Section 11.1: Functions of Several Variables

Practice HW from Stewart Textbook (not to hand in)
p. 746 # 5-21 odd

Functions of More Than One Variable

So far most of our experience has been working with functions of one variables. Some examples are: $f(x) = x^2$, $g(x) = \ln x$, $h(x) = e^x$. In this section, we want to examine functions and variables of multivariate equations and functions, like $z = f(x, y) = x^2 + 4xy$ or $g(x, y, z) = \dfrac{xe^y}{z^2}$.

Example 1: Given $f(x, y) = x^2 + 4xy$, find $f(-1,2)$.

Solution:

Example 2: Given $g(x, y, z) = \dfrac{xe^y}{z^2}$, find $g(-2,0,3)$.

Solution:

Function of Two Variables – Domain and Range

A function of two variables associates with each ordered pair (x, y) a <u>unique</u> (one and only one) number $z = f(x, y)$.

Informally, the *domain* of a function of two variables is the set of ordered pairs (x, y) where the function $f(x, y)$ is defined. The *range* is the set of z values output by $f(x, y)$.

Example 3: Describe the domain and range of $f(x, y) = \sqrt{4 - x^2 - 4y^2}$

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Solution:

Graphing a Function of Two Variables

Graphically, a function of two variables gives a 3-D surface. It can be useful in some cases to recognize the quadric surface and cylinder graphs studied in Section 9.6 when graphing functions of two variables.

Example 4: Make a rough sketch of the surface $f(x, y) = \sqrt{4 - x^2 - 4y^2}$.

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Solution:

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Level Curves and Contour Maps

Level Curves gives a way of representing the behavior of a 3D surface using 2D curves in a projection like fashion. Formally, the level curves of a function f of two variables are the curves with equations $f(x, y) = k$, where k is a constant in the range of f.

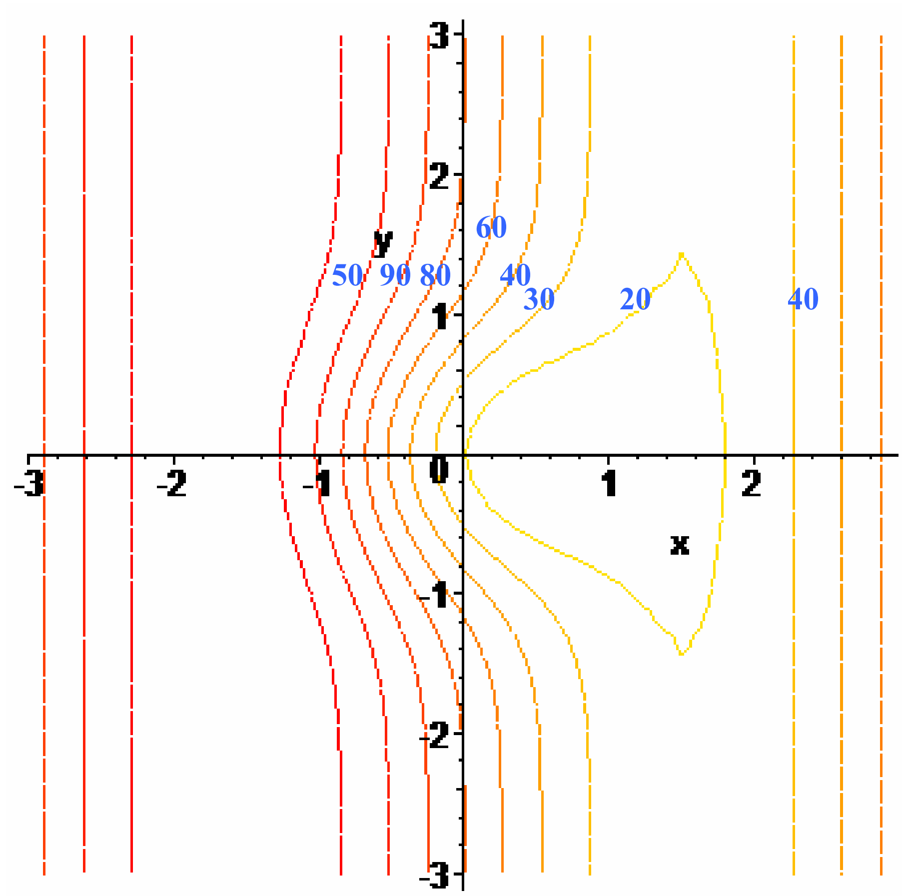
All the level curves sketch together form a *contour* map for a function.

Example 5: Sketch a contour map of the function $f(x, y) = x^2 + y^2$ and show the relationship to the 3D surface it represents.

Solution:

Note: A surface $z = f(x, y)$ is steep when the level curves are close together. It is somewhat flatter when they are further apart (see Figure 5 and 6 on p. 742).

Example 6: Use the following contour map to estimate $f(2, 1)$ and $f(0, 1)$.



Solution:

Example 7: Draw a contour map of the function $f(x, y) = x^2 - y$

Solution:

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Example 8: Draw a contour map of the function $f(x,y)=\sqrt{x^2-y^2}$

Solution: