Section 13.1: Vector Fields

Practice HW from Stewart Textbook (not to hand in)
p. 910 # 1-15 odd

Vector fields are functions that assign vectors to a specified point. We can define a vector field as a function of two or three variables as follows.

Definition: Given functions $P = P(x, y)$ and $Q = Q(x, y)$ of two variables. A vector field in 2D space assigns an order pair (x, y) a two dimensional vector $\mathbf{F}$ given by

$$\mathbf{F}(x, y) = P(x, y)\mathbf{i} + Q(x, y)\mathbf{j}$$

Given functions $P = P(x, y, z)$, $Q = Q(x, y, z)$, $R = R(x, y, z)$ of three variables. A vector field in 3D space assigns an order pair (x, y, z) a three dimensional vector $\mathbf{F}$ given by

$$\mathbf{F}(x, y, z) = P(x, y, z)\mathbf{i} + Q(x, y, z)\mathbf{j} + R(x, y, z)\mathbf{k}$$

Example 1: Sketch several representative vectors in the vector field $\mathbf{F}(x, y) = \mathbf{i} - \mathbf{j}$.

Solution:

Example 2: Sketch several representative vectors in the vector field $\mathbf{F}(x, y) = y\mathbf{i} + \mathbf{j}$.

Solution:

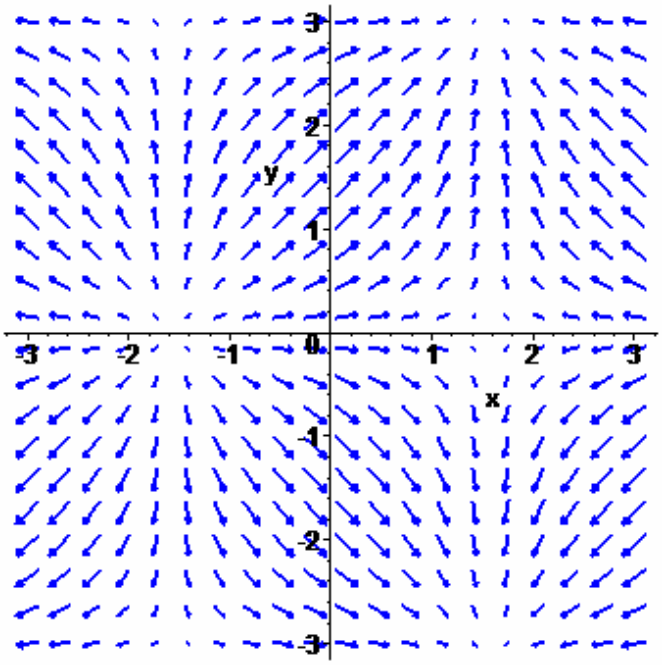
Example 3: Sketch several representative vectors in the vector field $\mathbf{F}(x, y) = \frac{x}{2}\mathbf{i} + y\mathbf{j}$.

Solution:

Example 4: Use Maple to plot the vector field $\mathbf{F}(x, y) = \cos x\,\mathbf{i} + \sin y\,\mathbf{j}$.

Solution: To plot this vector field, we can use the **fieldplot** command, which is part of the **plots** package. The following commands give a plot of this vector field.

```
> with(plots):
> fieldplot([cos(x), sin(y)], x = -3..3, y = -3..3,
thickness = 2, color = blue, arrows = SLIM);
```

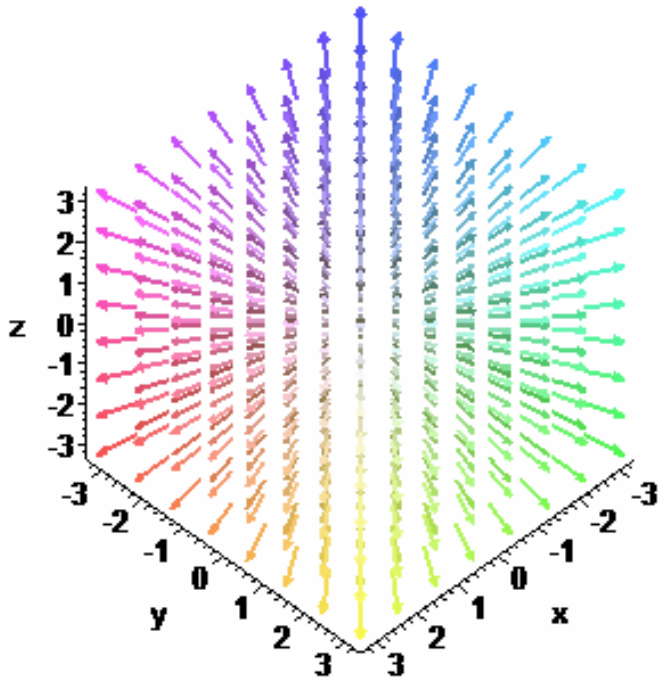


In the previous command, the components of the vector field are placed in brackets, the next two parameters specify the *x* and *y* axis ranges (-3..3) of where the vectors will be plotted, *thickness* = 2 specifies the how thick the arrows are (can be either 1, 2, 3), *color* specifies the color of the arrows (blue in this case), and *arrows* specifies how the arrow style (can be LINE, THIN, SLIM and THICK with THIN being the default).

Example 4: Use Maple to plot the vector field $\mathbf{F}(x, y) = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$.

Solution: To plot this vector field, we can use the **fieldplot3d** command, which is part of the **plots** package. The following commands give a plot of this vector field.

```
> with(plots):
> fieldplot3d([x, y, z], x = -3..3, y = -3..3, z = -3..3,
thickness = 2, arrows = SLIM, axes = FRAMED);
```



In the previous command, the components of the vector field are placed in brackets, the next two parameters specify the *x*, *y* and *z* axis ranges (-3..3) of where the vectors will be plotted, *thickness* = 2 specifies the how thick the arrows are (can be either 1, 2, 3), *arrows* specifies how the arrow style (can be LINE, THIN, SLIM and THICK with THIN being the default), and *axes* specifies that a FRAMED axes with be used (can be BOXED, FRAMED, NORMAL, or NONE).