z + 33 = 0$$

Solution:

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