Parallel Lines, Transversals, and Angles: What's the connection?

UNIT OVERVIEW & PURPOSE:

This unit will review basic geometric vocabulary involving parallel lines, transversals, angles, and the tools used to create and verify the geometric relationships. This unit will investigate the relationship between angles formed when parallel lines are cut by a transversal. This unit will explore four methods for proving lines are parallel when given angle relationships.

I. UNIT AUTHOR:

Pins 16, 14, 8

II. COURSE:

Mathematical Modeling: Capstone Course (the course title might change)

III. CONTENT STRAND:

Geometry

IV. OBJECTIVES:

The student will use the relationships between angles formed by two lines cut by a transversal to a) determine whether two lines are parallel; b) verify the parallelism, using algebraic and coordinate methods as well as deductive proofs; and c) solve real-world problems involving angles formed when parallel lines are cut by a transversal.

V. MATHEMATICS PERFORMANCE EXPECTATION(s):

Students will apply inductive and deductive reasoning skills to make and test parallel lines and the relationship between transversals and angles. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid.

MPE. 32 Use the relationships between angles formed by two lines cut by a transversal to
a) determine whether two lines are parallel;
b) verify the parallelism, using algebraic and coordinate methods as well as deductive proofs; and
c) solve real-world problems involving angles formed when parallel lines are cut by a transversal.

VI. CONTENT:

This unit will review basic geometric vocabulary involving parallel lines, transversals, angles, and the tools used to create and verify the geometric relationships. This unit will investigate the relationship between angles formed when parallel lines are cut by a transversal. This unit will explore four methods for proving lines are parallel when given angle relationships.

VII. REFERENCE/RESOURCE MATERIALS:

Resources/materials used will include: compass, protractor, straightedge, calculator, graph paper, geometric software (Geometer's Sketchpad, Geogebra, or TI-Inspire), journal (composition notebook), teacher handouts, and student handouts.

VIII. PRIMARY ASSESSMENT STRATEGIES:

The student will use the relationships between angles formed by two lines cut by a transversal to
a) determine whether two lines are parallel:
b) verify the parallelism, using algebraic and coordinate methods as well as deductive proofs:
c) solve real-world problems involving angles formed when parallel lines are cut by a transversal:

IX. EVALUATION CRITERIA:
Rubrics and other rating scales are attached to each lesson.

X. INSTRUCTIONAL TIME:
This unit will take roughly 7 – 10 days for a 90 minute block class or no more than 10 – 14 days for a 45 minute class. Time will vary dependent upon the depth the teacher decides to pursue.

Lesson 1 Title

Strand
Geometry

Mathematical Objective(s)
This lesson will review basic geometric vocabulary involving parallel lines, transversals, angles, and the tools used to create and verify the geometric relationships. This lesson will explore the relationship between parallel lines, transversals, and angles.

Mathematics Performance Expectation(s)
Students will apply inductive and deductive reasoning skills to make and test parallel lines and the relationship between transversals and angles. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid.

MPE. 32 Use the relationships between angles formed by two lines cut by a transversal to
a) determine whether two lines are parallel;
b) verify the parallelism, using algebraic and coordinate methods as well as deductive proofs; and
c) solve real-world problems involving angles formed when parallel lines are cut by a transversal.

Related SOL
G.2 The student will use the relationships between angles formed by two lines cut by a transversal to
a) determine whether two lines are parallel;
b) verify the parallelism, using algebraic and coordinate methods as well as deductive proofs; and
c) solve real-world problems involving angles formed when parallel lines are cut by a transversal.

NCTM Standards List all applicable NCTM standards related to each lesson.
- use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations;
- investigate conjectures and solve problems involving two- and three-dimensional objects represented with Cartesian coordinates.
- draw and construct representations of two- dimensional geometric objects using a variety of tools;
- use geometric models to gain insights into, and answer questions in, other areas of mathematics;
- use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture.

Materials/Resources

- Protractors
- Class set of graphing calculators
- Straight edge
- Geogebra or other equivalent software
- Student/Teacher handouts

Assumption of Prior Knowledge

- In high school geometry, students study the angle relationships formed when two lines are cut by a transversal in more depth. Although, students have prior knowledge of this information, it is necessary to review the important vocabulary with the students before proceeding to the application portion of the unit.

- For optimal student success, van Hiele level 2 (Analysis) should have been previously achieved and students should be moving towards level 3 (Abstraction).

- Students will identify congruent angles and supplementary angles immediately in the form of vertical angles and linear pairs. To express congruent angles, the students will use words like "the same measure", "congruent", or "equal angles", etc. To convey supplementary angles, the students will use words like "one is acute and one is obtuse", "add up to 180 degrees", "supplementary", or "linear pair", etc.

- Students may remember that angle relationships exist, but may not remember the specific names. Students may think that just because the alternate interior angles are not congruent that the angle relationship does not exist.

- Students should be aware of the various types of angles, and proper notation for naming angles and lines.

- This lesson builds on the Analysis level (level 2) of parallel lines. It is designed to get them to the Abstraction level (level 3) if they are not already there.

- This is an introductory lesson so at this point students gain a working knowledge of the different tools used by architects, mathematicians, carpenters, etc. to create parallel lines. In the next lesson, we will build upon that knowledge and delve into why it is important to have parallel lines in society.

Introduction: Setting Up the Mathematical Task

Day 1 Warm-Up Activity Here (student handout)

- In this lesson, you will investigate various geometric tools and their purpose in society. You will also use a protractor to measure angles formed when two lines are cut by a transversal as a method of reviewing this topic.
- The warm-up will take about five minutes for students to complete. The discussion that will follow will take about fifteen minutes.
- The students will be handed a sheet with pictures of different geometric tools. The student will name each item and state what they think it is used for in society.

- The teacher will have posters placed around the room that showcase various professions that use geometric tools.
- After two minutes, if any students are struggling with the warm-up the teacher will prompt those students by asking questions like the following: 1) Have you ever been lost and needed to use a map to find your way? How do you think the map was created? 2) Have you ever been to Lowe's or Home Depot and noticed any workers wearing tool belts? What are the uses for those tools? 3) Have you ever had your windows wrapped or replaced in their home? What types of tools were used?
- Once five minutes is up, the students will be instructed to discuss their answers using the teachers' choice of grouping strategies. (Think-Pair-Share is an example of an effective grouping strategy that would work in this situation.)
- Each student will have used a compass, straightedge, and protractor to explore the given diagrams and discuss with other students.
- Once the students have had the chance to speak with one another regarding the various geometric tools used in the real world, the teacher will tell the students to be ready to do their own preliminary investigation using a protractor as a guide. Just as a carpenter uses a square as a guide they will use the protractor as a guide to let them know the various angle relationships formed by the two lines being cut by the transversal.
- Students will discuss their ideas and findings with other students and finally with the teacher and the class.

Student Exploration 1: Students explore the relationship between parallel lines, transversals and congruent angles.

Activity #1: Parallel Lines? (Student handout)

Student/Teacher Actions:

- Students should work in partner pairs on Activity #1. Each student will measure the angles in each figure. Once all the angles in a figure are measured, the students will describe any angle relationships that exist. They will also describe the relationship lines *a* and *b* have in relation to line *t*.

- The teacher should be walking around the room to listen for pertinent vocabulary as this is a review lesson.

- It is possible that students will not remember the special names for the angle relationships formed when two or more lines are cut by a transversal, so the teacher can give them clues like "same position" for corresponding. Or "jumping across" or "what belt does your car use again?" for alternate interior, etc. Ask students if it matters how long the lines are? Can the lines *a* and *b* be rotated in different directions?

- In the next activity, you will be able to explore these questions using Geogebra software.

Monitoring Student Responses

- Students will have 10 minutes to work with a partner on Activity #1.
- After 10 minutes is up, the students will get back into groups of four and on a clean sheet come to a consensus about the assignment.

- The teacher will circulate the room to listen for appropriate vocabulary.
- If any students are having difficulties with the material, the teacher will ask them if they are comfortable using the protractor. Then the teacher will ask the student if they see any similarities among the angle measures that have identified. Hopefully, this will spark their memory.
- If groups are ready to move on, they may pick up the next assignment, but must work individually initially. (Think-pair-share)

Student Exploration 2: Understanding the Components of Parallel Lines Investigation Worksheet

Activity #2: Understanding the Components of Parallel Lines (Student handout)

Student/Teacher Actions:

- Students will work collaboratively to create and determine pairs of congruent angles created by the intersection of a transversal and a pair of parallel lines.
- Teachers should watch for appropriate student interaction while providing assistance to groups during the investigation.
- Students will work collaboratively to create and determine pairs of congruent angles created by the intersection of a transversal and a pair of parallel lines.
- Teachers should watch for appropriate student interaction while providing assistance to groups during the investigation.

Monitoring Student Responses

- Students will communicate their thinking and knowledge on the Activity #2 student handout.
- Students will continue to work in groups as they maneuver through the Geogebra software.
- Students will record any questions or "aha" moments on their handout so they can share those questions/ideas with their peers.
- The teacher will provide for individual assistance while facilitating learning for the group. The teacher will also request that if students finish early, they may assist the students having difficulties.
- Students who are ready to move forward will be instructed to create another construction as detailed on Activity #2.

- How do you plan to summarize your lesson? Describe it here.
  - Students will complete a journal writing prompt in the form of a ticket out. This will let the teacher know what the students understand or do not understand from the explorations.
  - Students' work will be recorded on the various student handouts used throughout the lesson

Assessment

Ticket Out Journal Activity (student handout)

Ticket Out Scoring Rubric

CATEGORY	4	3	2	1	0	Score
Requirements	Student noted one similarity, one difference and one real-world application.	Student noted two of the three requirements but not the third.	Student noted one of the three requirements.	Student did not note any of the three requirements.	Journal entry/ reflection was not attempted.	
Completion	The journal entry/ reflection was complete.		The journal entry/ reflection was incomplete.		The journal entry/ reflection was not attempted.	
Total						____/8

Journal/writing prompts

- Explain the similarities and differences between the angle relationships formed from the pre-printed figures on Activity #1 and the figures you created on Activity #2. Describe some situations that require the use of parallel lines in the real world. Think back to the warm-up. Be creative!
- Accommodations for the "ticket out" activity could include: reading of the prompt, verbal responses instead of written responses, or responses typed on the computer instead of hand-written.
- The "ticket out" assessment will give full credit for students noting at least one similarity, one difference, and one real-world application idea.

Extensions and Connections

- The students can be challenged to develop other ways of proving/disproving lines parallel in student activity #1.
- The students can use the lines given in student activity #1 and find each equation.
- The teacher can also bring in solving systems of linear equations using student activity #1.

Strategies for Differentiation

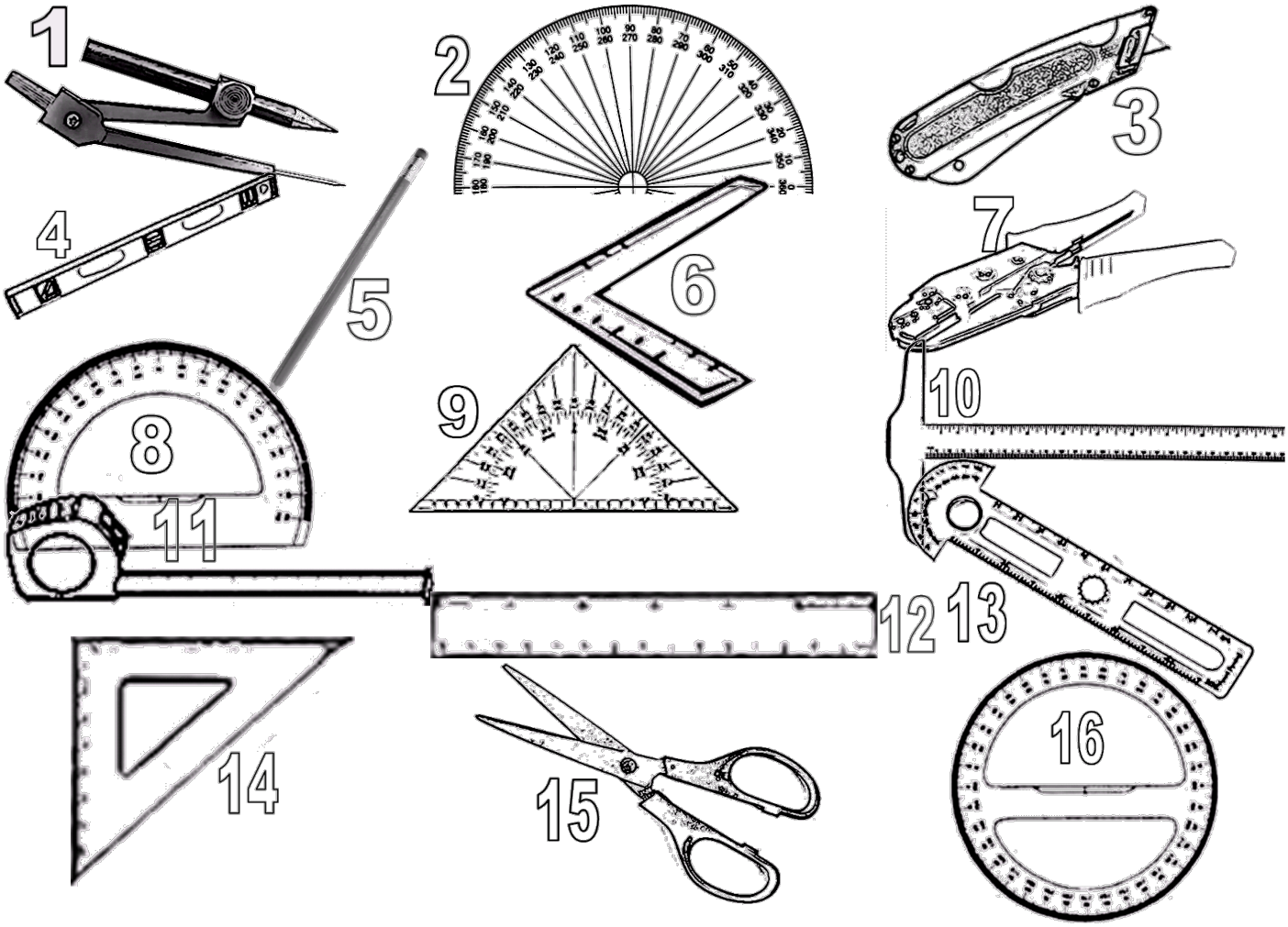
- The activities in Day 1 are designed with many learner types in mind. Kinesthetic students will have the ability to manipulate the geometric tools, auditory and visual learners will be able to hear and see instructions and demonstrations.
- Students with processing, memory, or motor issues will benefit from the group work in the first and second activities. The third activity, using Geogebra, may require written instructions for students with processing or memory issues. Students with motor challenges may benefit by completing the Geogebra activity with a partner.
- English language learners (ELLs);

- high-ability students (Ideas for providing depth "deep" doesn't mean "more").

Day 1 Name_____

Warm-Up Activity

Directions: Identify each item as it relates to geometry and state some examples of how each one is used in real – life. Take about 5 minutes to complete this activity before our discussion.

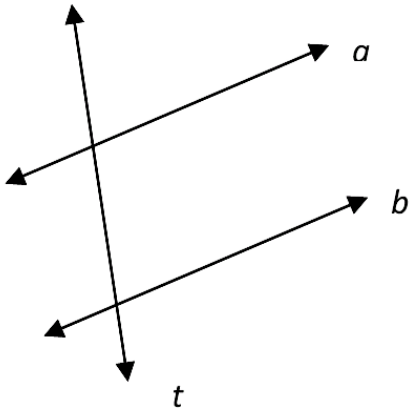


Parallel Lines, Transversals, and Angles: What's the connection?

Parallel Lines? Activity #1 Name_____

Directions: a) Use a protractor to measure each angle in the figures below. b) Describe any angle relationships that exist. c) What do you know about lines *a* and *b* in relation to line *t*?

1)



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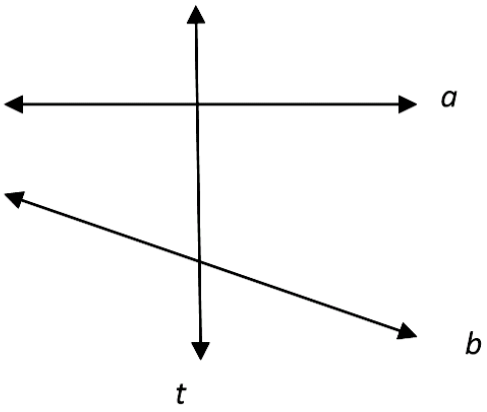

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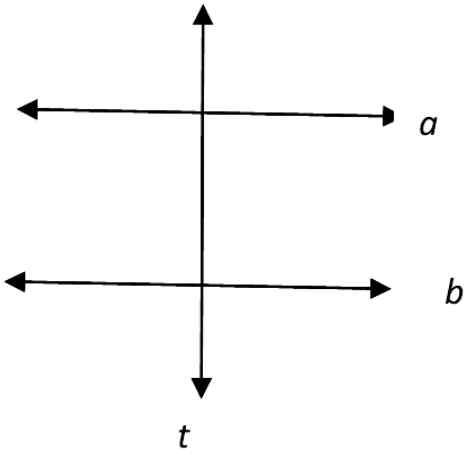

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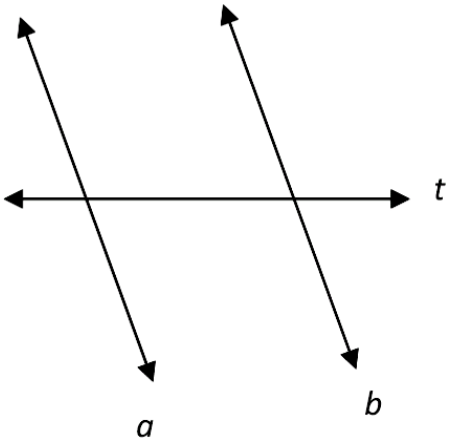
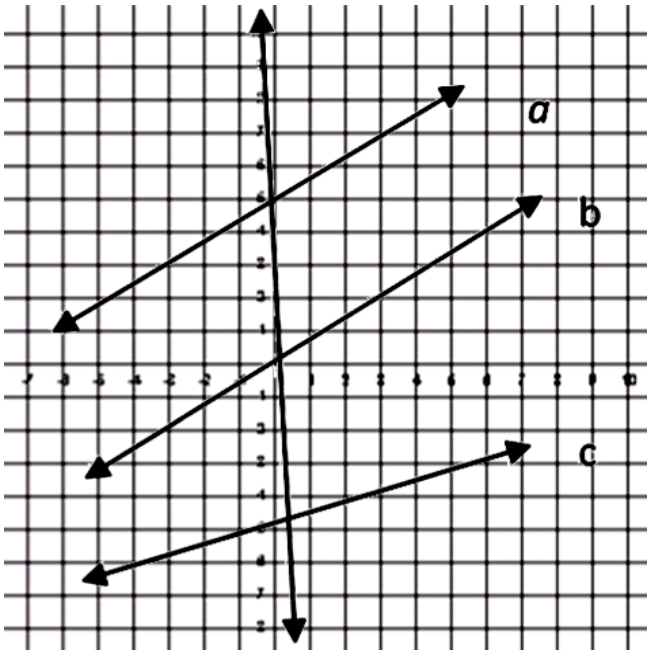
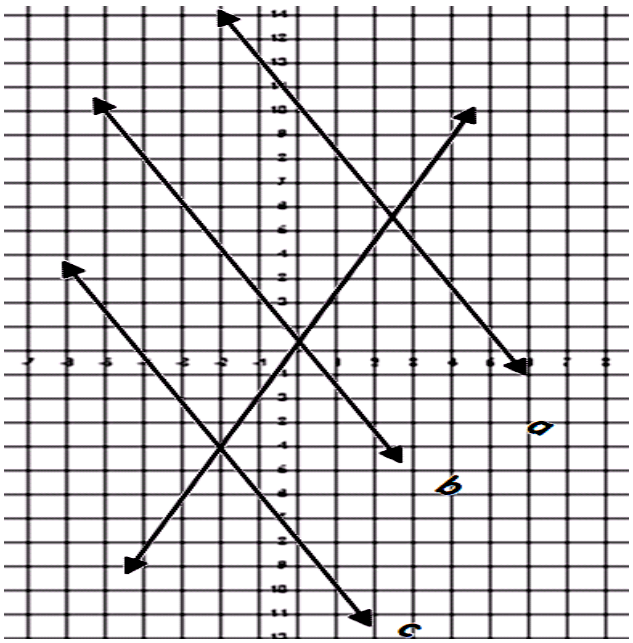


Challenge Problems

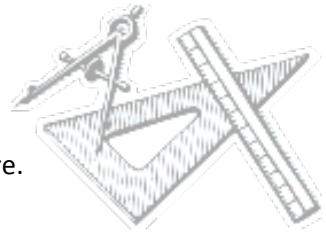
5. Determine two distinct ways to determine which lines, if any, are parallel. Show your work. 6.





Once your group is done with Activity #1, pick up Activity #2. When you are done with Activity #2, complete the "ticket out" activity.

Exploration 2 Handout Name _____



Understanding the Components of Parallel Lines

Materials: For this activity, students will need the student handout and Geogebra software.

Procedure:

1) Each student should construct a set of parallel lines using the Geogebra software. Label your lines. Make this drawing as neat and precise as possible as you will be using the diagram to make conjectures.

2) Now, construct a transversal through the two parallel lines. No perpendicular lines please.

3) Using the Geogebra software, number and measure your newly created angles.

4) Using the Geogebra measuring tools, figure out which pairs of angles, are equal.
Describe your methods for comparing which angles are equal. What is another method you could have used to go about this inquiry?

5) Using the Geogebra measuring tools, figure out which pairs of angles, are supplementary.
Describe your methods for finding which angles are supplementary. What is another method you could have used to go about this inquiry?

6) Using the vocabulary of **corresponding angles, alternate interior angles, alternate exterior angles, vertical angles, consecutive interior angles** name the angles that were equal or supplementary.

Ticket Out Journal Handout Name _____

Ticket Out Activity

Explain the similarities and differences between the angle relationships formed from the pre-printed figures on Activity #1 and the figures you created on Activity #2. Describe some situations that require the use of parallel lines in the real world. Think back to the warm-up. Be creative!

Lesson 2

Strand
Geometry

Mathematical Objective(s)
The student will explore angle relationships formed when lines are cut by a transversal in city planning models.

Mathematics Performance Expectation(s)
Students will apply inductive and deductive reasoning skills to make and test parallel lines and the relationship between transversals and angles. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid.

MPE. 32 Use the relationships between angles formed by two lines cut by a transversal to
a) determine whether two lines are parallel;
b) verify the parallelism, using algebraic and coordinate methods as well as deductive proofs; and
c) solve real-world problems involving angles formed when parallel lines are cut by a transversal.

Related SOL

G.2 The student will use the relationships between angles formed by two lines cut by a transversal to
a) determine whether two lines are parallel;
b) verify the parallelism, using algebraic and coordinate methods as well as deductive proofs; and
c) solve real-world problems involving angles formed when parallel lines are cut by a transversal.

NCTM Standards List all applicable NCTM standards related to each lesson.

- investigate conjectures and solve problems involving two- and three-dimensional objects represented with Cartesian coordinates.
- draw and construct representations of two- and three-dimensional geometric objects using a variety of tools;
- use geometric models to gain insights into, and answer questions in, other areas of mathematics;
- use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture.

Additional Objectives for Student Learning (include if relevant; may not be math-related):
WG.1
The student will use maps, globes, satellite images, photographs, or diagrams to
a) obtain geographical information about the world's countries, cities, and environments;
b) apply the concepts of location, scale, map projection, or orientation;
d) create and compare political, physical, and thematic maps;
WG.11

The student will analyze the patterns of urban development by
c) describing the unique influence of urban areas and some challenges they face.

Materials/Resources

Describe the materials and resources (including instructional technology) you plan to use in each lesson.

- Teacher Computer – used to show Google maps and short video clips on-line
- Laminated classroom set of maps from various areas around the country or an Atlas book or a Computer lab
- CD Player

Assumption of Prior Knowledge

- Students have a strong foundation in the area of parallel lines and transversals. An extensive review was completed in the last lesson and the students are ready to move forward and do some exploration on their own.
- For optimal student success, van Hiele level 2 (Analysis) should have been previously achieved and students should be moving towards level 3 (Abstraction).
- Students will know the various types of angle relationships associated with parallel lines cut by a transversal. They will use such vocabulary as corresponding angles, alternate interior angles, etc. with confidence.
- Students will have talked about the various geometric tools used to create pictures, maps, buildings, bridges, and other structures.
- In this lesson, we will discuss sustainability and city planning. We will also make mention of natural disasters as they relate to city planning.

Introduction: Setting Up the Mathematical Task

- In this lesson, you will investigate maps of different cities around the country and discuss any geometric influences you see. In particular, you should look for parallel and perpendicular lines, as well as angle relationships formed when two lines are cut by a transversal.
- This is the first day in a series of lessons that will take approximately two weeks.
- As students enter the classroom, the song "Wake up America" by Miley Cirus will be playing in the background. The teacher will return papers from last class (ticket out). Once everyone is settled the teacher will pull up the following clip from you tube.
  So what is sustainability?
  http://www.youtube.com/watch?v=7mxmPlUhWqI&feature=related

- Once the video clip has ended, the teacher will begin to explain the Island Project that the students will spend the next two weeks completing. The teacher will request that certain students read their ticket out aloud to the class. (These would be chosen based on relevant responses that relate to the island project.)
- If you were to build a city, what would you have to incorporate in your design? Would you include any special structures/buildings? How would the roads look? Why? How does the location of the city affect your planning?

- Activity one is a think/pair/share activity. Activity two is a partner/pair activity.
- In the first activity the teacher is modeling what the students will be doing in Activity Two.
- Indicate how you will you invite students to draw upon their prior knowledge?
  Students will include their knowledge of angle relationships within the writing prompt and geometric vocabulary.
- How will you help them to understand the task?
- In what way will students make their mathematical thinking and understanding public?

Island Project

- **In an effort to prove to the world that a sustainable environment is possible, your task is to create a "green" island.**
- **Temperature: ranges from 60 to 95 degrees**
- **Weather: North west end of the island is prone to earthquakes. The entire island regularly experiences high winds and flooding in lower lying areas.**
- **Size: (will discuss later)**
- **Accessibility to and from mainland: (will discuss later)**
- **See rubric for more project details.**

Student Exploration 1:

The first activity will be a group discussion about downtown Richmond, VA. The teacher will pull the Google maps website and locate Virginia Commonwealth University. (This gives a nice view of the City of Richmond.) The teacher will then instruct the students to look for geometric patterns in the design of the city.

Student/Teacher Actions:

- What should students be doing? Example: Function matching game where students work with a partner to pair different function representations (e.g., $y = x^2$ matched with a graph of that function or with a list of data that fits this pattern).
- The teacher will ask students if they have ever driven in downtown Richmond. (Students will take about a minute or two to discuss their experience(s) with their partner pair.)
- In an effort to promote student thinking, the teacher will ask the class if it is relaxing or frustrating to drive in Downtown Richmond. Why? Do you think there is a specific reason for the arrangement of the streets

in Downtown Richmond, VA? What might the reasons be? During the discussion, the teacher will share his/her own experiences with driving in different places around the country. (Students will have 2 minutes to discuss their answers with their partner before the teacher begins calling on students for answers. The teacher will record the answers on the board for later use.)

- Include ideas for technology integration or cooperative/collaborative learning within the student/teacher actions.

Monitoring Student Responses

- Describe how you expect:
  - Students will informally communicate their thinking to their partner pair.
  - Students will have ample time to quietly discuss their thoughts when prompted.
  - After students have been given time to share their thoughts with their partner they may be called upon to share those ideas with the class.
  - Since it is a whole class discussion, the teacher may find it helpful to allow other students to explain concepts or assist them in seeing connections they are having difficulty understanding.
- At the conclusion of the whole class discussion, the teacher will pass out Activity #2 which will be completed in partner pairs. The teacher will state that from the discussion we realize that city planning does have a lot of geometric influence including parallel lines and perpendicular lines. There are even other geometric shapes that are pleasing to the eye that are incorporated in such planning. One interesting observation about city planning is the use of parallel, perpendicular and one way streets as a means of military defense.
- The teacher will record responses on the board.

Student Exploration 2:

Same as above, if more than one exploration is included in the lesson plan.

Small Group Work (if relevant)

Activity #2: (partner pairs)

Directions: Take about 20 minutes to research maps of different cities around the world using the Atlas, travel maps, or internet. Write a one paragraph paper discussing any geometric influences you notice in the planning of the city you choose to research. In your discussion, make mention of parallel and perpendicular lines. Also, include any angle relationships formed when two lines are cut by a transversal. (Use a protractor to measure the angles on the map.)

Parallel Lines, Transversals, and Angles: What's the connection?

Student/Teacher Actions:

- Students should be researching a city of their choosing and reporting any geometric influences they notice that may have gone into the planning of the city. Students can use an Atlas, travel map, or the internet.
- After about five minutes, the teacher will begin to circulate the room to make sure that all partner pairs have chosen a city map and have begun formulating ideas about the design of the city.

Monitoring Student Responses

- Describe how you expect:
    - Students should use appropriate vocabulary with each other.
    - The teacher will remind students of the concepts explored last class and reiterate finding a small picture in a big design.
    - If any students are struggling, the teacher will help them to see smaller figures (ie. Corresponding angles) in the bigger picture (the maps)
    - If a group is ready to move on, they can continue their research by finding another city with a similar street pattern. They can then discuss why they think certain places have a "grid like" design and why others do not.
- How do you plan to summarize your lesson? Describe it here.
- During the last 10 minutes of class, the teacher will play a short clip (a little over 3 minutes) from the following website http://geometricdesignsmartcity.com/eng/index.html Click on "Why this project?" Immediately after watching the clip, students will respond to the writing prompt in the Ticket Out activity.
- The "Ticket Out" works as a summary for the lesson by having students brainstorm about what they have learned and use their knowledge to think ahead.
    - **Journal/writing prompts**

TICKET OUT

Keeping in mind today's discussion, write a short response to the following writing prompt.

If you were to design a city, what would it look like? Also, how would you incorporate geometry into your design? Be sure to use your mathematical vocabulary!

Assessment

Grading Rubric for Map Research paper	0 (no mention)	2 (partially mentioned)	4 (fully mentioned)	Points
1) Geometric influence				
2) Angle pair relationships				
3) Parallel & Perpendicular lines				
TOTAL POINTS				=

Extensions and Connections (for all students)

- The ticket out activity extends the lesson and prepares the students for the following lessons in the unit.
- Within this lesson there are connections to social studies because of the discussion of the history of maps and science because of the discussion of sustainability, and finally a connection to writing with the Ticket Out activity.

Strategies for Differentiation

- List ideas for addressing needs of a diverse population of students such as:
  - The activities in this lesson are designed with many learner types in mind. Kinesthetic students will have the ability to manipulate the geometric tools, auditory and visual learners will be able to hear and see instructions, demonstrations, music, and video clips.
  - English Language Learners have the ability to change the language on their internet search, if they desire.
  - Students with processing, memory, or motor issues will benefit from the group work throughout the activities.

- Each group can assign particular jobs to group members. Each group member can create a checklist of their required responsibilities.

Lesson 3

Strand
Geometry

Mathematical Objective(s)
The student will explore angle relationships formed when lines are cut by a transversal in city planning models.

Mathematics Performance Expectation(s)
Students will apply inductive and deductive reasoning skills to make and test parallel lines and the relationship between transversals and angles. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid.

MPE. 32 Use the relationships between angles formed by two lines cut by a transversal to
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c) solve real-world problems involving angles formed when parallel lines are cut by a transversal.

Related SOL
G.2 The student will use the relationships between angles formed by two lines cut by a transversal to
a) determine whether two lines are parallel;
b) verify the parallelism, using algebraic and coordinate methods as well as deductive proofs; and
c) solve real-world problems involving angles formed when parallel lines are cut by a transversal.

NCTM Standards
In grades 9–12 all students should–

- use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations;
- investigate conjectures and solve problems involving two- and three-dimensional objects represented with Cartesian coordinates.
- draw and construct representations of two- and three-dimensional geometric objects using a variety of tools;
- use geometric models to gain insights into, and answer questions in, other areas of mathematics;
- use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture.

Additional Objectives for Student Learning (include if relevant; may not be math-related):
World Geography
WG.1 The student will use maps, globes, satellite images, photographs, or diagrams to
a) obtain geographical information about the world's countries, cities, and environments;

b) apply the concepts of location, scale, map projection, or orientation;
d) create and compare political, physical, and thematic maps;
WG.2 The student will analyze how selected physical and ecological processes shape the Earth's surface by
a) identifying regional climatic patterns and weather phenomena and their effects on people and places;
b) describing how humans influence the environment and are influenced by it;
c) explaining how technology affects one's ability to modify the environment and adapt to it.

WG.11 The student will analyze the patterns of urban development by
c) describing the unique influence of urban areas and some challenges they face.

Materials/Resources

- Ruler
- Markers
- Pencils
- Paper 8 ½ by 14 or larger
- Colored pencils

As part of the previous lesson's homework, students were asked to compile a list of things that they would include in a city of their creation.

Assumption of Prior Knowledge

- Students have been exposed to parallel lines, transversals and the angles that are created.
- For optimal student success, van Hiele level 2 (Analysis) should have been previously achieved and students should be moving towards level 3 (Abstraction).
- Students will know the various types of angle relationships associated with parallel lines cut by a transversal. They will use such vocabulary as corresponding angles, alternate interior angles, etc. with confidence.
- Identify/ predict what students may find difficult or confusing or have misconceptions about?
- Students will have talked about the various geometric tools used to create pictures, maps, buildings, bridges, and other structures.
- What stages (levels) does this lesson build on?
- In this lesson, we will discuss sustainability and city planning. We will also make mention of natural disasters as they relate to city planning.

Introduction: Setting Up the Mathematical Task

- In this lesson, you will create a basic city plan using information that involves parallel lines and transversals.

- Depending upon the level of detail involved in this activity, it should take approximately two days from brainstorming to completion.

- Students will begin class by discussing their homework from the previous night. In that assignment, students were asked to think about what a city of their design would look like. Students will then be given the opportunity to be creative and design a basic city design based upon what they feel is important to include in their community.

- Students will discuss the concept of urban planning and the needs of the communities' inhabitants. Some time should be spent on developing student's ideas of the importance of planning for environmental disasters such as earthquakes or hurricanes while also having evacuation routes for non environmental events if necessary.

- If you were to design a city, what would you have to incorporate in your design? Would you include any special structures/buildings? What structures should be included so that the inhabitants have a healthy and meaningful life. How would the roads look? Why?

- Students will complete this activity with a partner. Students will use the ideas from the ticket out and incorporate their ideas into one island project.

- What activities (if any) move students toward the stated objectives/goals?

- Indicate how you will you invite students to draw upon their prior knowledge?

- How will you help them to understand the task?

- Students will show mastery of geometry topics while using visual arts skills to represent real world situations in a fictitious environment.

Student Exploration 1:

The teacher will remind students of the importance of urban planning and the issues surrounding designing and building a functional and green community.

Student/Teacher Actions:

- What should students be doing? Example: Function matching game where students work with a partner to pair different function representations (e.g., $y = x^2$ matched with a graph of that function or with a list of data that fits this pattern).

- The teacher will direct a brief discussion on the needs of the community but allow students to explore what is needed. Once students begin sketching, teachers are asked to monitor student progress.

- As this is an open exploration there are no correct or incorrect solutions. Ask students how the inhabitants would evacuate a city in the case of a natural disaster? How would the design of the streets and buildings facilitate this evacuation if necessary?

- Include ideas for technology integration or cooperative/collaborative learning within the student/teacher actions.

Monitoring Student Responses

- Describe how you expect:
    - Students will informally communicate their thinking/ ideas with the group or partner.
    - After students have had an opportunity to brainstorm their ideas for their community they may share them with the class.
    - For students having difficulties creating their own community, teachers can have a map or atlas available to start the creative process.
    - teacher to extend the material for students that are ready to move forward.

- How do you plan to summarize your lesson? Describe it here.
    - At the end of the activity students should have a sense of the geometry that is needed to plan and create a community from scratch.
    - The teacher may record information on the board as necessary.

Student Exploration 2:

Introduction: Setting Up the Mathematical Task

- In lesson 3 students will learn how to create their previously created graph-paper map using Geogebra.
- The timeline for this activity can be as short or as long as the teacher feels is necessary. It can be time-consuming to correctly construct objects in Geogebra. For this reason the teacher may choose for students to create only a portion of their map, possibly the most geometric. Or, the teacher can put the students into pairs or small groups and have each student create a portion of the map so that when all students are finished the entire city map is completed.
- In previous lessons students explored city designs and created their own geometric map on graph paper. This exploration teaches students how to use Geogebra to create their map using the computer. Included with this lesson are two tutorials to help teachers become comfortable with the Geogebra software. One tutorial focuses on creating a parking area using points, segments, parallel lines, and polygons. The second tutorial focuses on creating a traffic circle using points, circles, arcs, segments, and parallel lines. It is suggested that the teacher practice creating these elements, or elements of their choice, before having students begin their work in Geogebra.
- The teacher will need to demonstrate how to create objects in Geogebra for the class. The teacher can create a short tutorial worksheet or use the attached tutorial lessons for the students to learn the software.

- As the demonstration progresses the teacher may ask the entire class or individual students how they think they will create the next object (line, point, segment, parallel line, etc.). This will encourage students to continue to stay engaged in the demonstration. The teacher may also ask how we can tell when an object has been constructed versus just drawn.
- Whole class discussion will be necessary to introduce the activity. If the teacher has access to Smart Board technology they may want to have student volunteers create objects. The teacher could describe the object to be created, give pairs of students a few minutes to think about how it could be created in Geogebra, then have one or both students come to the board and create the object.
- Once the students become comfortable with the software their only limitation will be the amount of time allotted for the activity. It will be important for the teacher to have decided how long they want to spend on the activity and how much of the student's maps they want created in Geogebra.
- Students will draw upon their prior knowledge of Geometry when they construct objects in Geogebra and not simply draw objects. As they work within the software they review geometric vocabulary.
- The teacher will help students understand the task by demonstrating the Geogebra software, engaging the students within the demonstration, and helping the students with software issues throughout the activity.
- Students can create individual objects in separate files and then print at the end of the activity. Students should then be given the opportunity to cut out and piece together their new map.
- Students can make their mathematical thinking and understanding public when they demonstrate how to create objects for the class. Students can also express their mathematical thinking by sharing their Geogebra creations with the class, while it is still in progress. When a student is having trouble creating an object, have the student share the diagram with the entire class, using the Smart Board technology or saving the file to a thumb drive and displaying up on the teacher's computer using the projection system. Then, have the class brainstorm on how the student can work through the problem. If this process is repeated several times students will feel comfortable helping each other and feel like the difficulties are simply challenges to overcome.

In previous lessons students explored city designs and created their own geometric map on graph paper. This exploration teaches students how to use Geogebra to create their map using the computer. Included with this lesson are two tutorials to help teachers (and possibly students) become comfortable with the Geogebra software. One tutorial focuses on creating a parking area using points, segments, parallel lines, and polygons. The second tutorial focuses on creating a traffic circle using points, circles, arcs, segments, and parallel lines. It is suggested that the teacher practice creating these elements before having students begin their work in Geogebra.

Student/Teacher Actions:

- If students are working in groups they will need to decide who is creating which portions of their map. Once this issue has been decided students should begin working in Geogebra to create their portion of the map.
- As students begin working with Geogebra they may become frustrated, especially at first. The teacher needs to monitor frustration levels and notice if several students are stuck on creating the same type of object. If so, the teacher may want to walk the students through creating that type of object using the Smart Board or projector, so that the entire class may see and ask questions.
- When a student is having trouble creating an object, have the student share the diagram with the entire class, using the Smart Board technology or saving the file to a thumb drive and displaying up on the

teacher's computer using the projection system. Then, have the class brainstorm on how the student can work through the problem. If this process is repeated several times students will feel comfortable helping each other and feel like the difficulties are simply challenges to overcome.

Monitoring Student Responses

- Describe how you expect:
  - Students will communicate their thinking verbally with their peers and teacher. Students will also communicate their competency with Geogebra using the diagram of the map they create.
  - teacher and/or students to highlight and clarify the ideas being grappled;
  - When students have difficulties with creating their map in Geogebra they can raise their hand for the teacher to assist; or, the teacher may require the student to ask their group members (if they are working in groups) first before asking for help from the teacher. The teacher may have another process in place for the student to communicate that they need help; some teachers have students place a small red cup (or some other object) on the top of their computer to visually show that they would like the teacher's help. With this procedure students are not allowed to simply "call out" questions to the teacher.
  - There will definitely be students who work very quickly and others that work much slower. For the students that work quickly the teacher may have them create more of their map. The teacher may have the slower students create less of their map.
  - The most important portion of this lesson is not the map itself; it is the process of constructing objects using geometric software.
- How do you plan to summarize your lesson? Describe it here.
  When the allotted time for work in Geogebra has been completed the teacher will want to guide students to summarize their learning. Having students complete a journal entry about what they learned can be helpful. Then, the students can share their ideas with a nearby partner. Finally, the teacher can help the class compile a list of ideas that the students feel they have learned by working in Geogebra. This process will show evidence of students' knowledge of Geogebra, geometric concepts, working with the computer, working in groups, and other ideas that students may bring up.

Assessment

- **Journal/writing prompts**
  - What were some challenges you encountered working with Geogebra? How did you overcome each challenge?
  - Describe several things you disliked about working with Geogebra. Describe several things you liked about working with Geogebra.
  - Describe several things you learned, or relearned, while creating your map in Geogebra.
  - Describe your experience with Geogebra. Did you complete everything you wanted to complete? How does your final product make you feel?
- Accommodations or modifications may be necessary for some students. There may be students who have never worked with a computer and may require guidance at all times. This student may need to just work through one of the tutorials with help from the teacher when needed. The amount of the students' map created in Geogebra can be greater or smaller according to the abilities of the students. The teacher may need to pick groups and the amount of work to be completed by each student, instead of simply letting each group decide. Some students may need extended time to complete their portion of the activity.

1	2	3	4
Student does not complete their portion of the activity.	Student completes their assigned part of the map. Some objects behave as constructions.	Student completes their assigned part of the map. Most objects behave as constructions.	Student completes their assigned part of the map. All objects behave as constructions.

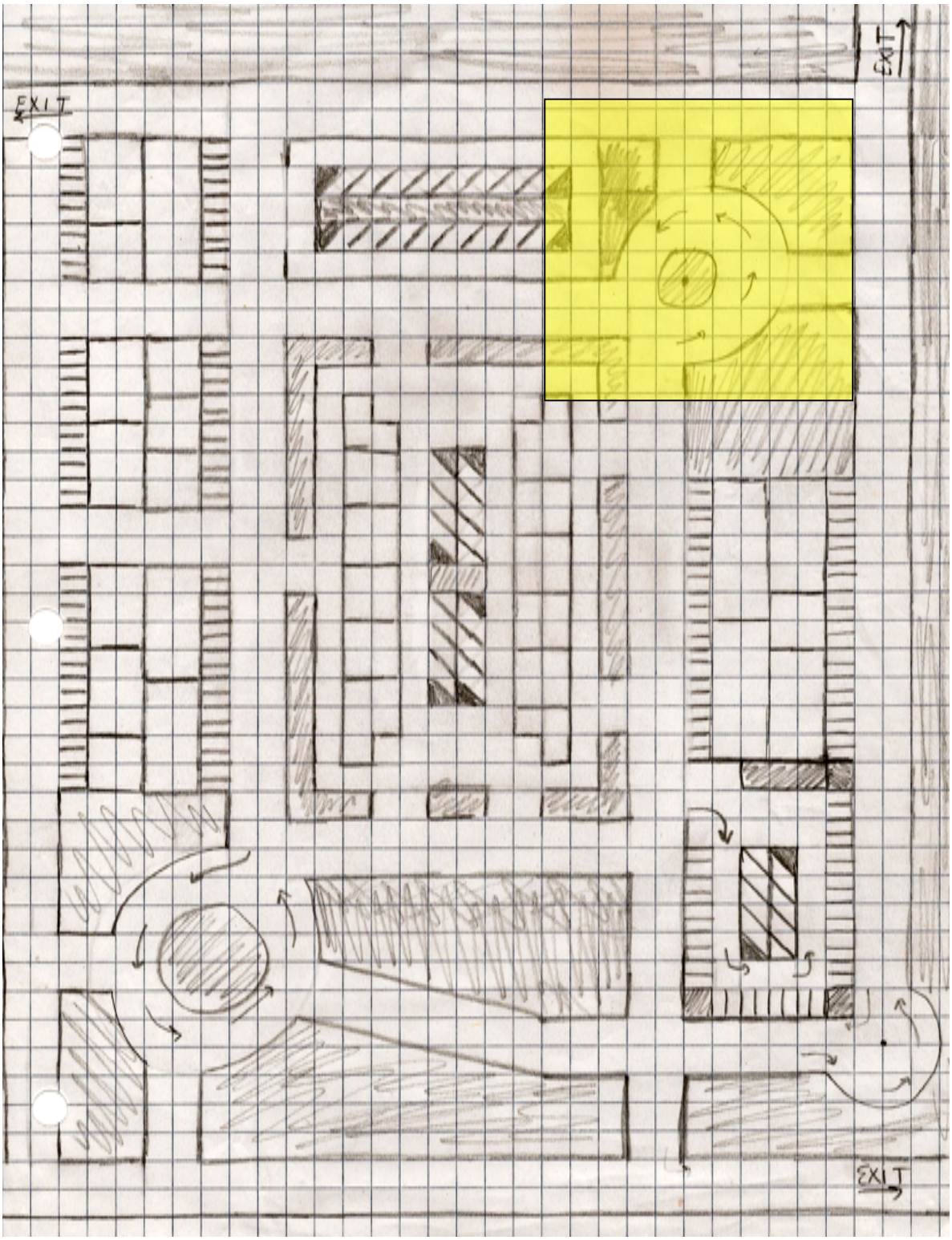
Extensions and Connections

- The lesson can be extended as long as the teacher has the time. If the teacher is comfortable with Google Sketch Up they can have students create 3D maps.
- Connections to history can be made by discussing the differences of creating maps hundreds of years ago and present day map making. Connections to writing can be made through the journal topics.

Strategies for Differentiation

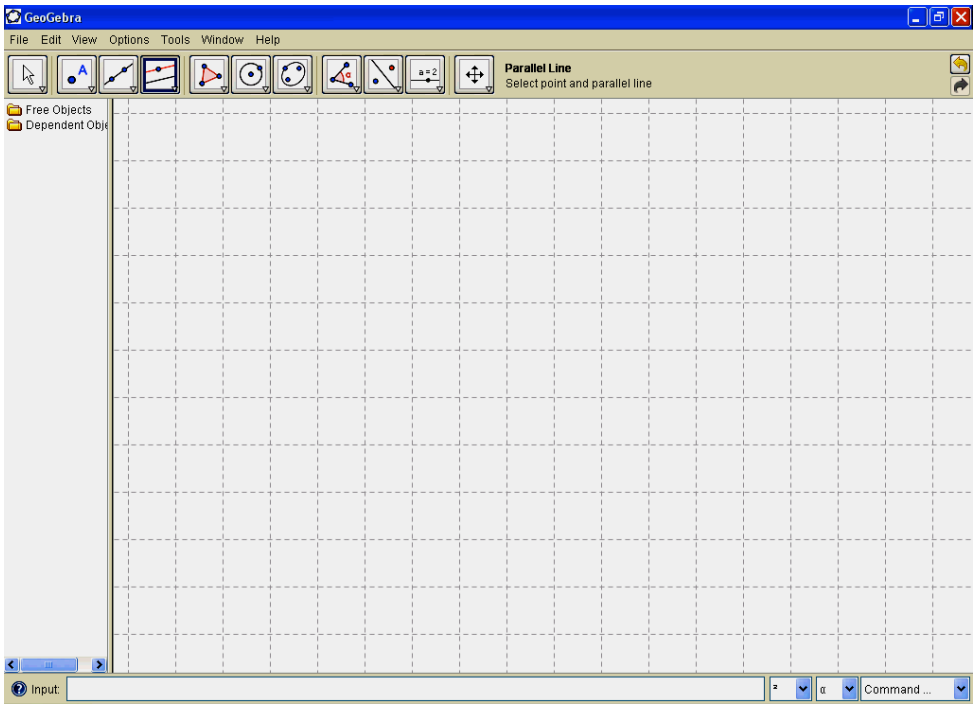
- List ideas for addressing needs of a diverse population of students such as:
    - This activity addresses many different learning types because it provides students the opportunity to work hands on with the computer, listen to verbal instructions from the teacher and other students, and demonstrate techniques visually.
    - Students with processing, memory, or motor issues may need to work through the tutorials instead of trying to create objects independently. Or, the student can take screenshots and paste into a Word document as they work to help them remember the steps they took to create a particular object. They can then use this document as they continue their work. Or, the teacher may need to provide explicit, written instructions on how to create the objects that they have been assigned to create.
    - English language learners should be comfortable throughout this assignment because Geogebra commands are shown using symbols. The teacher will need to monitor and help these students or modify the assignment if necessary.
    - High-ability students may have time to explore creating their map in Google Sketch Up, where they can create their maps in a 3D environment.
- If students are working in small groups they may want to make a copy of their original map and then cut out the portions that they need to create in Geogebra. Students can create individual objects in separate files and then print at the end of the activity. Students should then be given the opportunity to cut out and piece together their new map.

Example of a basic sketch of a portion of a city plan. The directions for creating the highlighted portions in Geogebra are included.

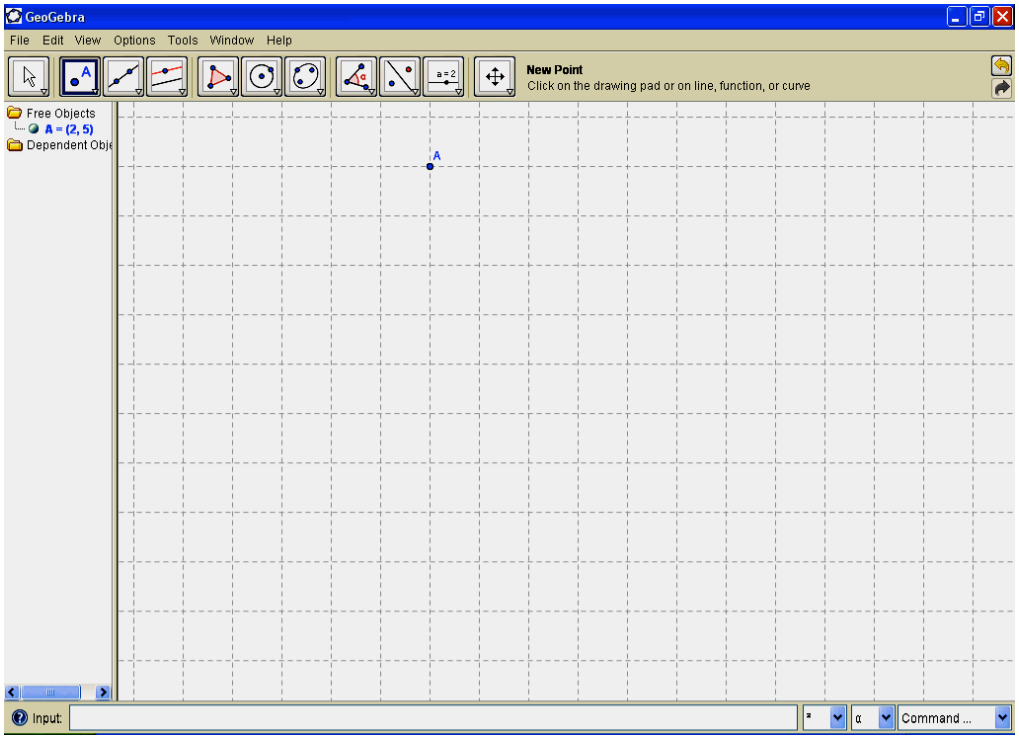

EXIT
EXIT
EXIT

Directions for creating the highlighted parking in the center of the sketch.

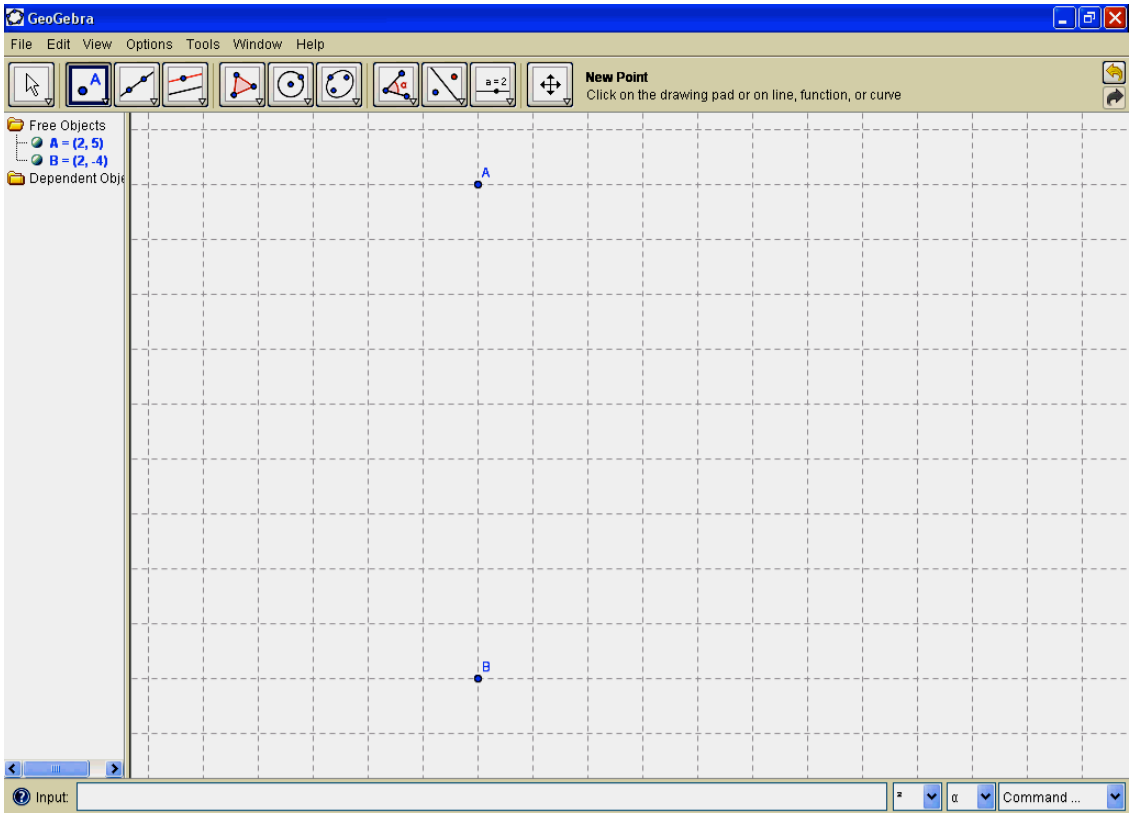
Begin creating our diagram by going to View and selecting "grid" and unselecting "axis".



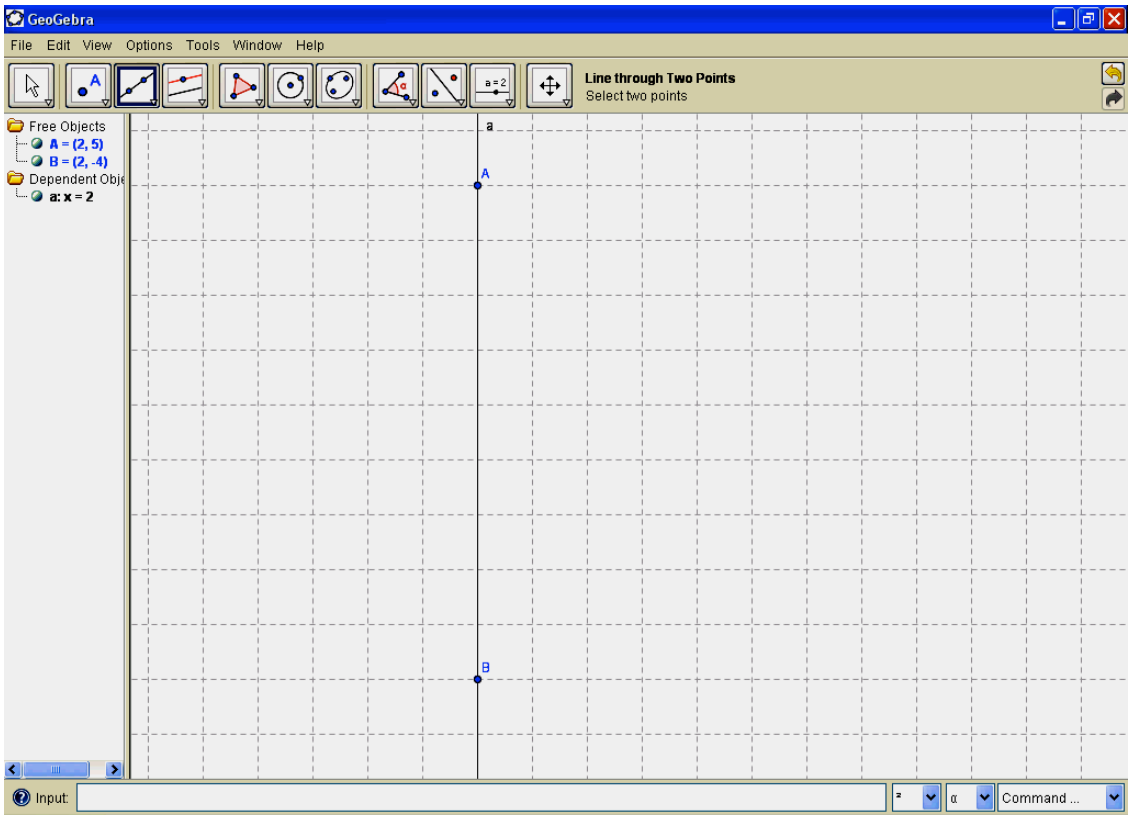
Start with creating a point



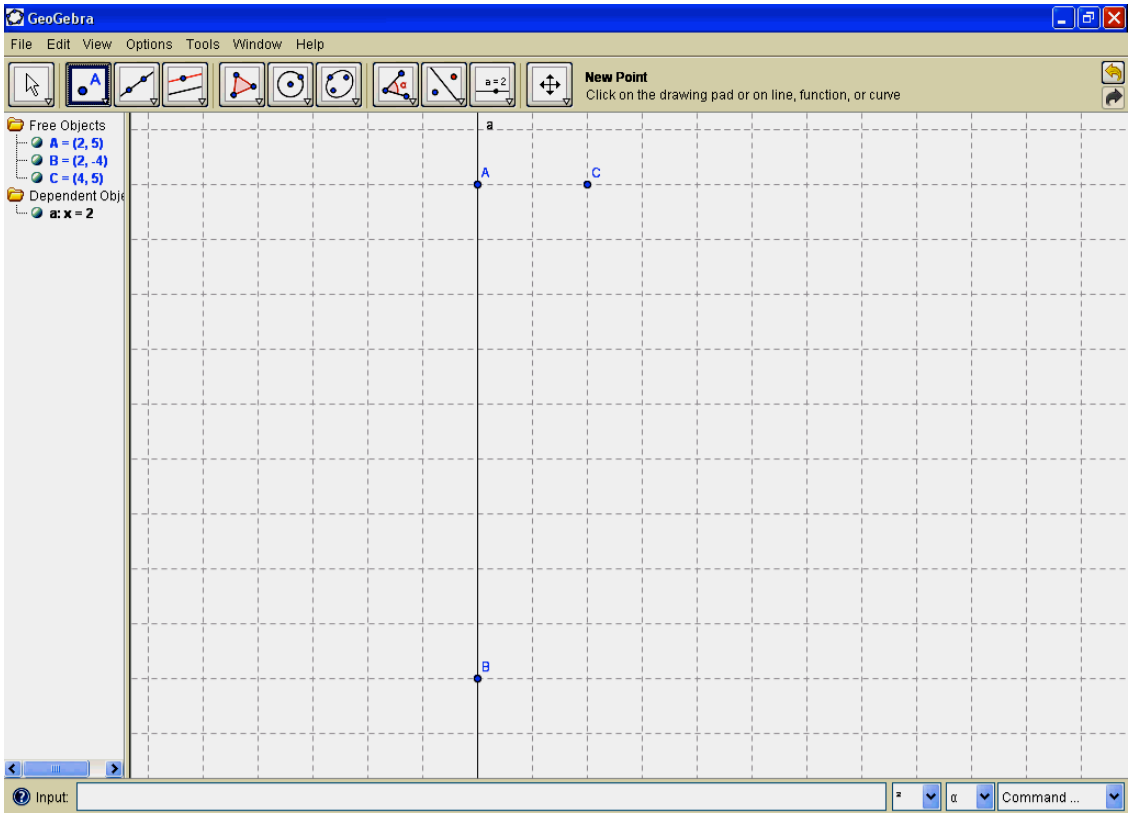
Create point B



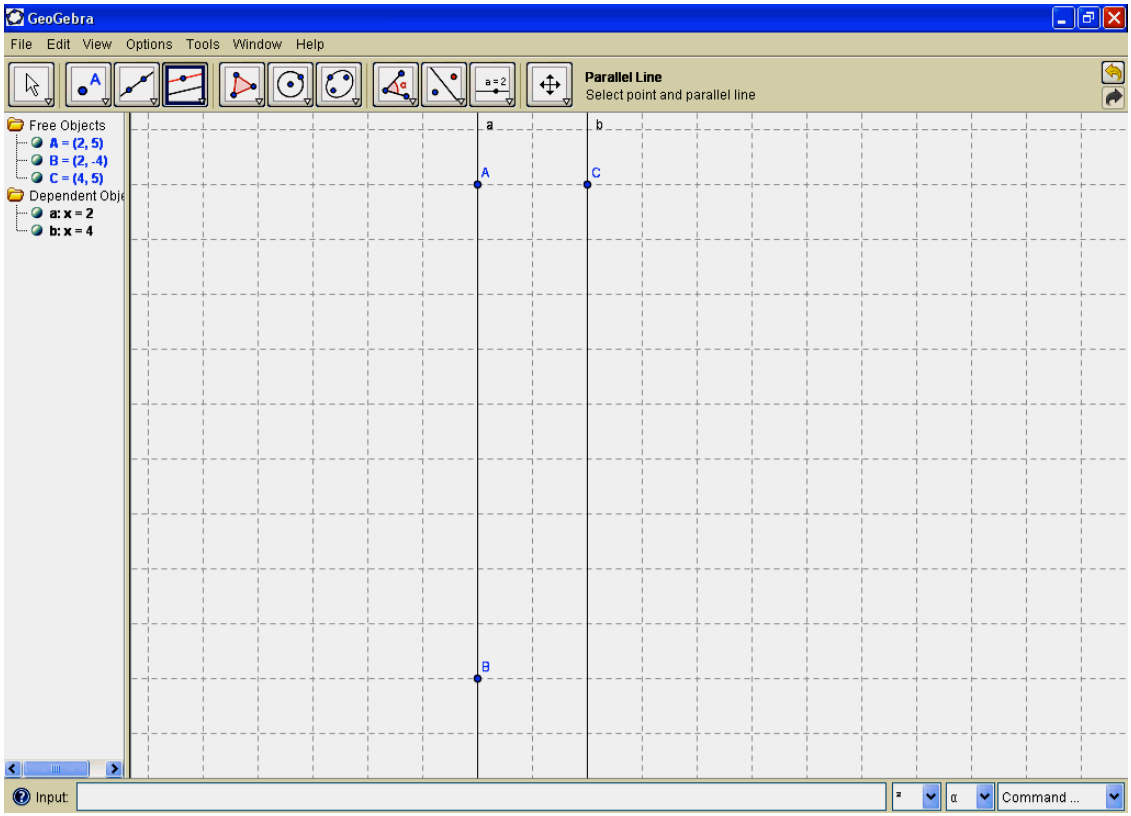
Create a line through points A and B



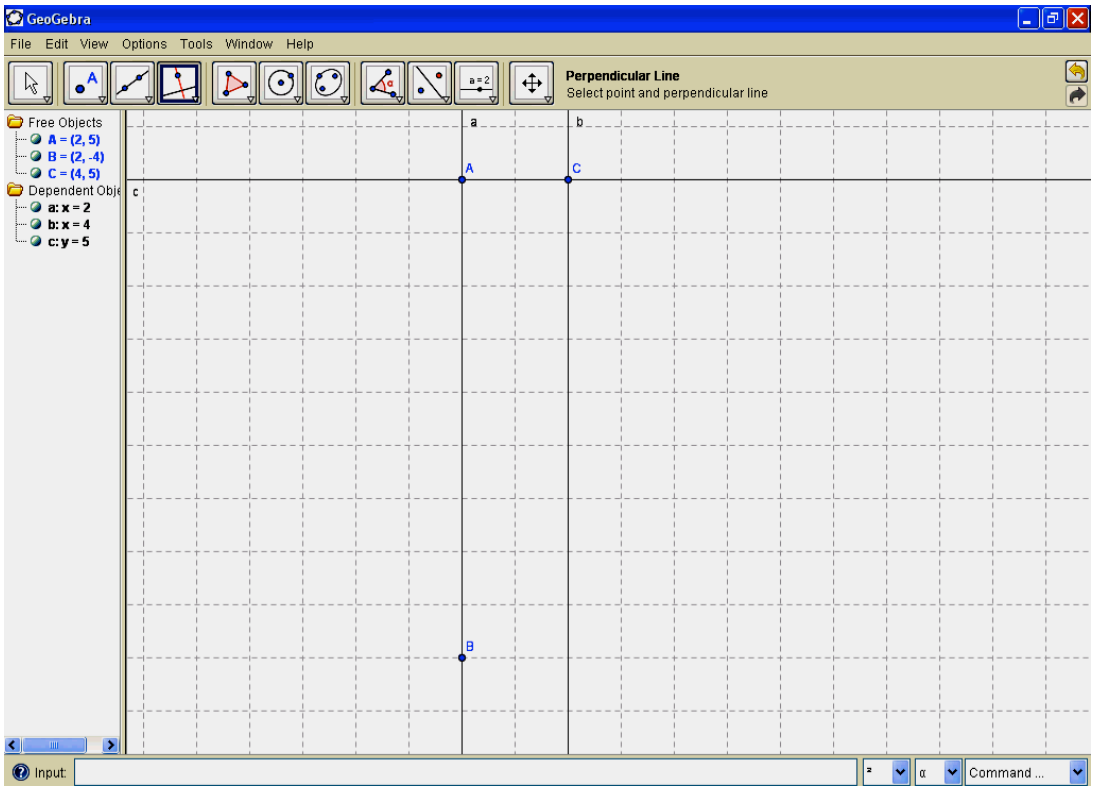
Notice the line through A and B is labeled as a. Create point C.



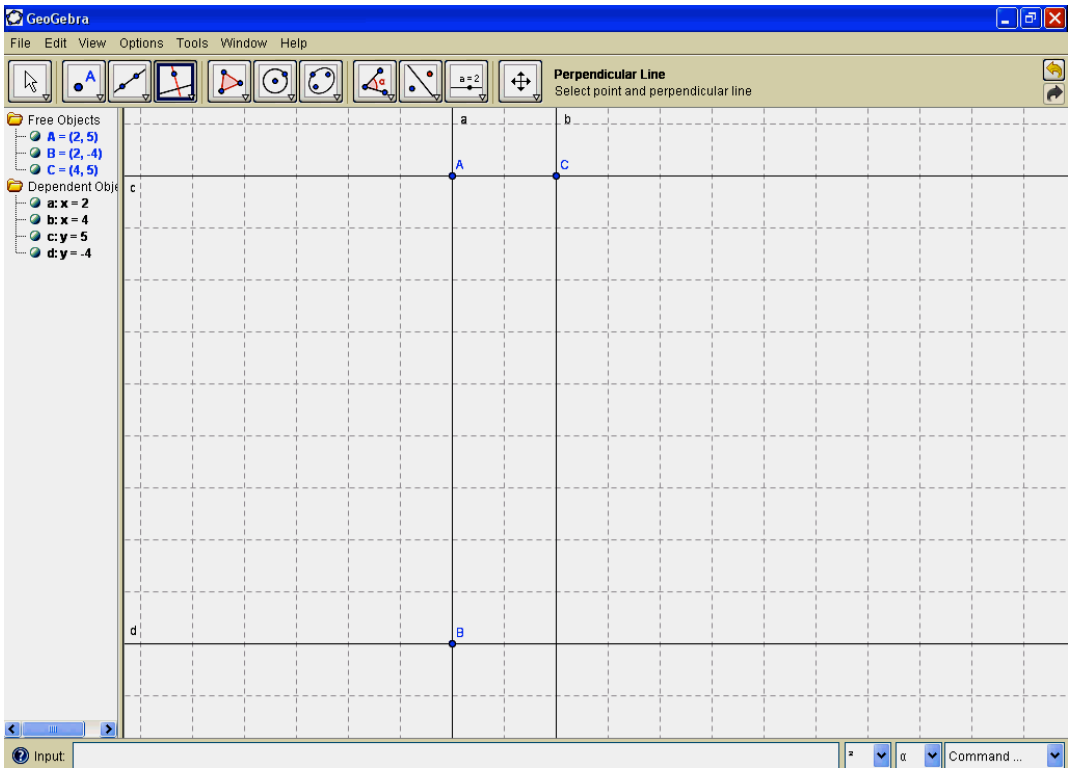
Using point C, create a line parallel to line a.



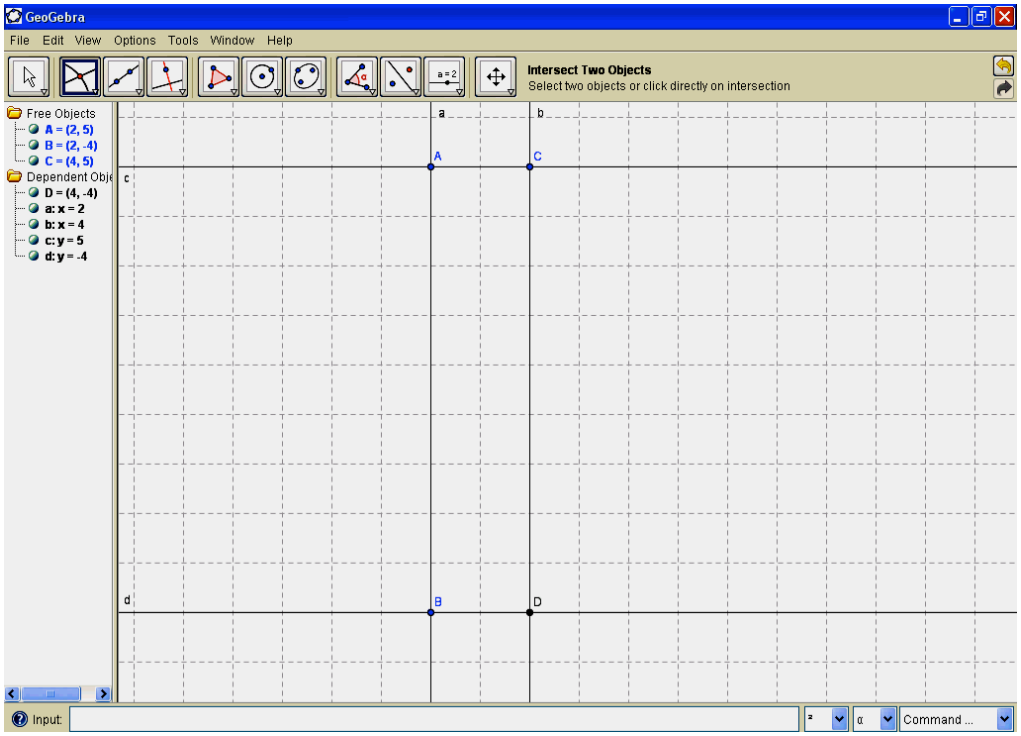
Create line c that goes through points A and C and is parallel to lines a and b.



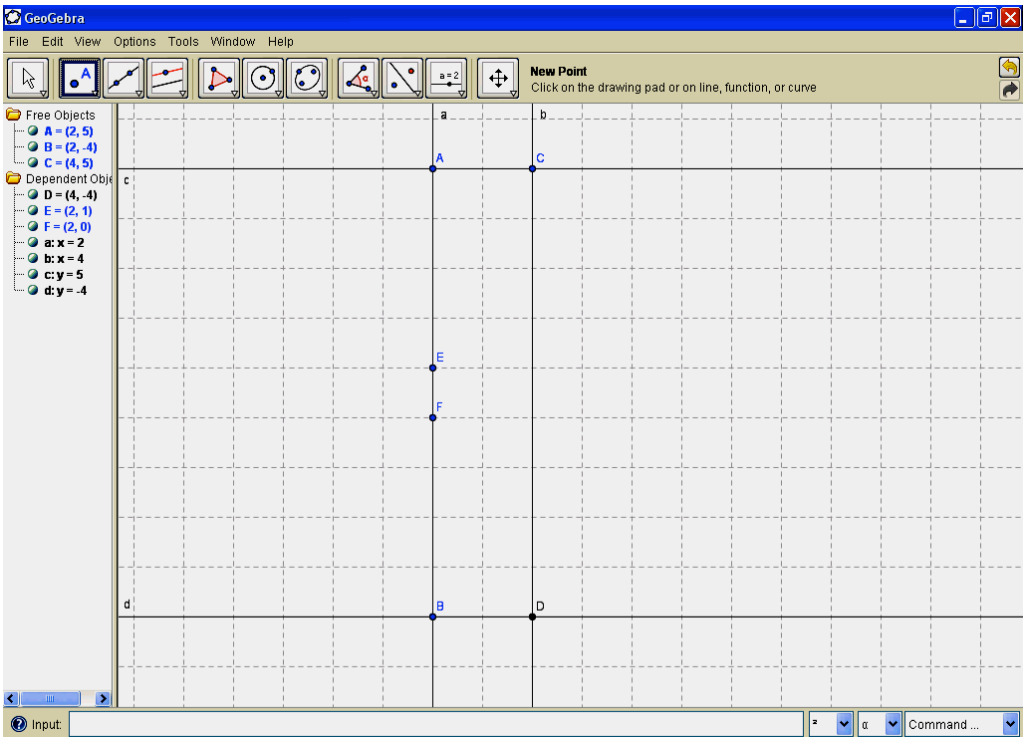
Create line d that goes through point B and is parallel to line c.



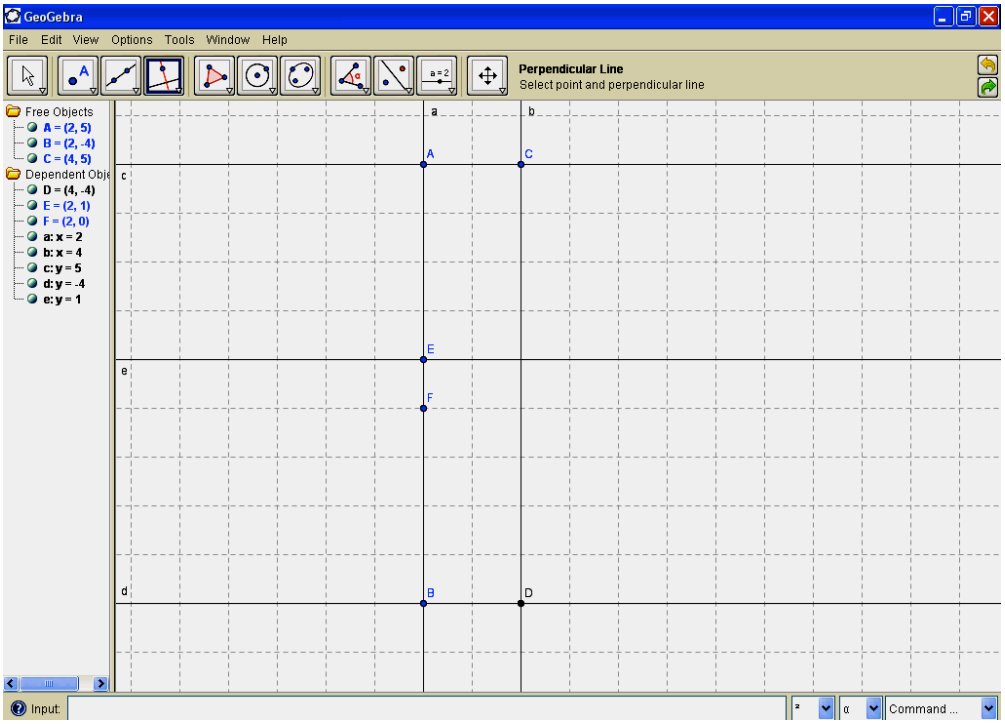
Using the intersection tool (found with the Point tool), label the intersection of lines b and d. Notice the intersection has been labeled as point D.



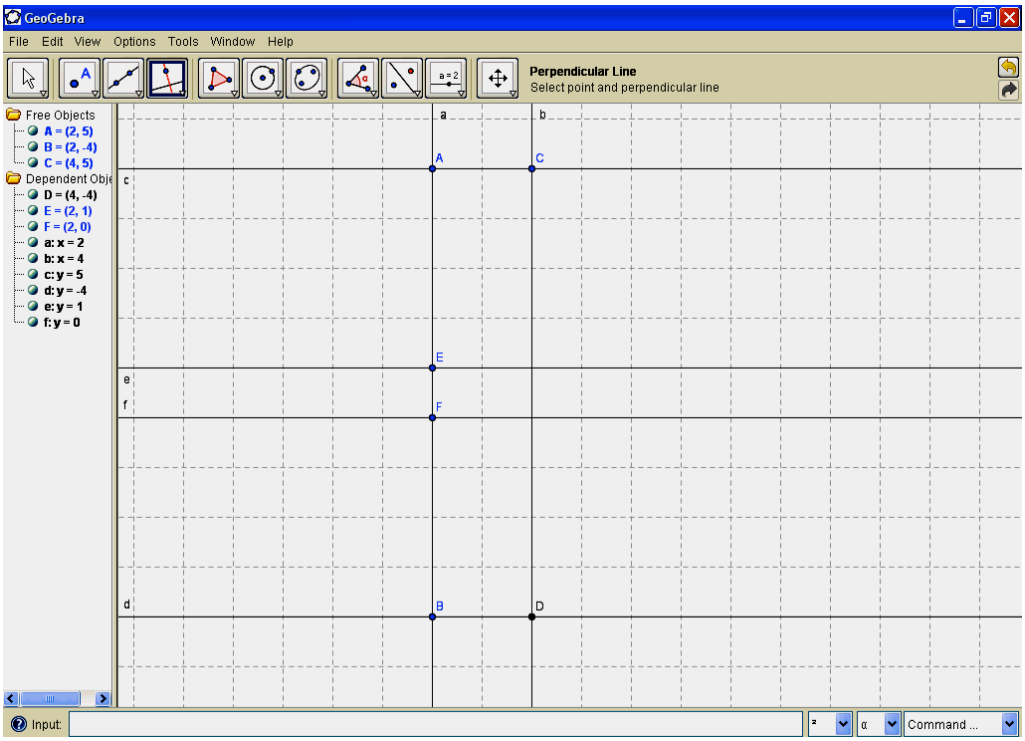
Create points E and F on line a, 4 units from point A and 4 units from point B.



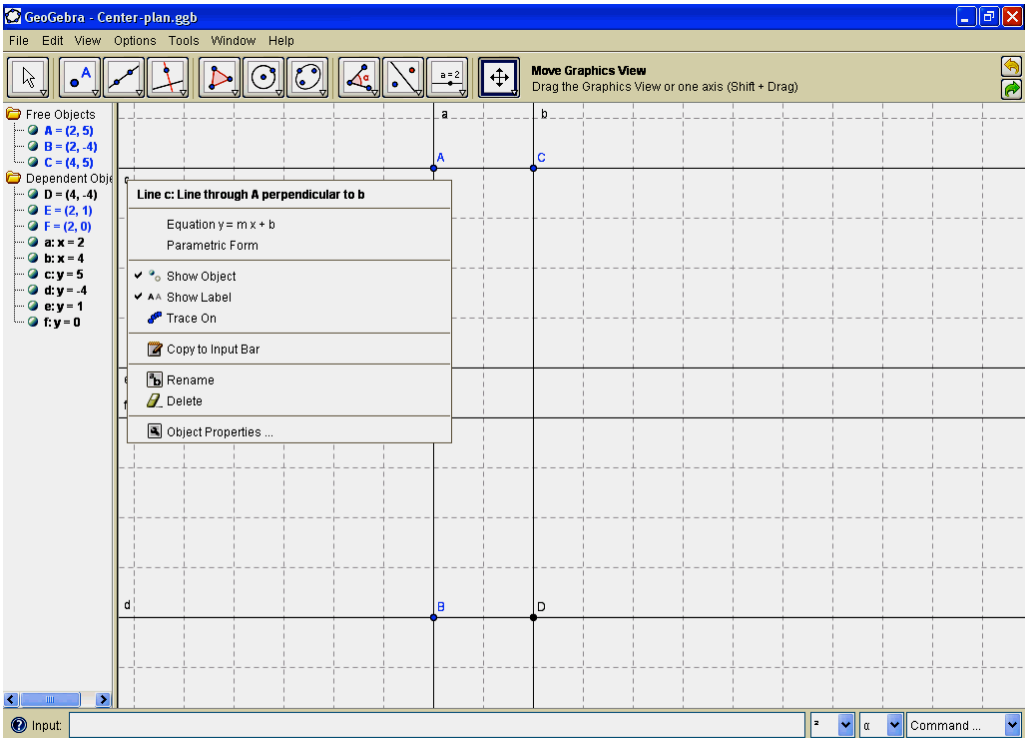
Create line e through point E parallel to line c.



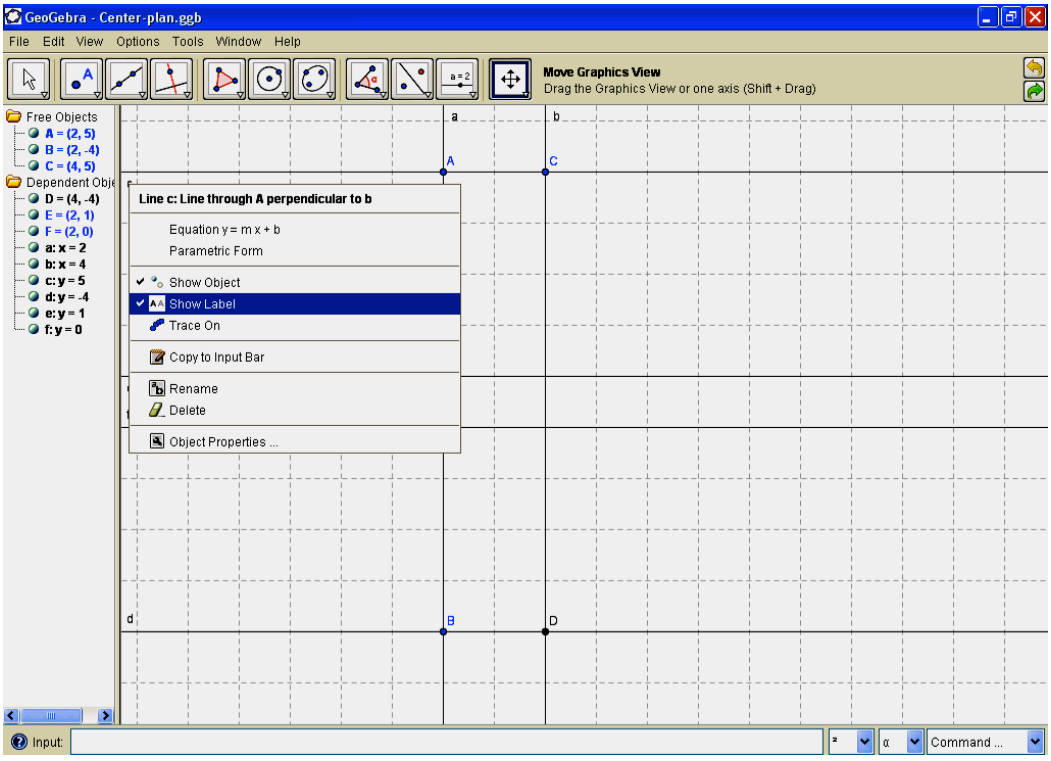
Create line f through point F parallel to line e.



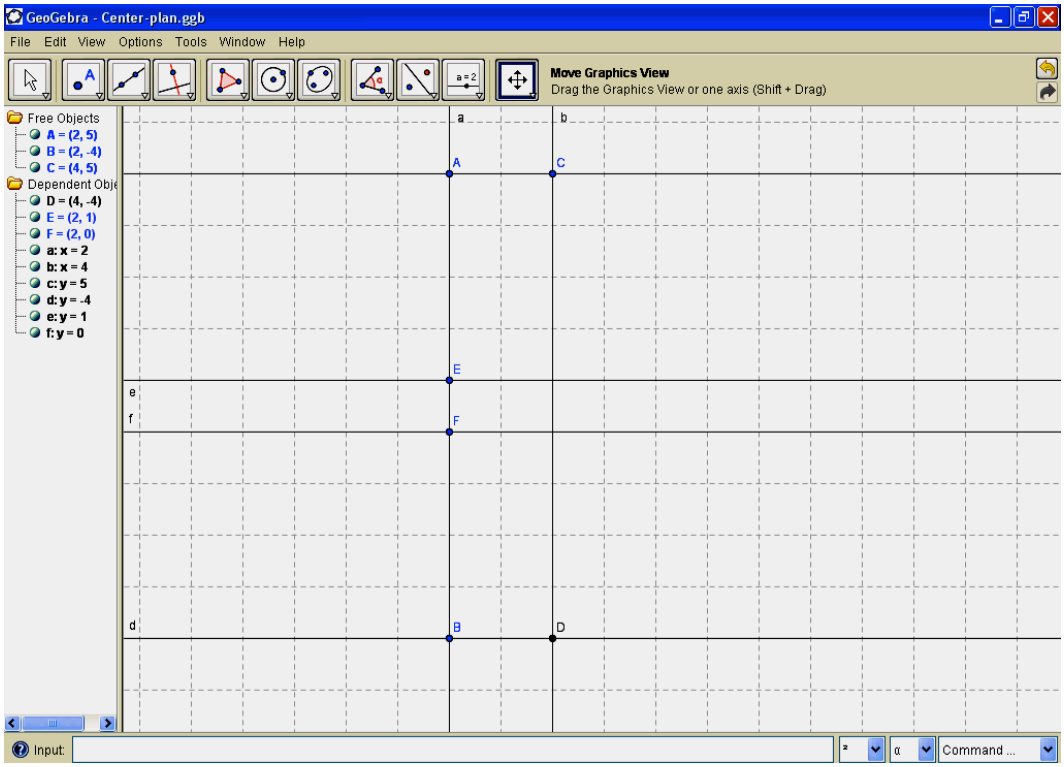
Clean up the diagram a bit by hiding some of the labels.



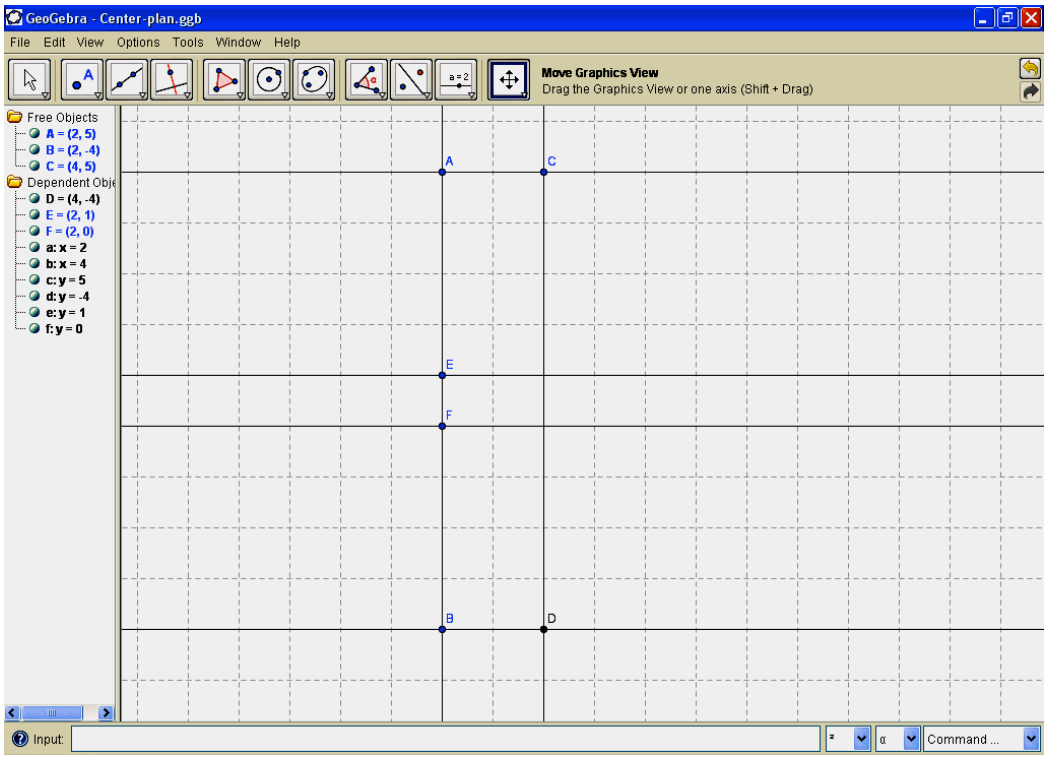
Right click on the label that you want hidden and unclick "Show Label".



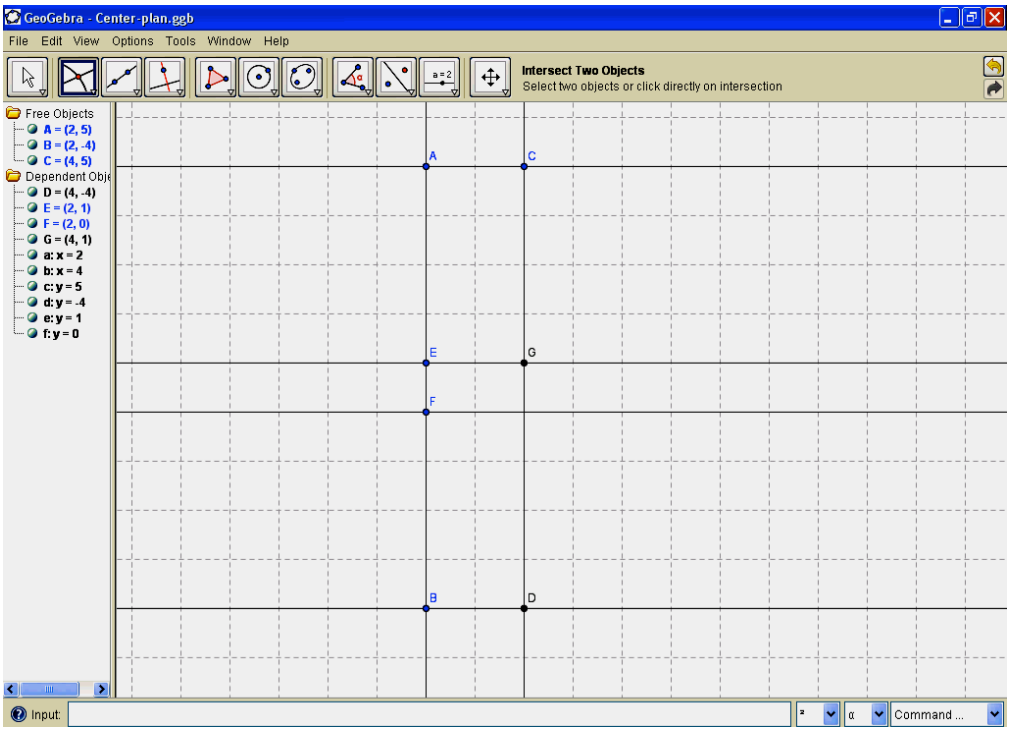
Notice that the label for line c is not showing. Information for line c is on the left under Dependent Objects.



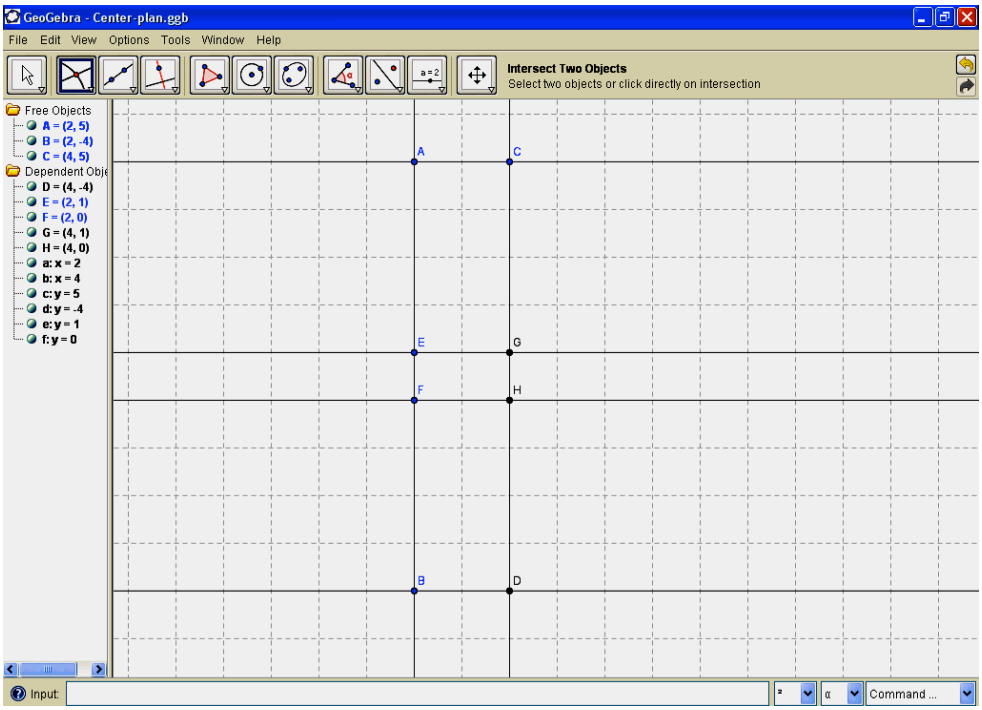
Continue hiding labels.



Using the intersection tool, label the intersection of the line that goes through point C and the line that goes through point E. Notice the point of intersection is labeled as point G.

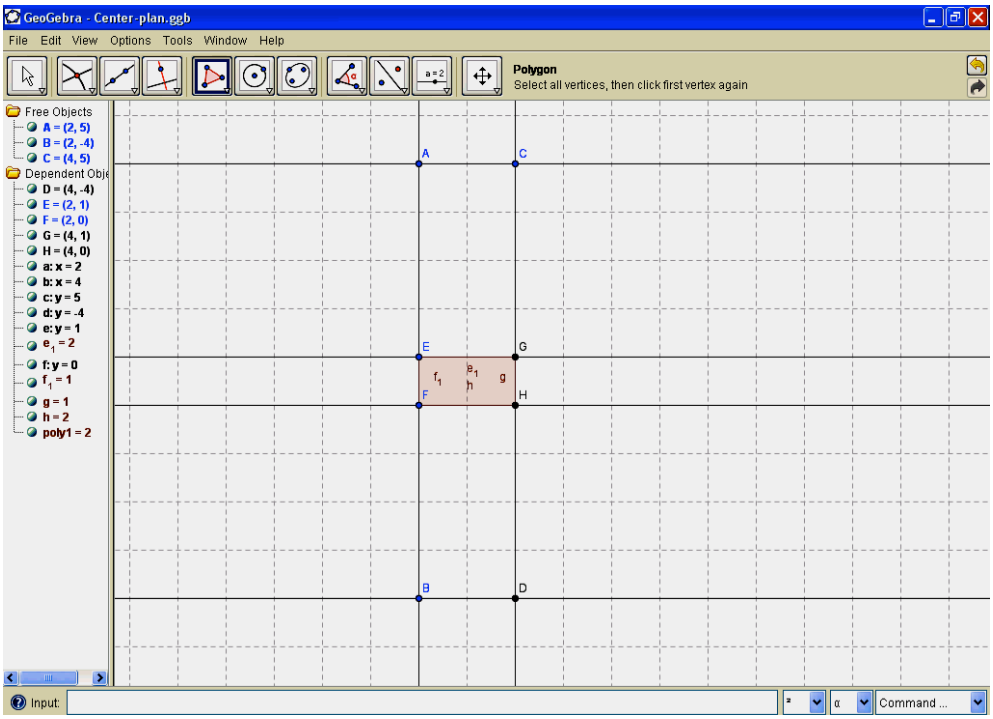


Repeat the process for the line through point G and the line through point F. The point of intersection is labeled as point H.

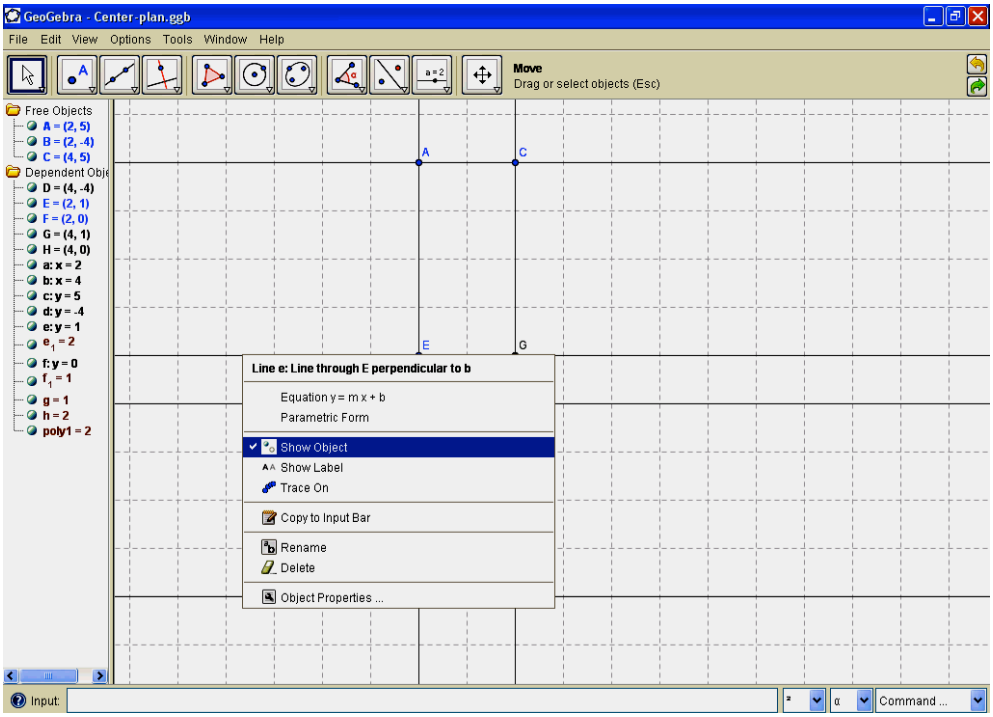


Parallel Lines, Transversals, and Angles: What's the connection?

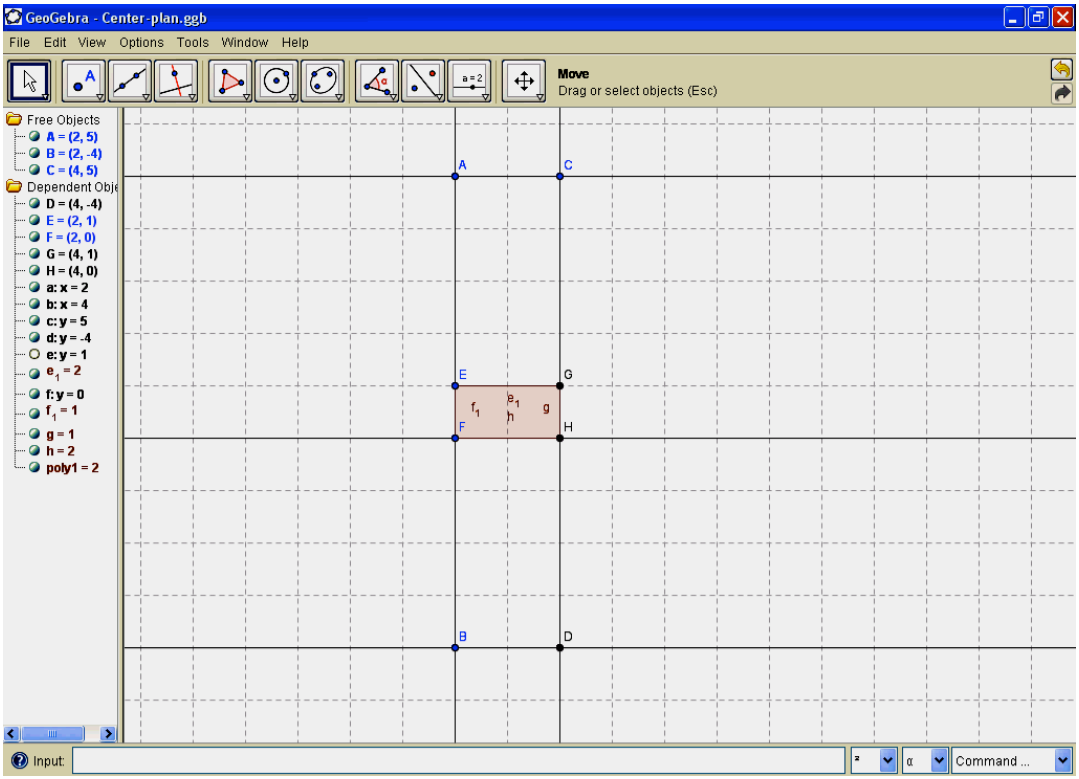
Using the polygon tool, create quadrilateral (rectangle) EGHF. Remember to click each point and then click the first point again. Notice that the rectangle is shaded in. This will represent "green space" in our diagram.



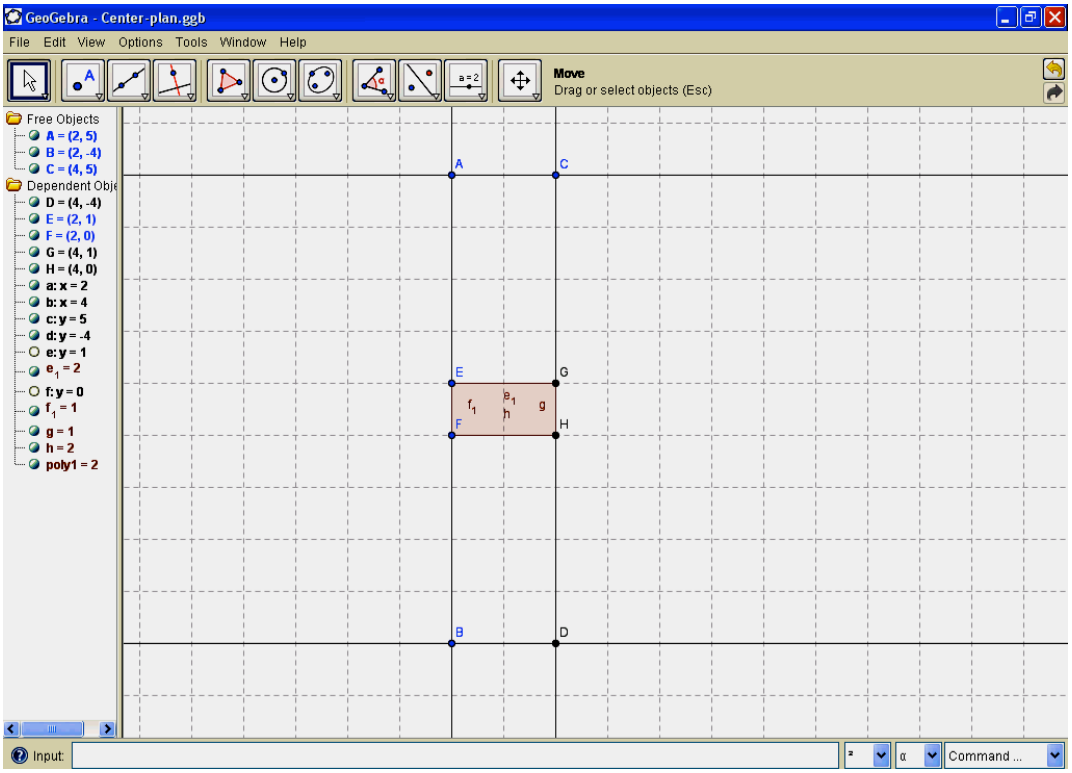
Now that we have created a polygon we can hide some of the lines in our diagram. Right click on any object you want to hide.



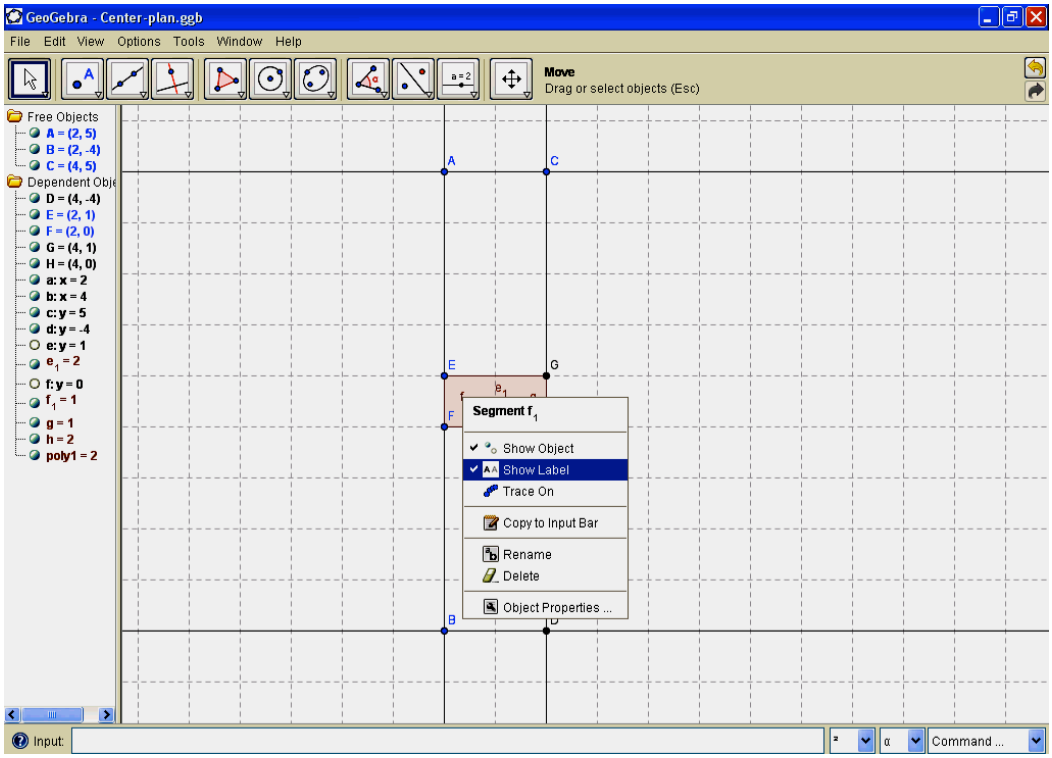
Notice, the line going through points E and G is hidden.



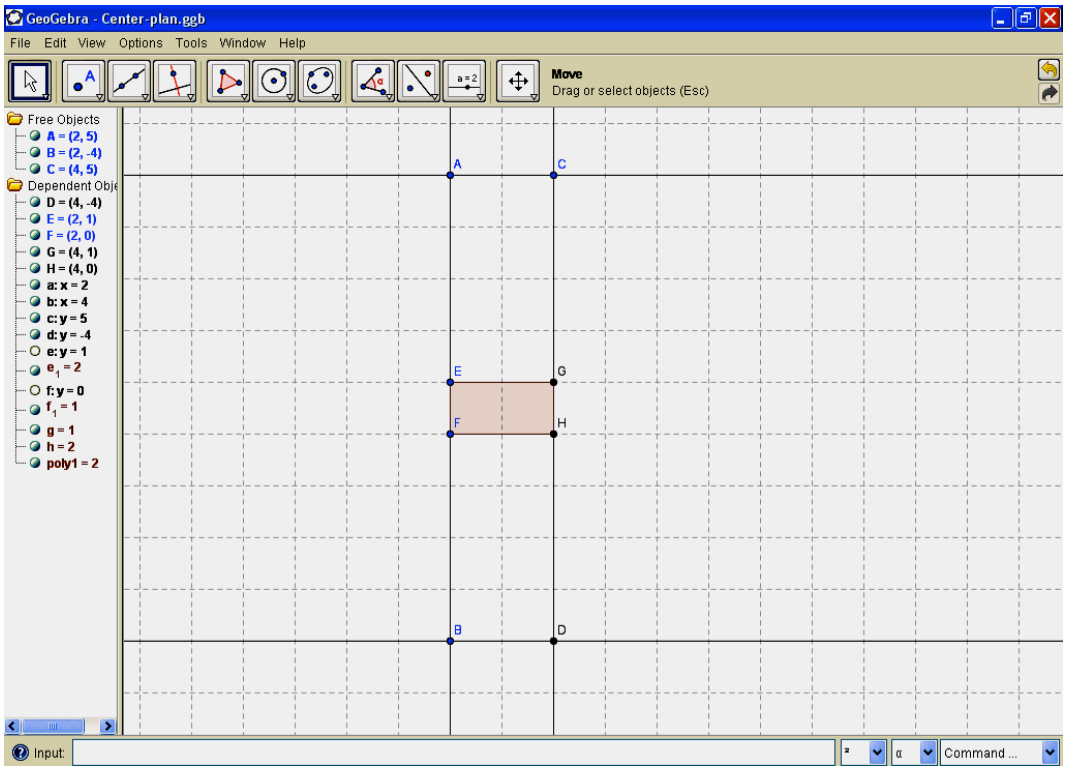
Repeat the process for the line passing through points F and H.



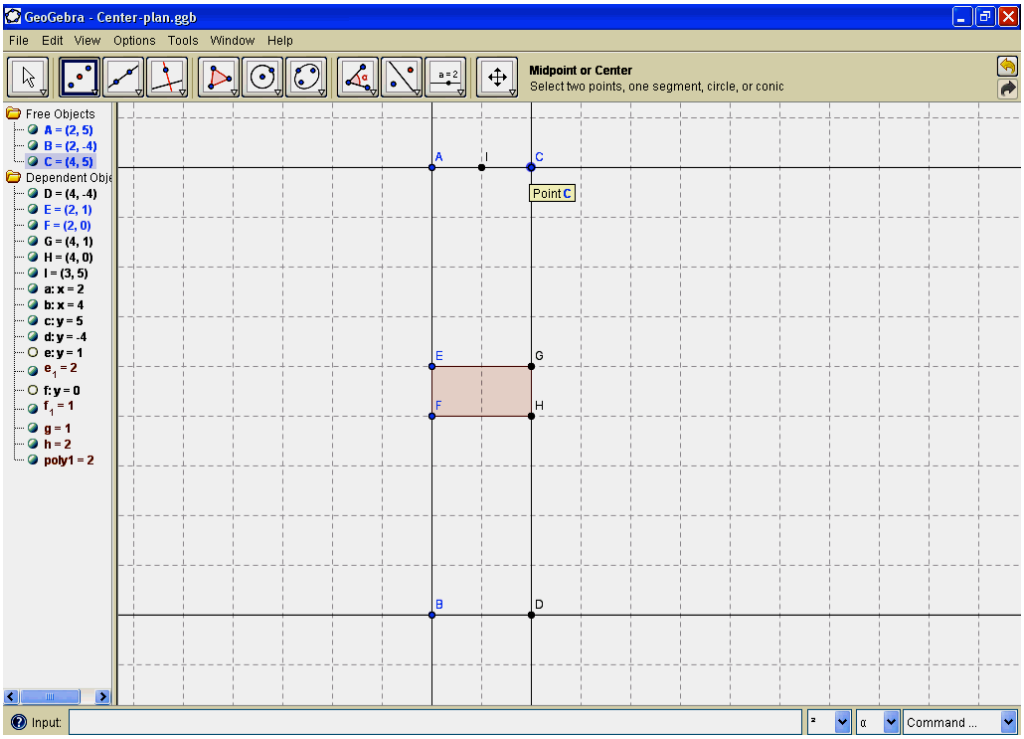
Clean up the diagram a bit by hiding more labels.



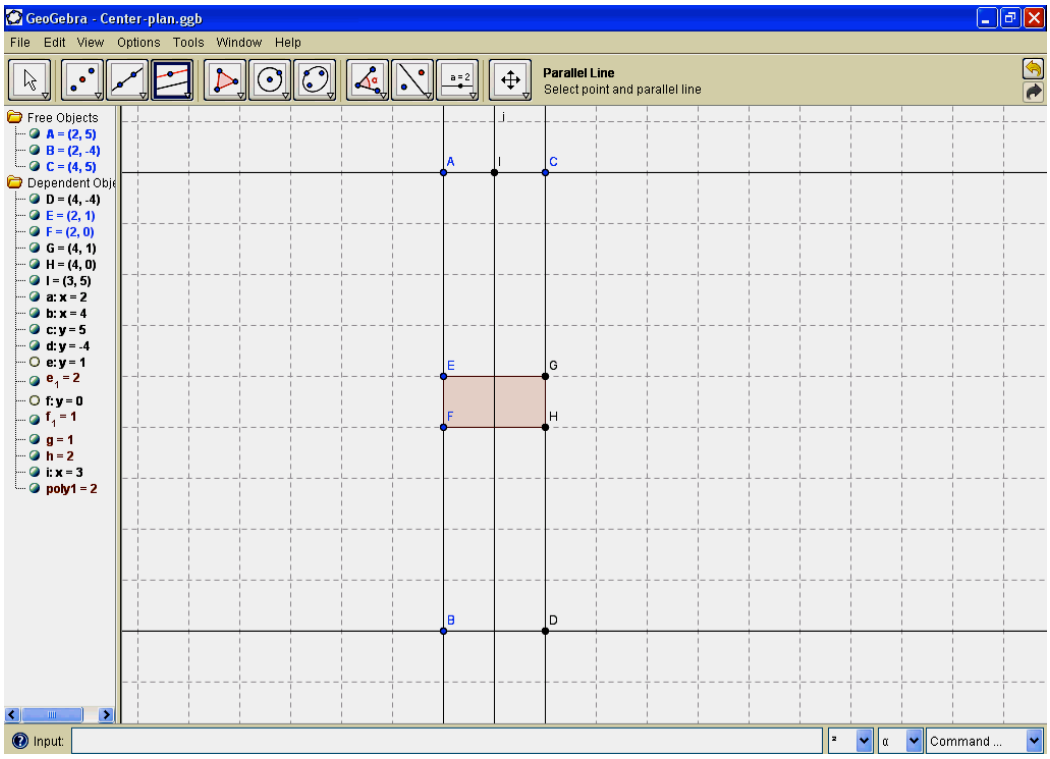
Notice the polygon labels are hidden.



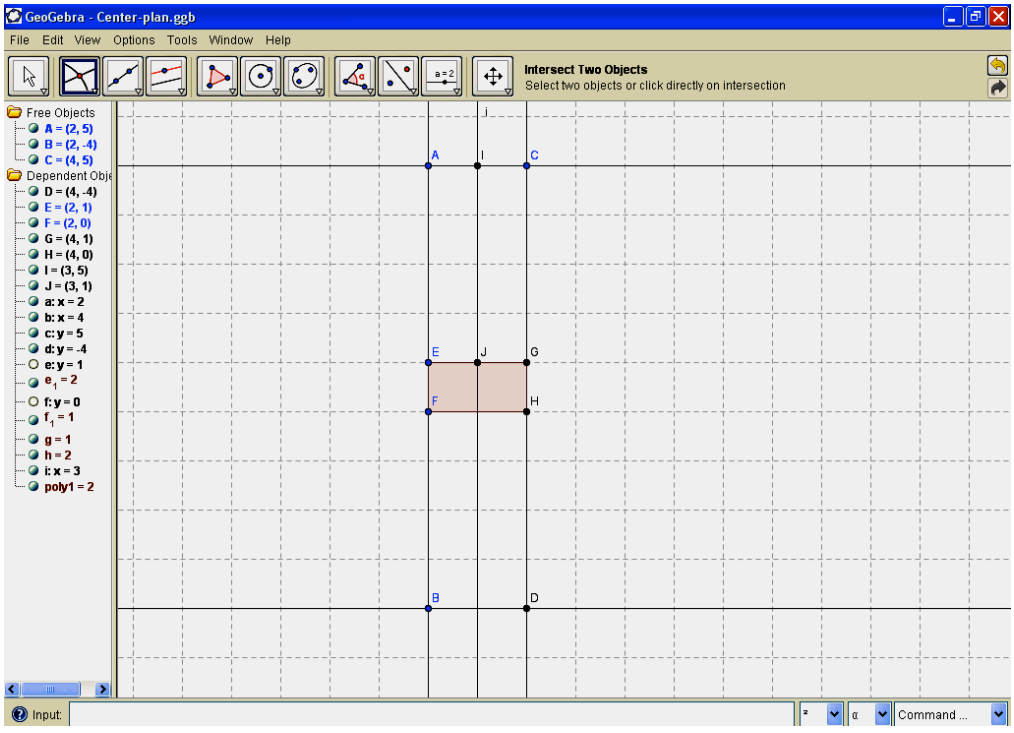
Using the midpoint tool (found on the Point tool) create the midpoint of segment AC. Notice point I is the midpoint of segment AC.



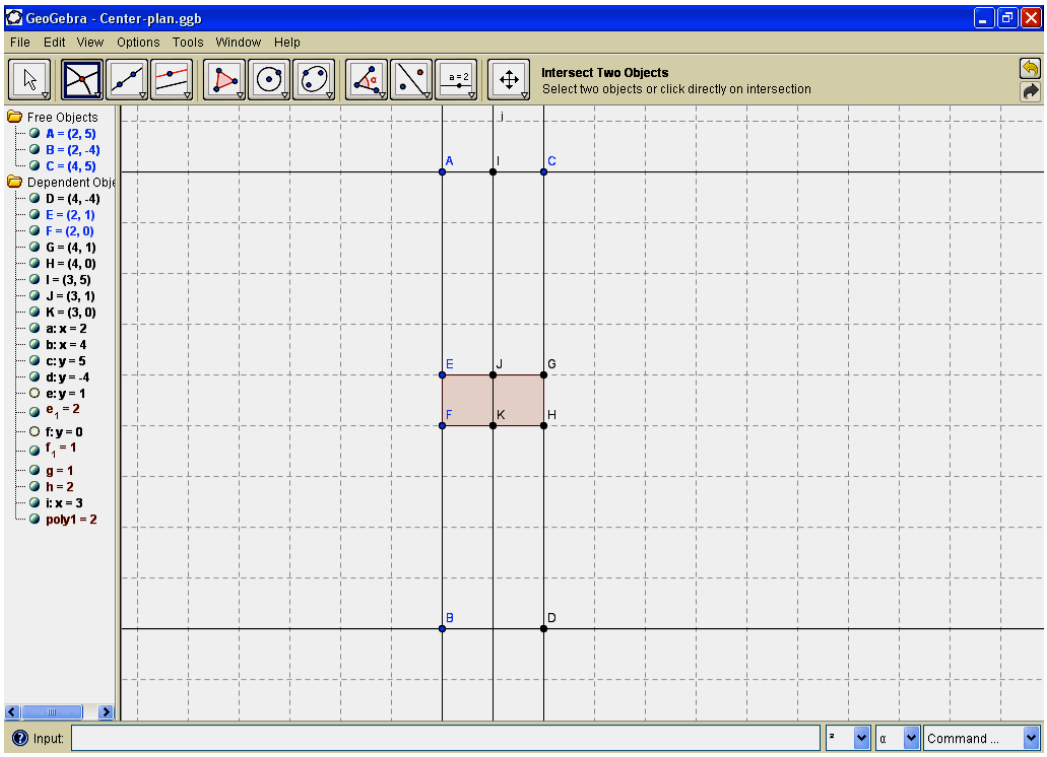
Create a line through point I parallel to the line passing through points C and G.



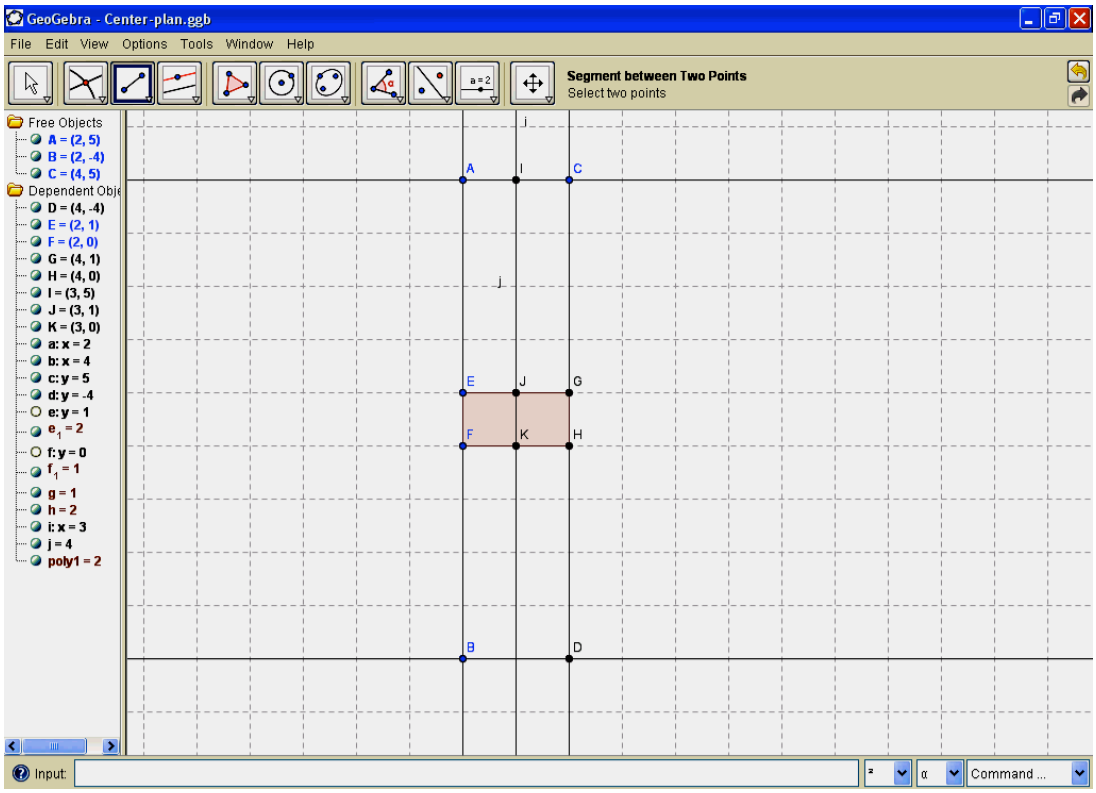
Using the intersection tool, find the intersection of line j and segment EG. Notice the intersection has been labeled as point J.



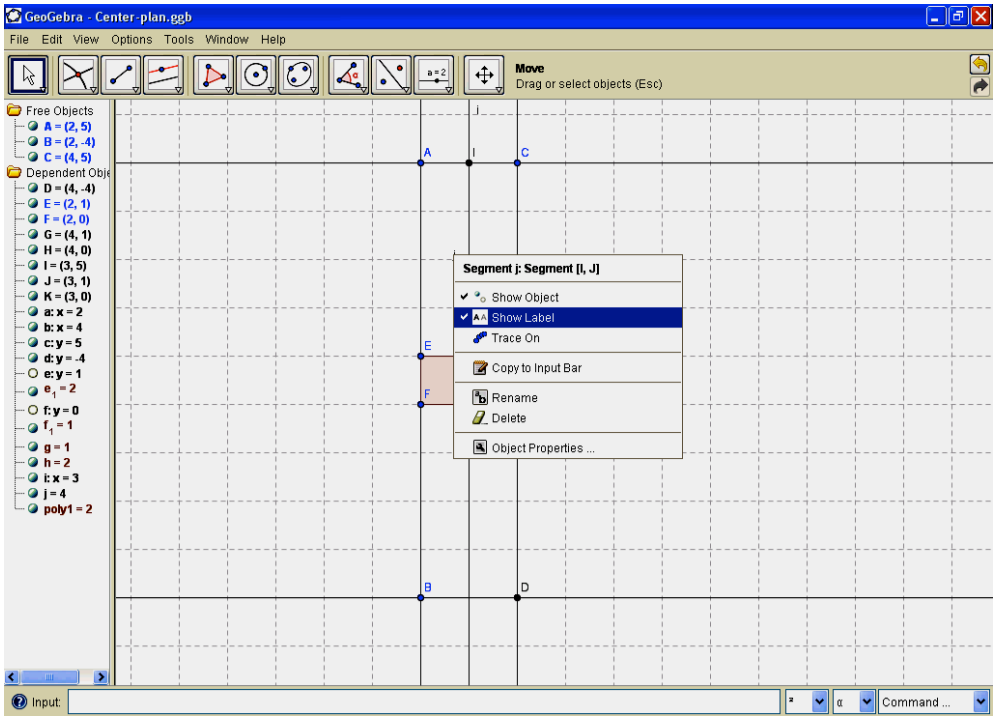
Repeat the process to find the intersection of line J and segment FH. Notice the intersection is labeled as point K.



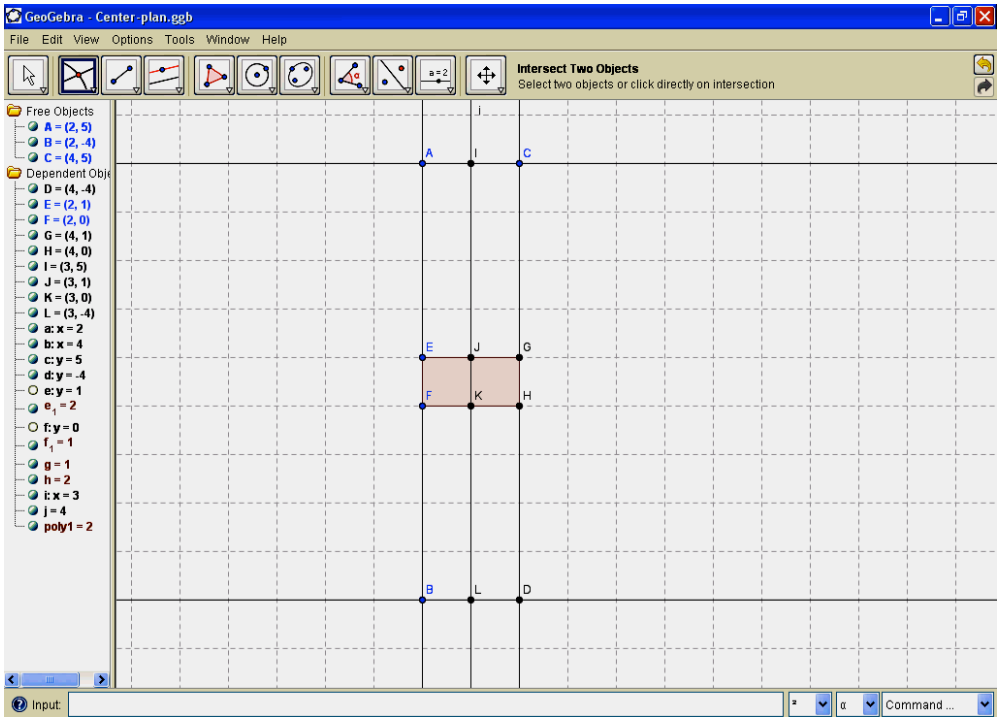
Using the Segment tool, create segment IJ. This will allow us to hide the unnecessary portions of line J.



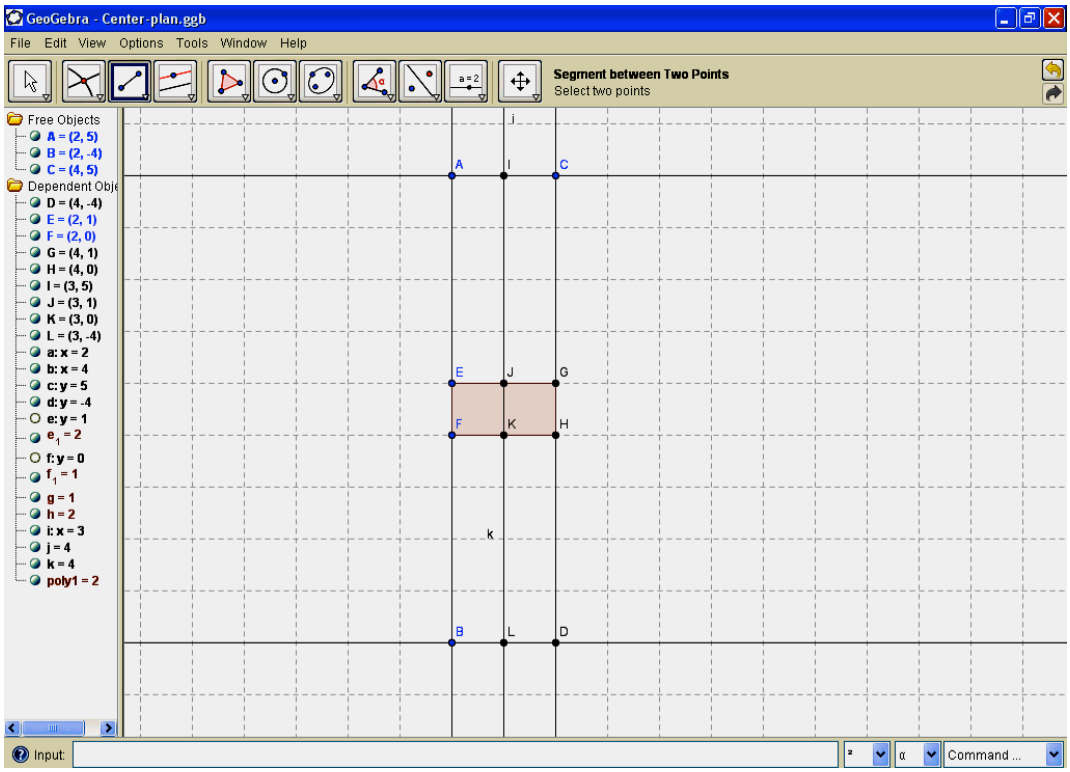
Hide the label for segment IJ.



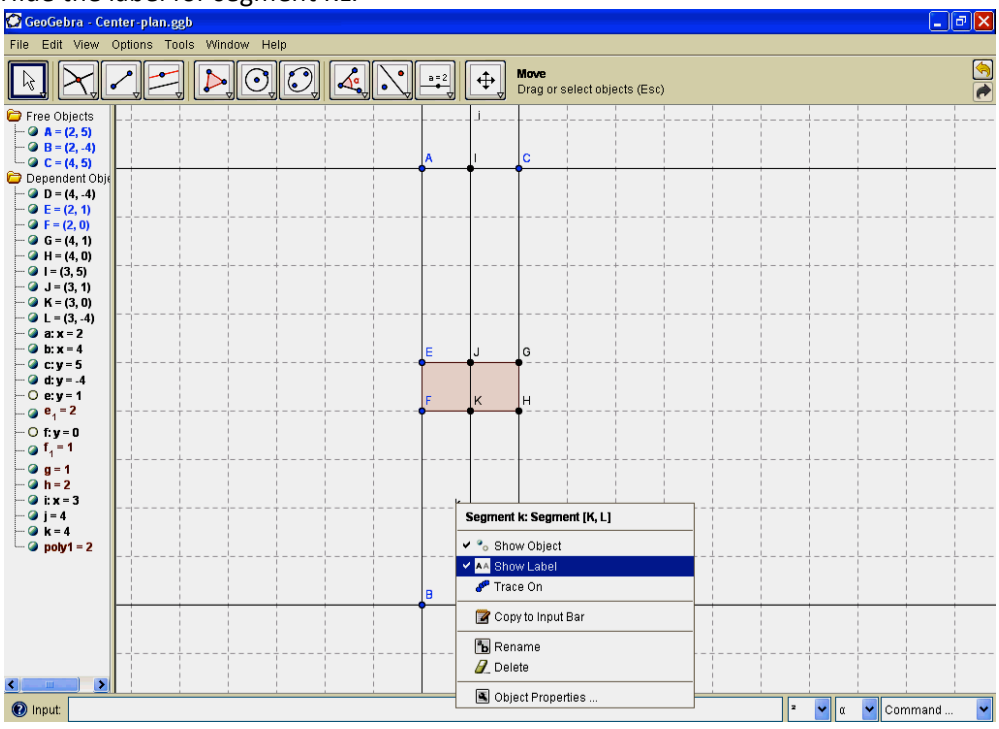
Notice the label for segment IJ is hidden.



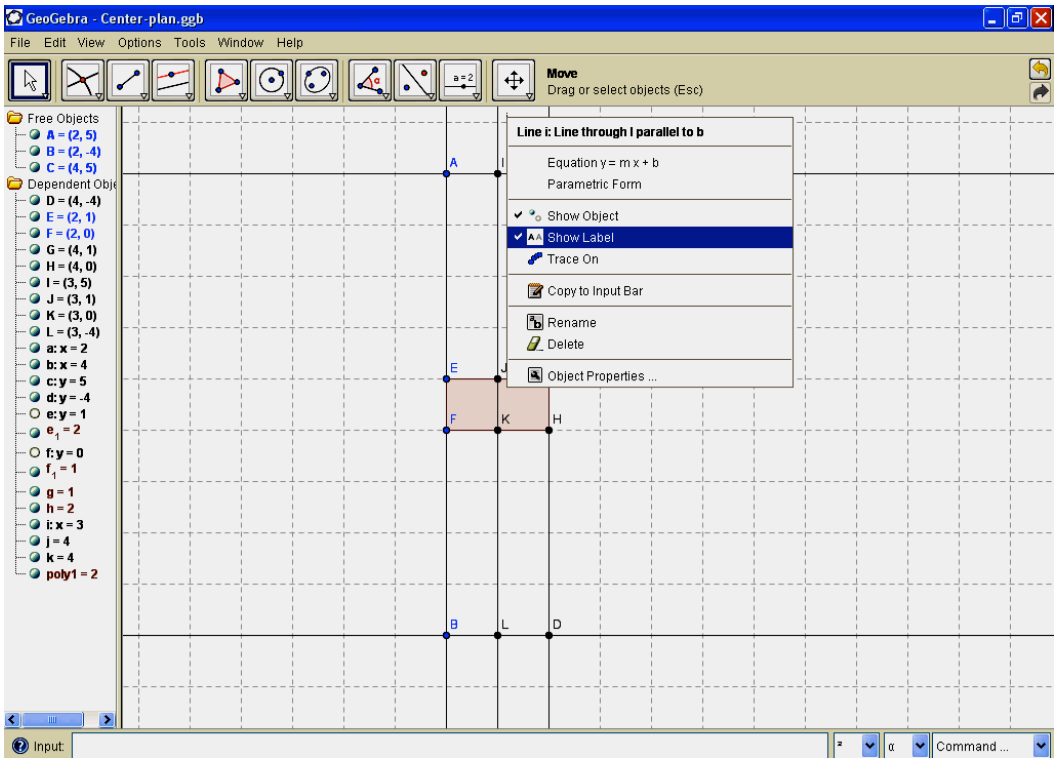
Using the segment tool, create segment KL.



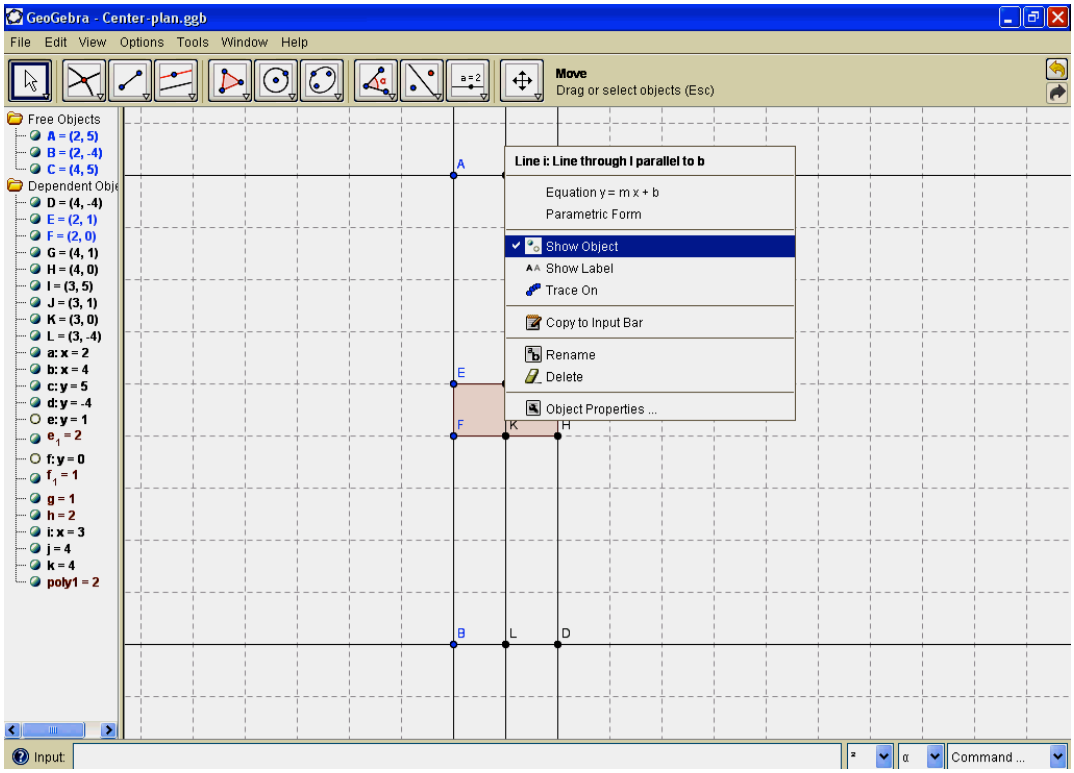
Hide the label for segment KL.



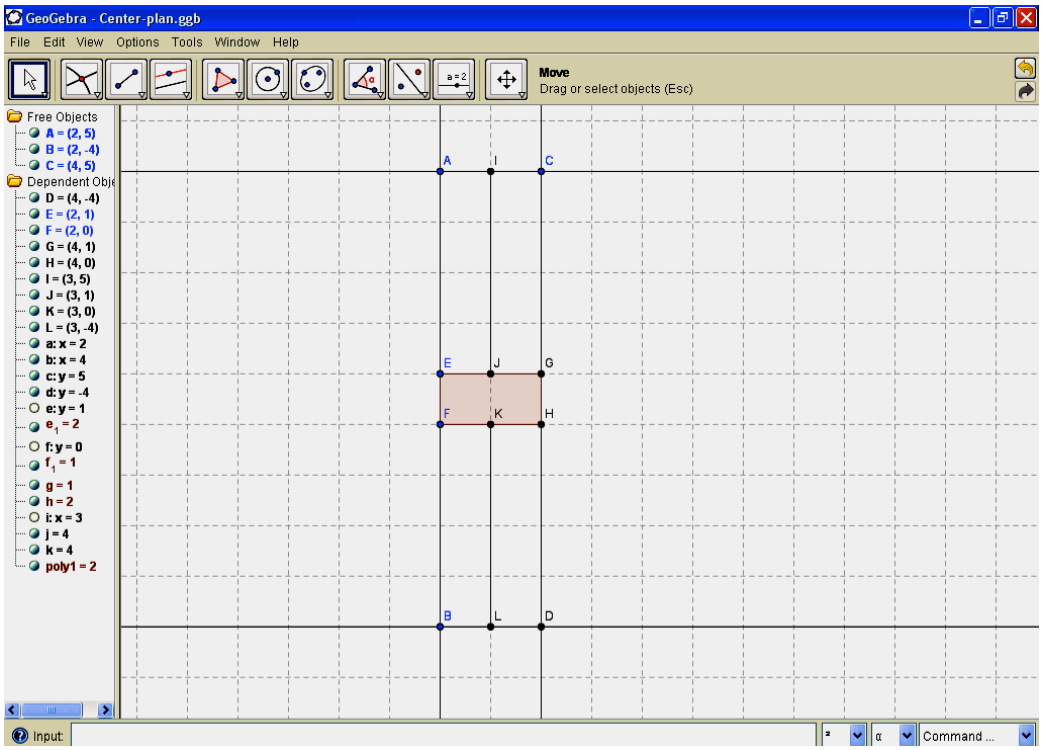
Continue to hide labels.



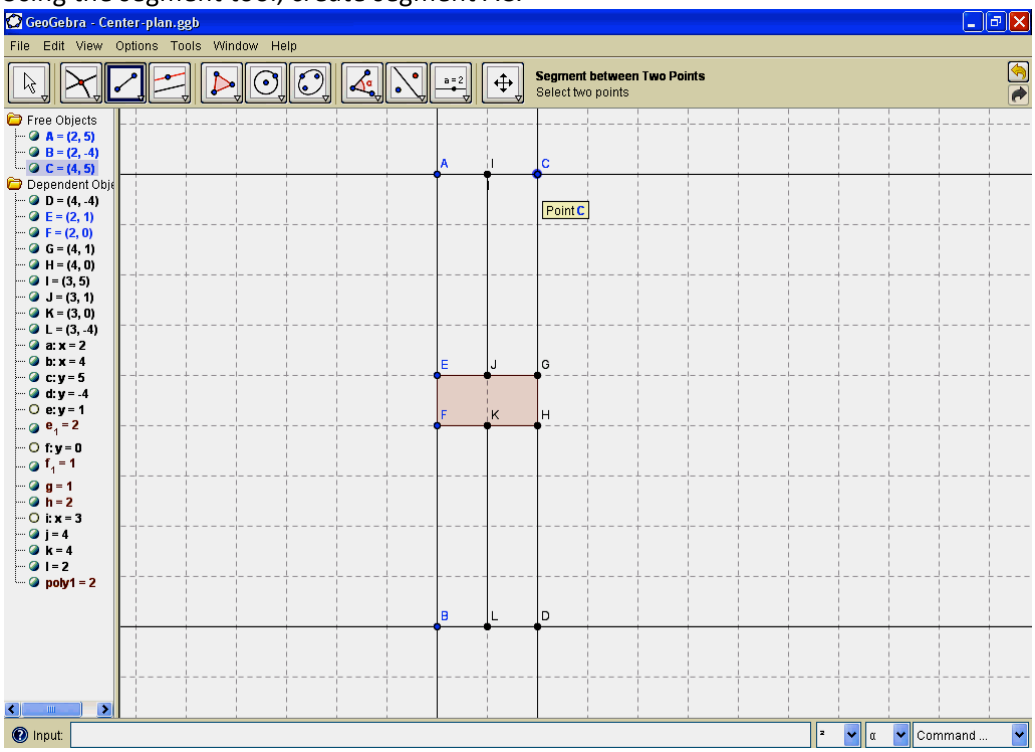
Once the segments have been created, we can hide line j and the needed portions will still be visible on our diagram.



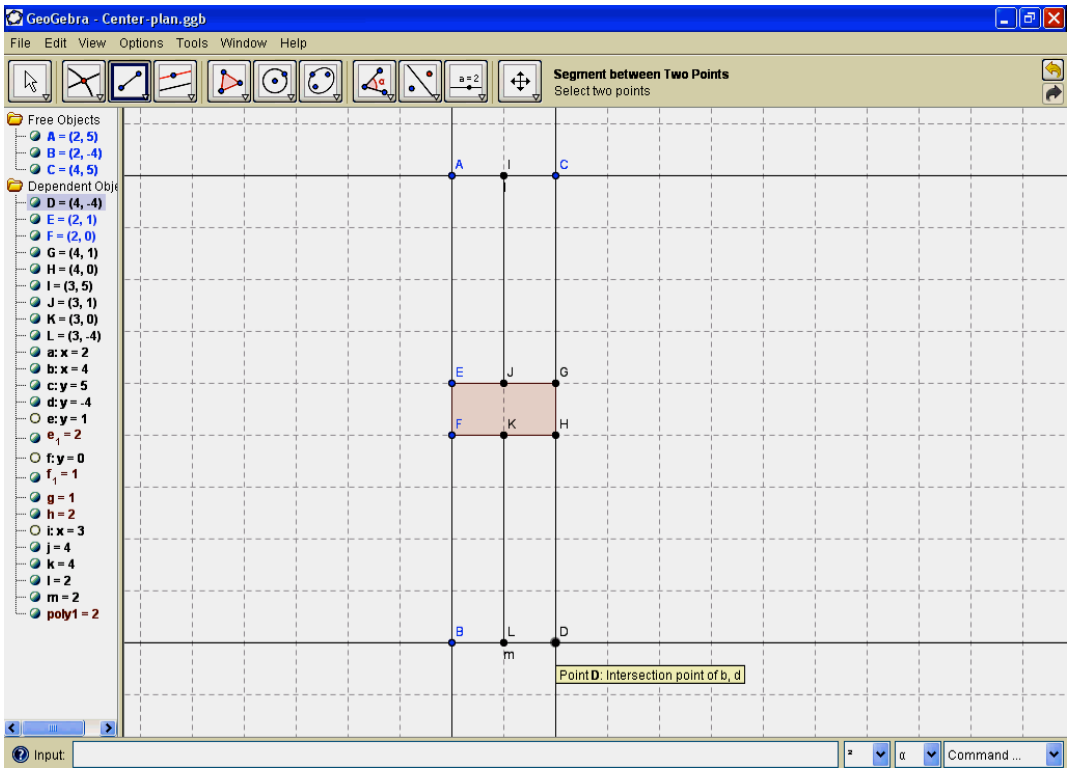
Notice line J is hidden.



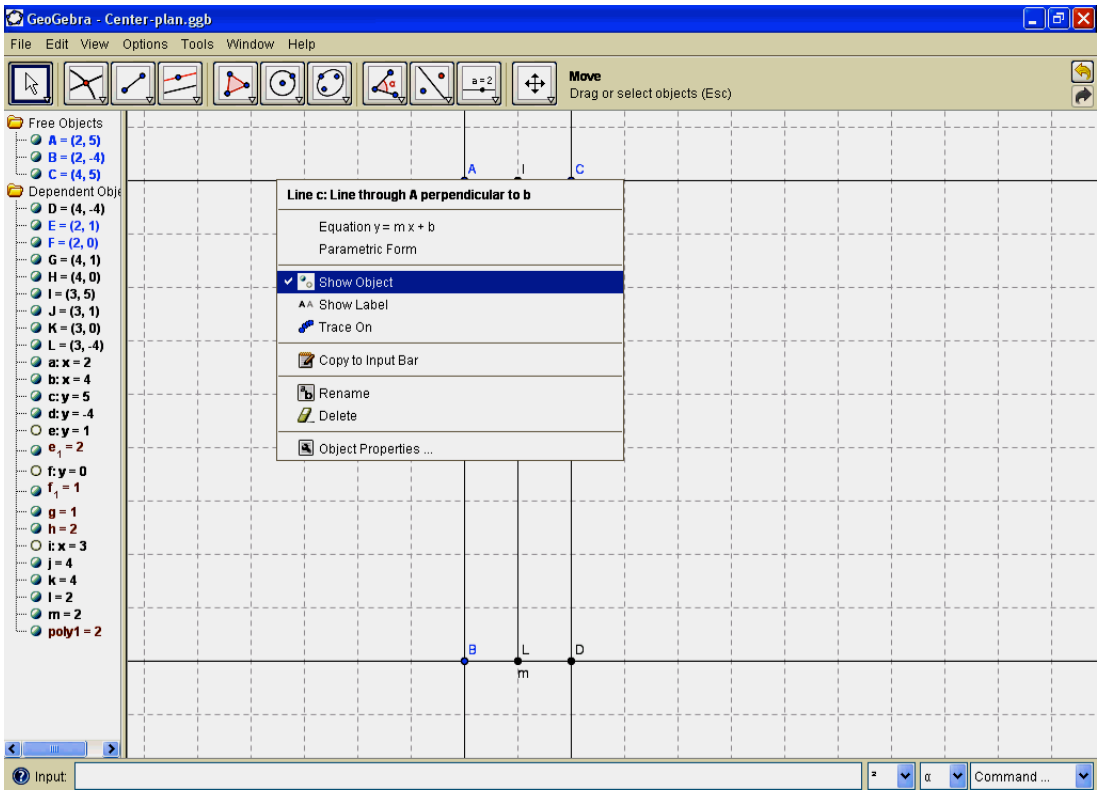
Using the segment tool, create segment AC.



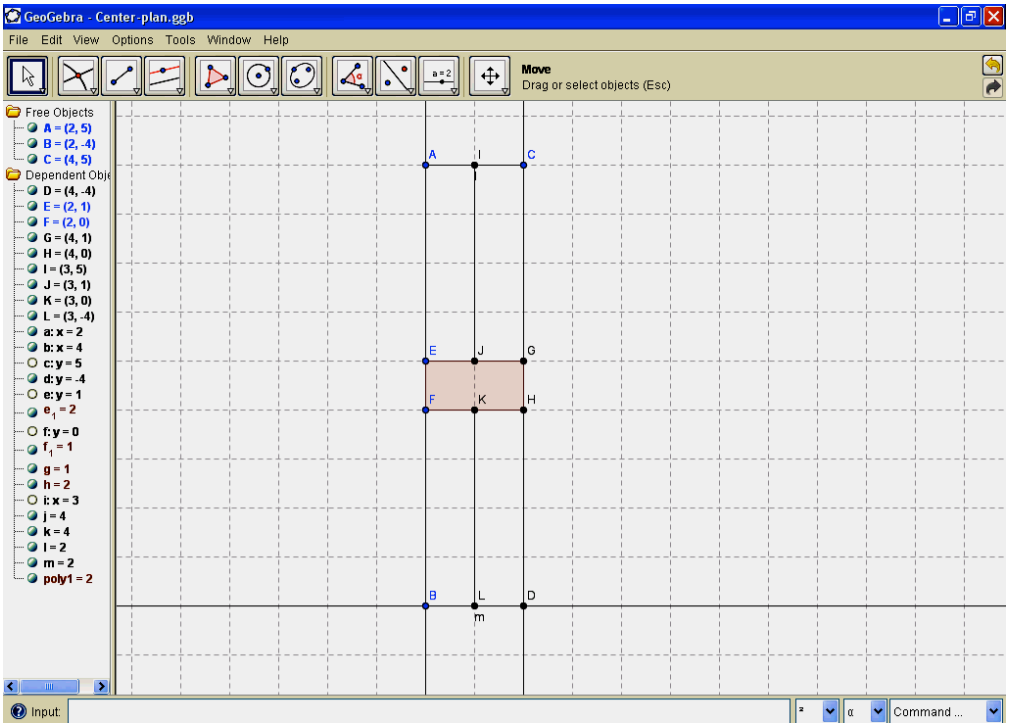
Segment AC has been labeled as I. The label for the segment is under point I and is difficult to see. When hiding the label for segment AC it will take a little patience to make sure you hide the right label.



Right click on the line passing through AC and hide the object.

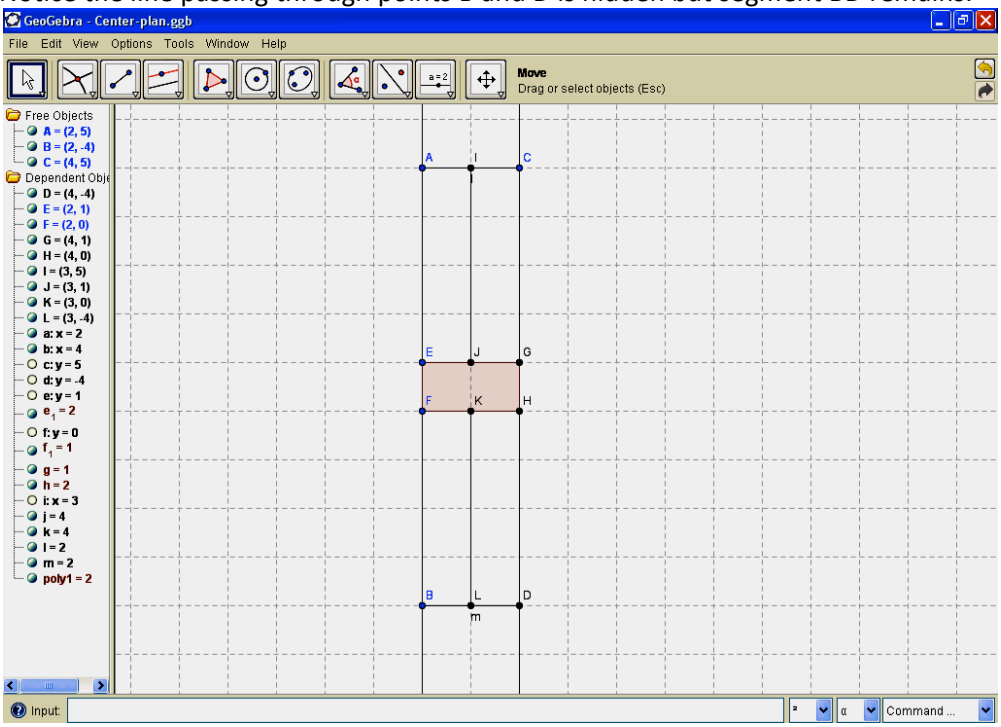


Notice, only segment AC remains.

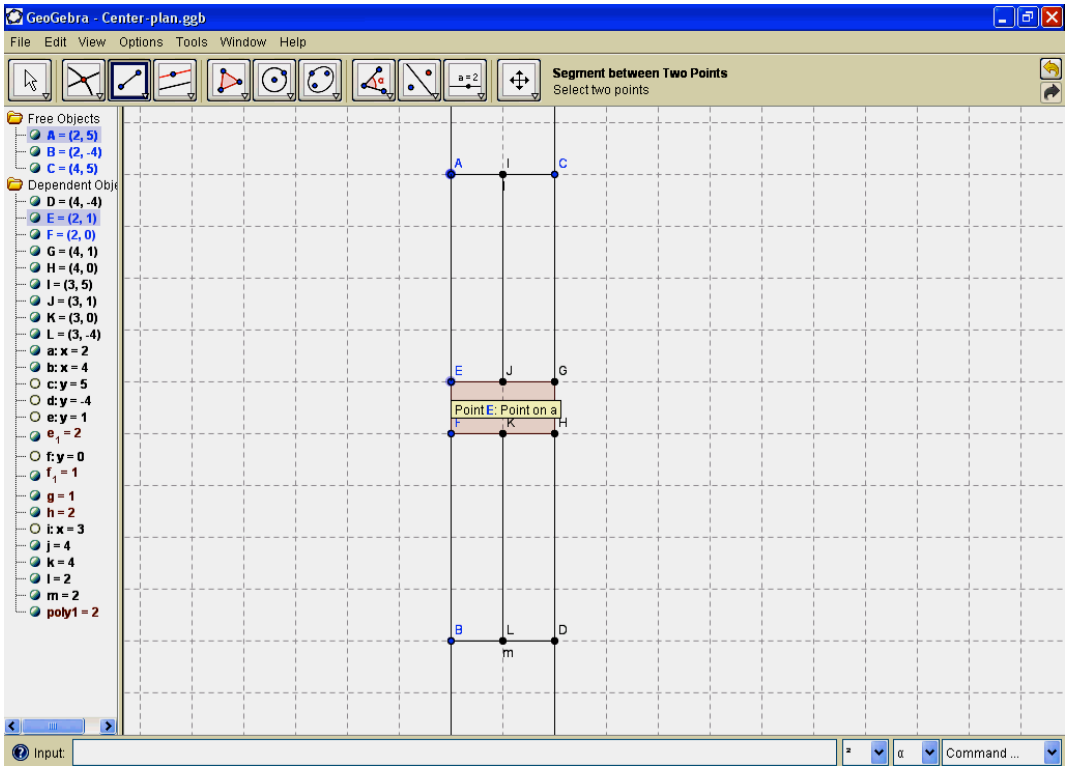


Continue the process of creating segments and hiding lines until only the needed portions remain.

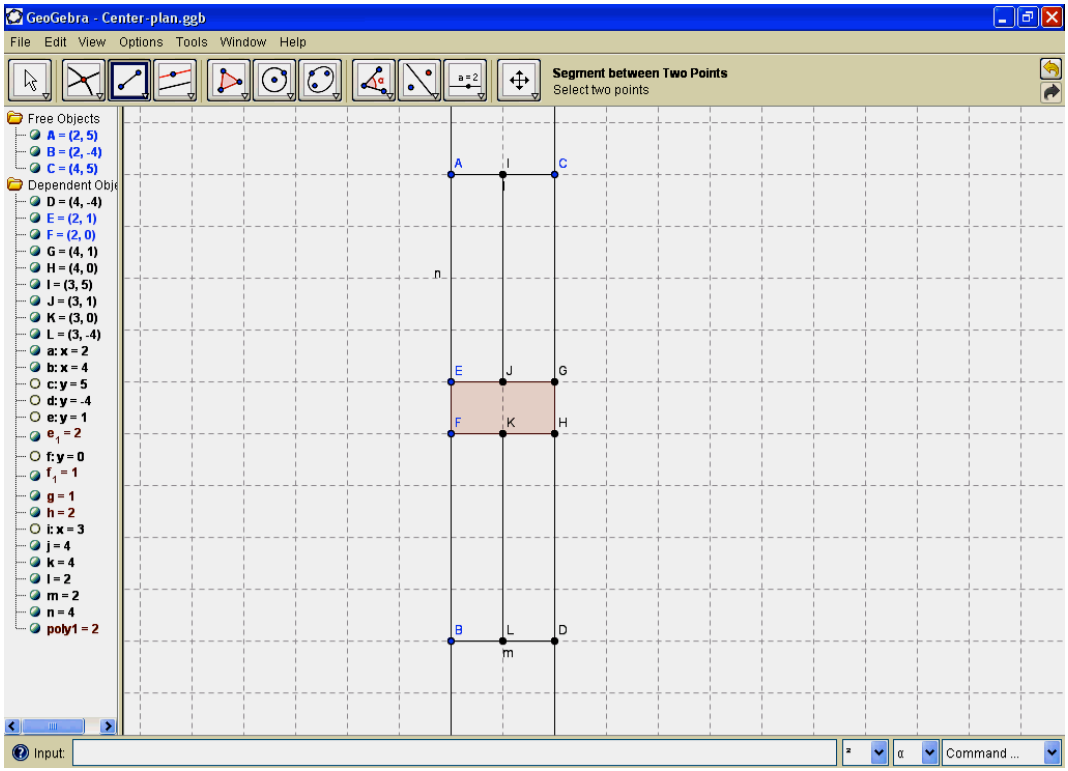
Notice the line passing through points B and D is hidden but segment BD remains.



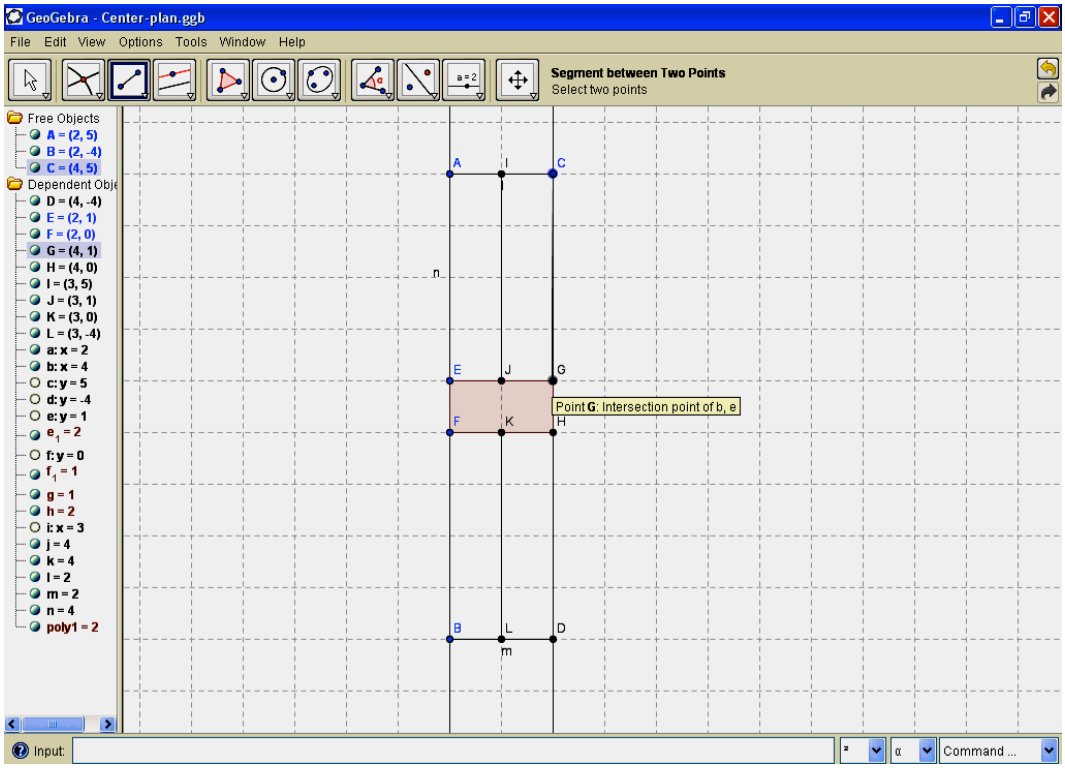
Using the segment tool, create segment AE.



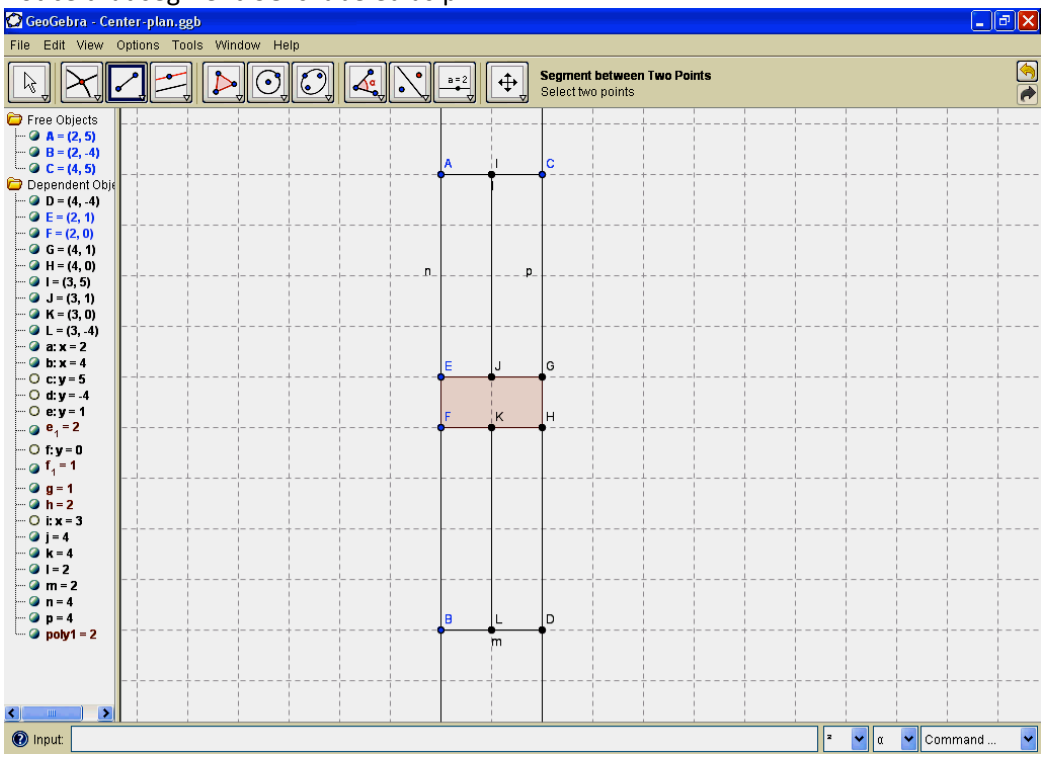
Notice that segment AE has been labeled n.



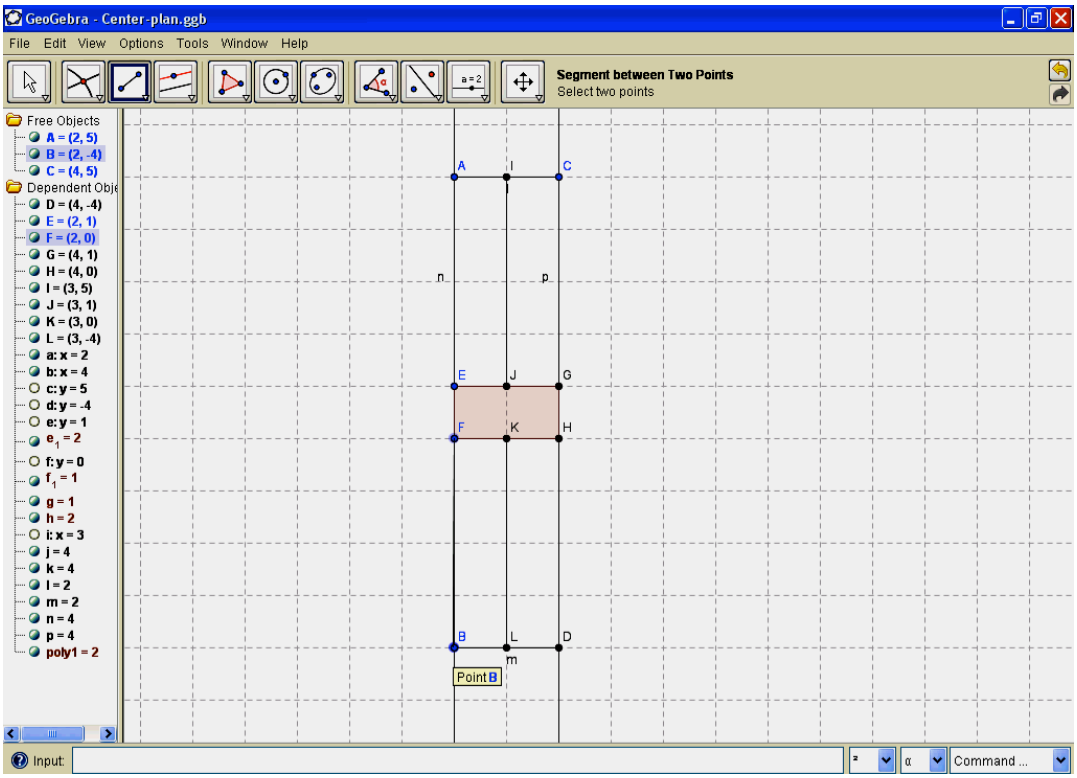
Create segment CG.



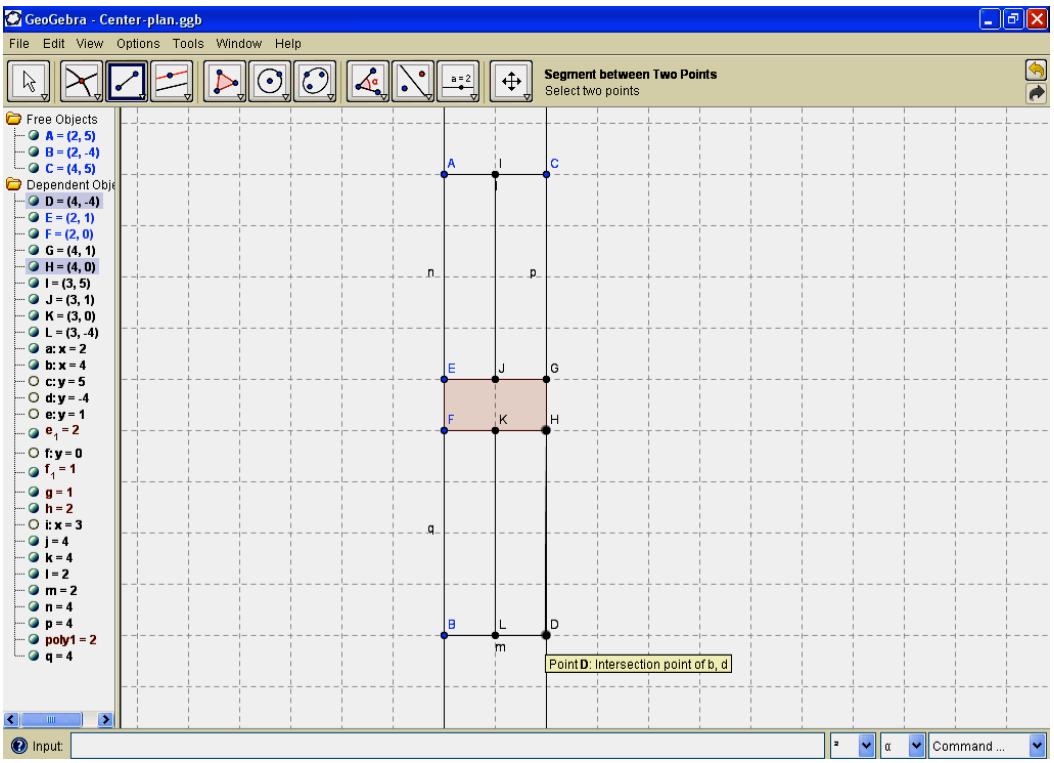
Notice that segment CG is labeled as p.



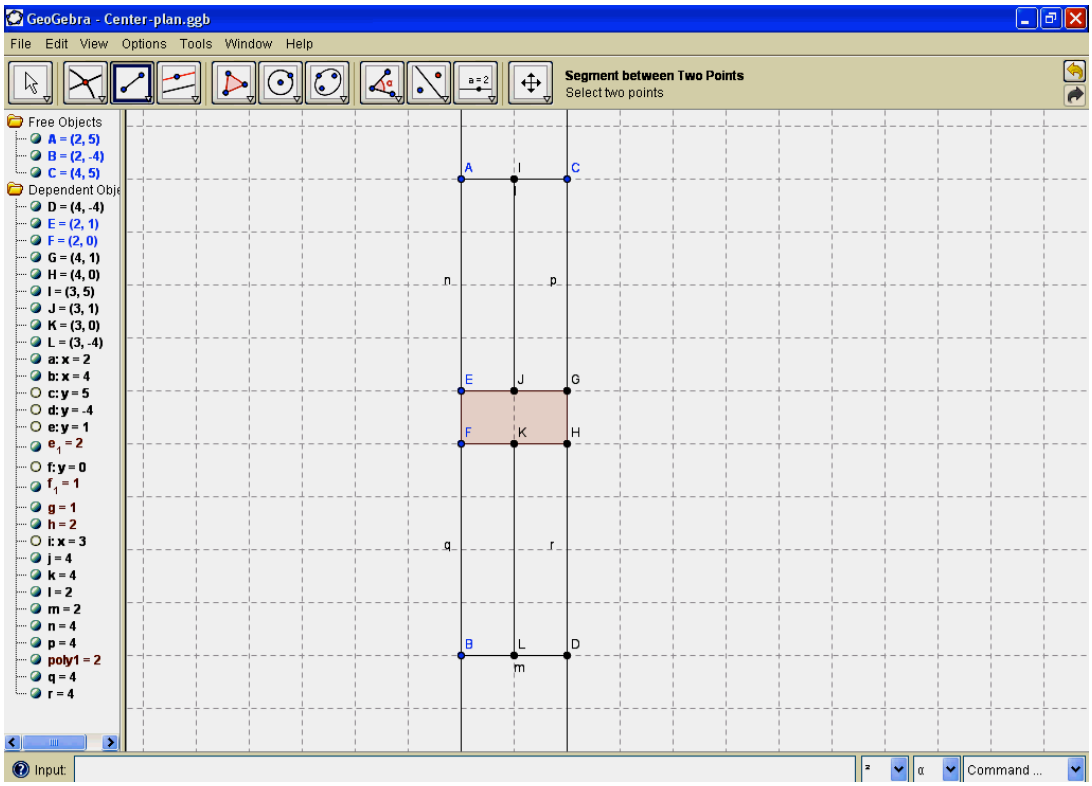
Create segment FB.



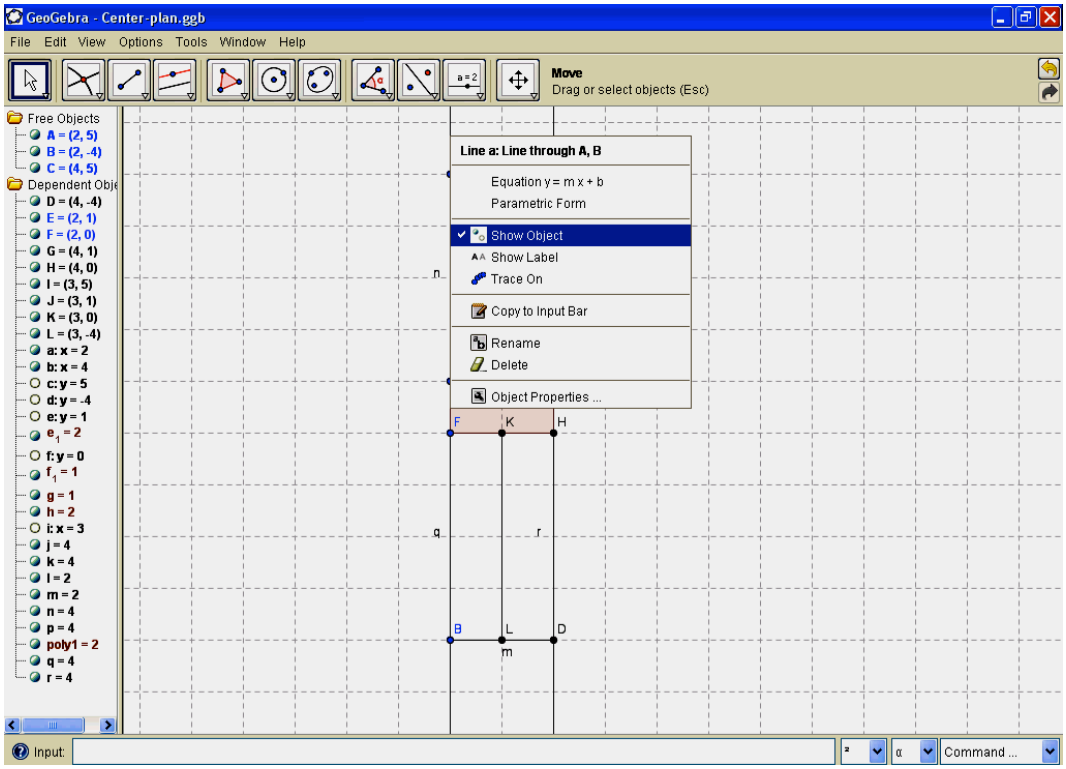
Create segment FB. Notice that segment FB has been labeled q.



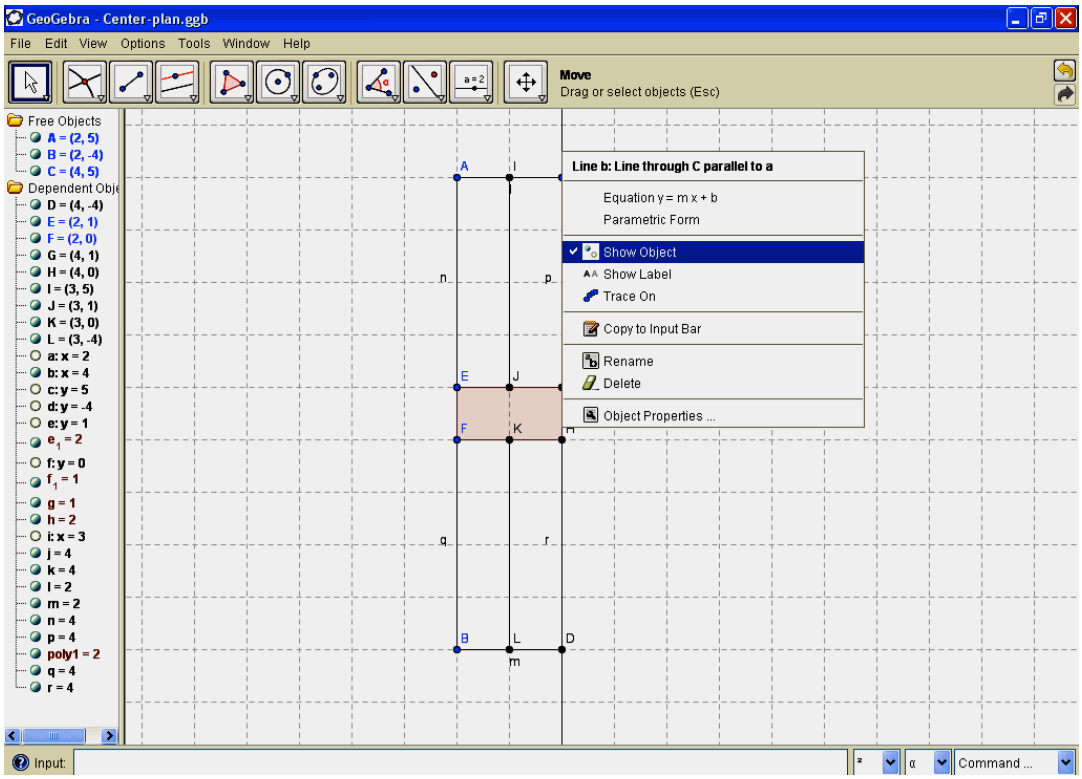
Create segment HD. Notice segment HD has been labeled r.



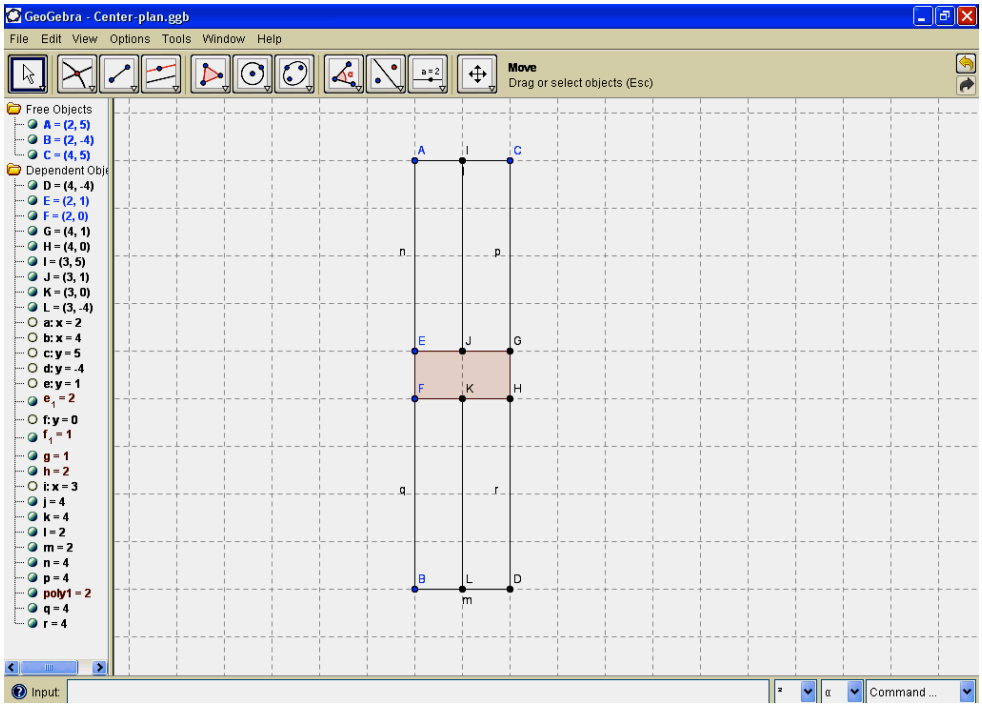
Hide the line extending through points A and B.



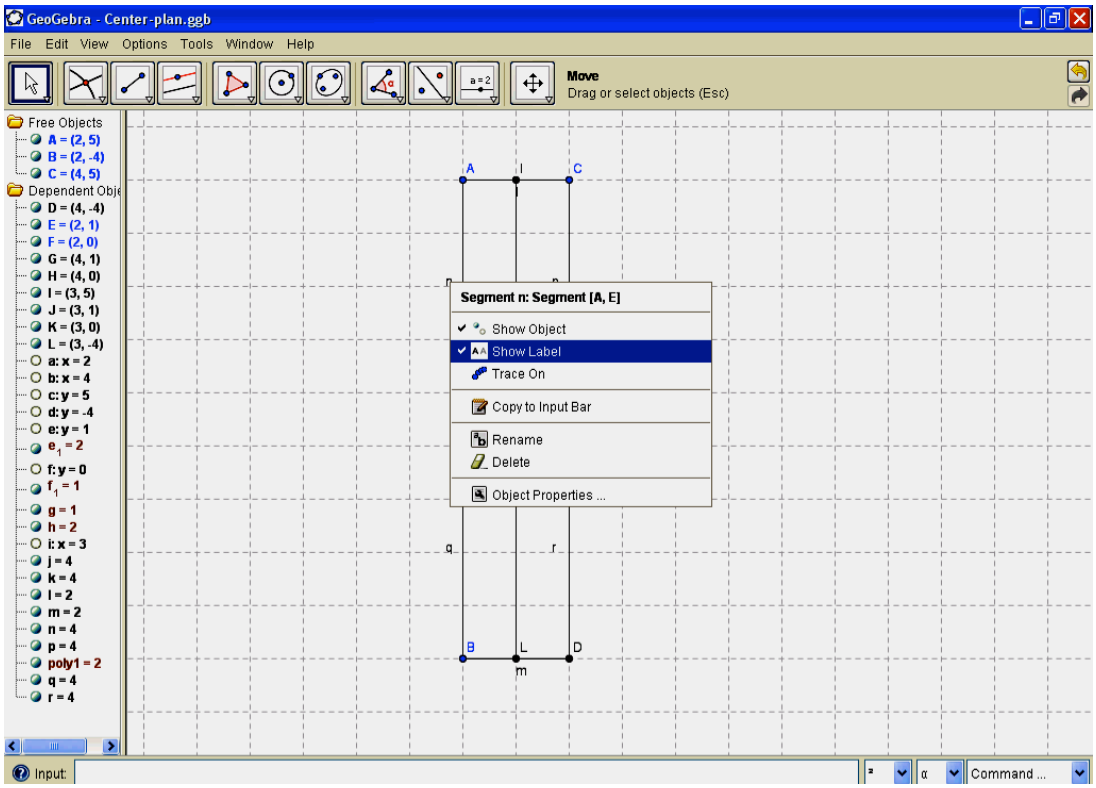
Hide the line that extends past points C and D.



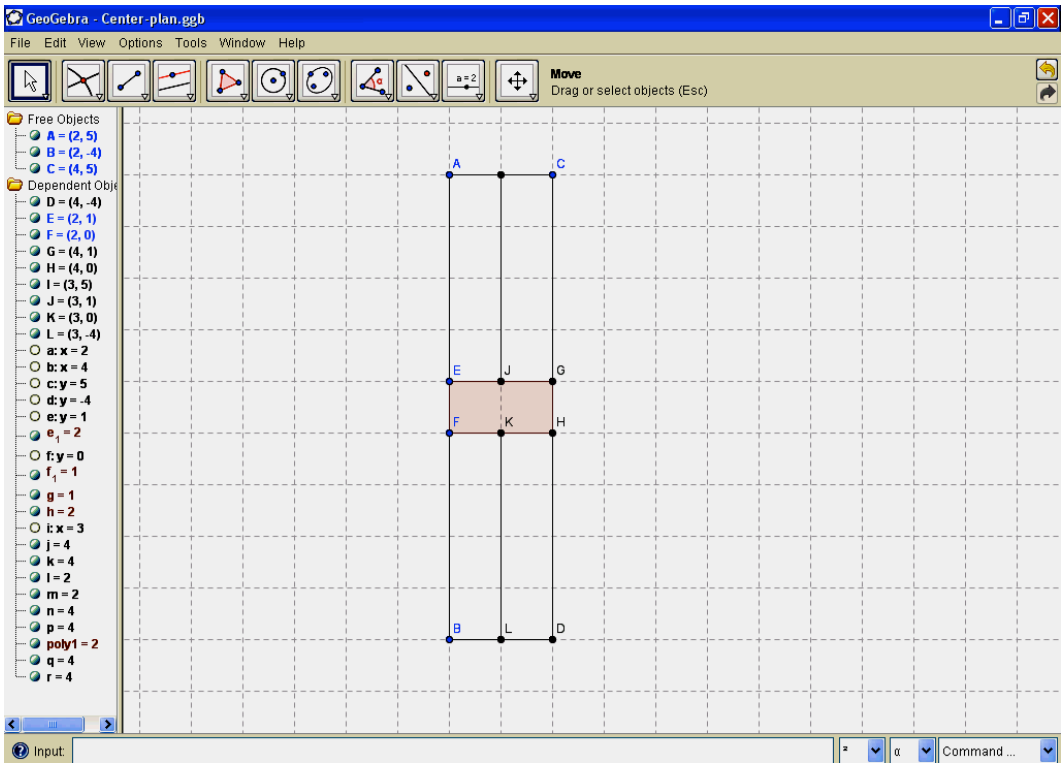
Notice how our diagram looks a little neater with the unnecessary objects hidden.



Continue cleaning up the diagram a bit by hiding some labels.

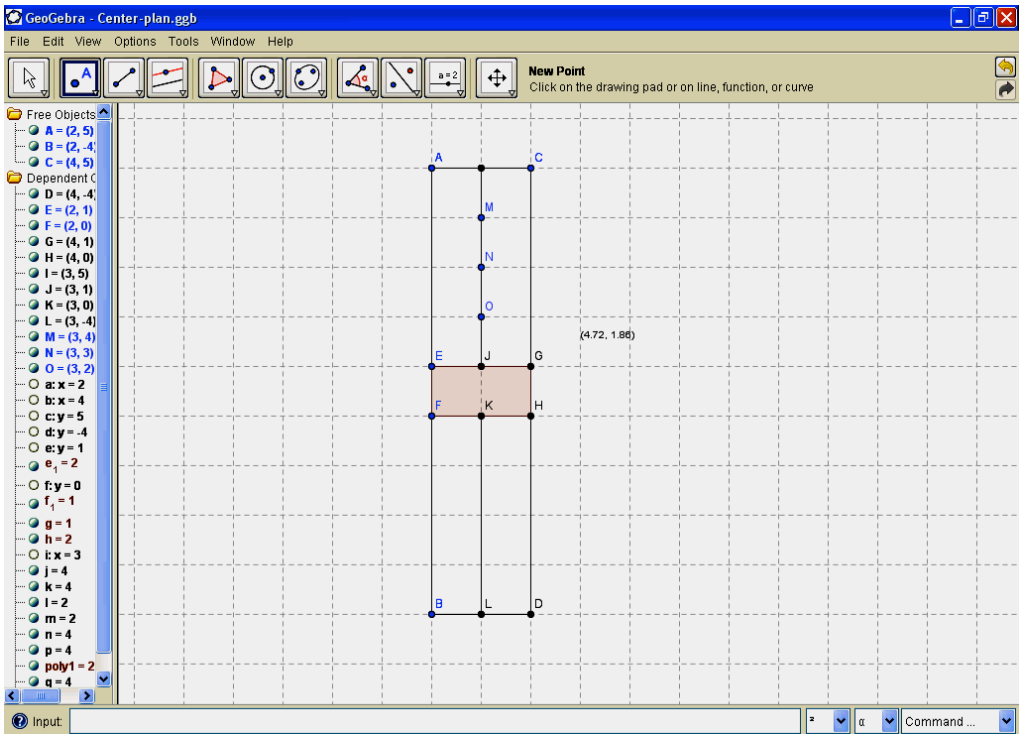


Notice that the label I is hidden.

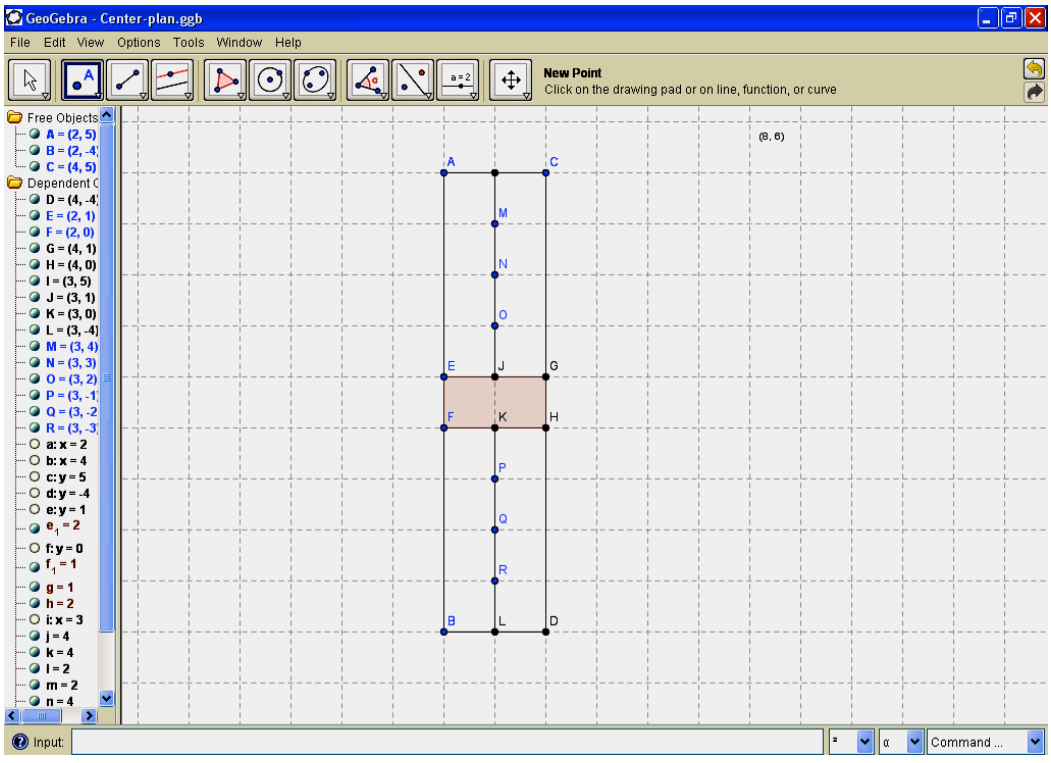


Parallel Lines, Transversals, and Angles: What's the connection?

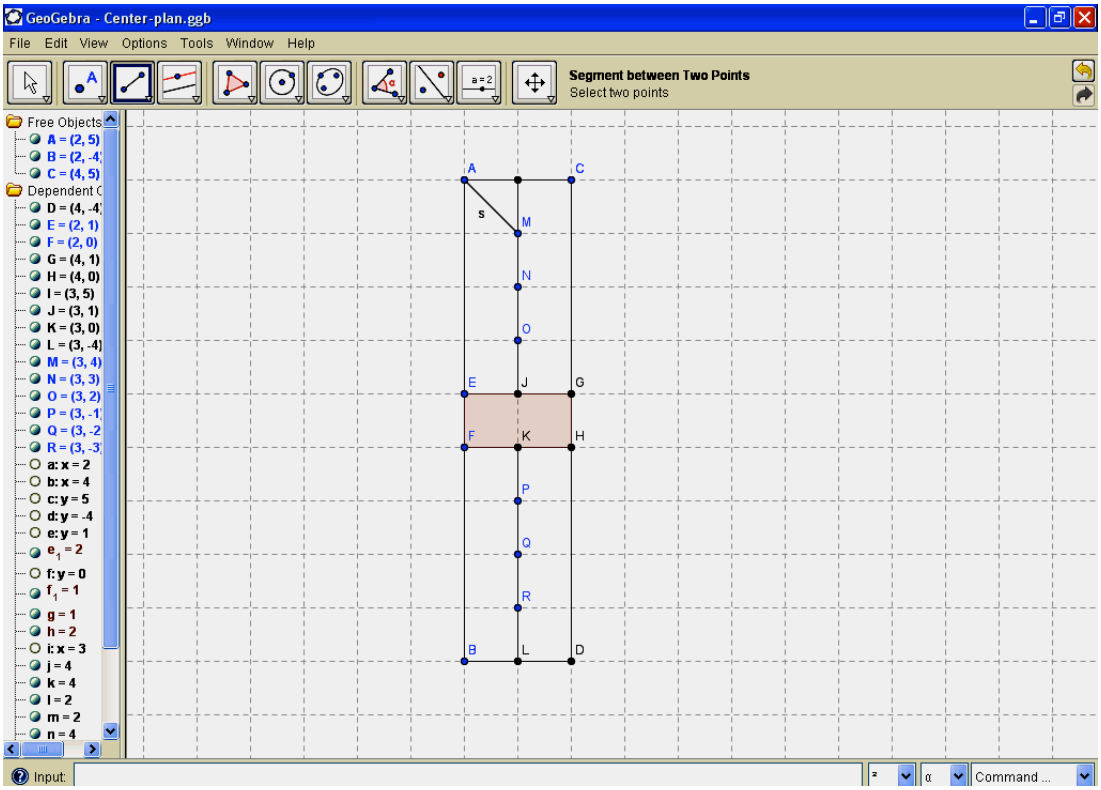
Using the point tool, create three points, M, N, and O. The actual labels don't matter at this point. If you have different letters you do not need to change the labels. If you want to change the labels though, right click on the label and rename.



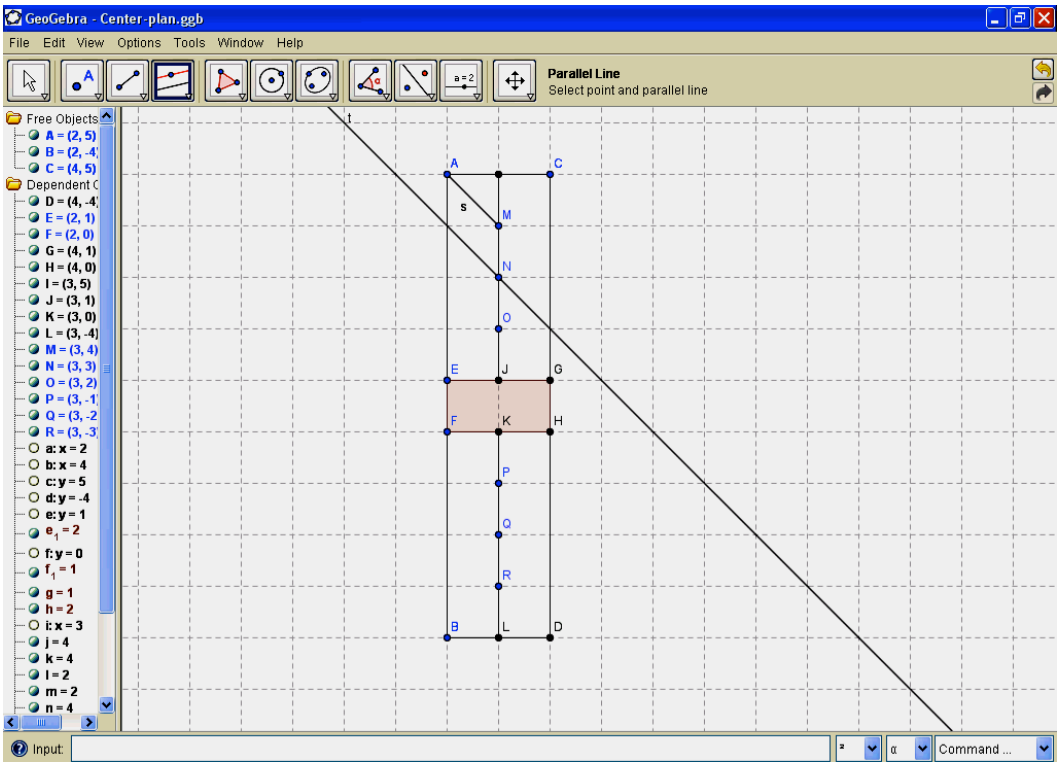
Create points P, Q, and R.



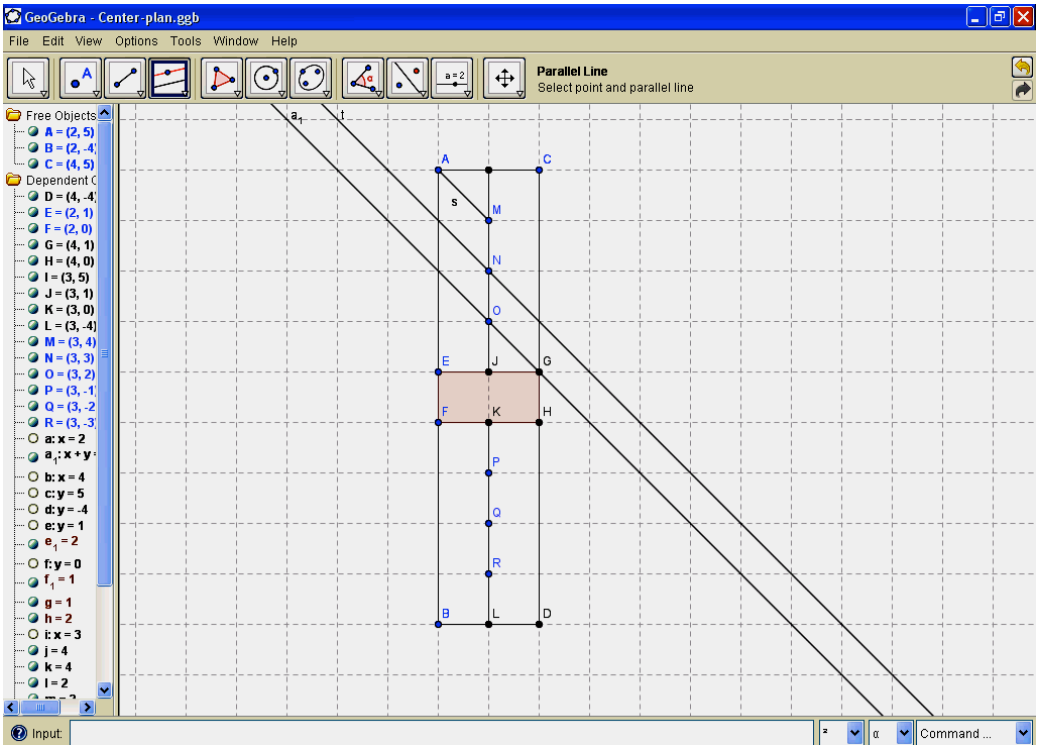
Using the segment tool, create segment AM.



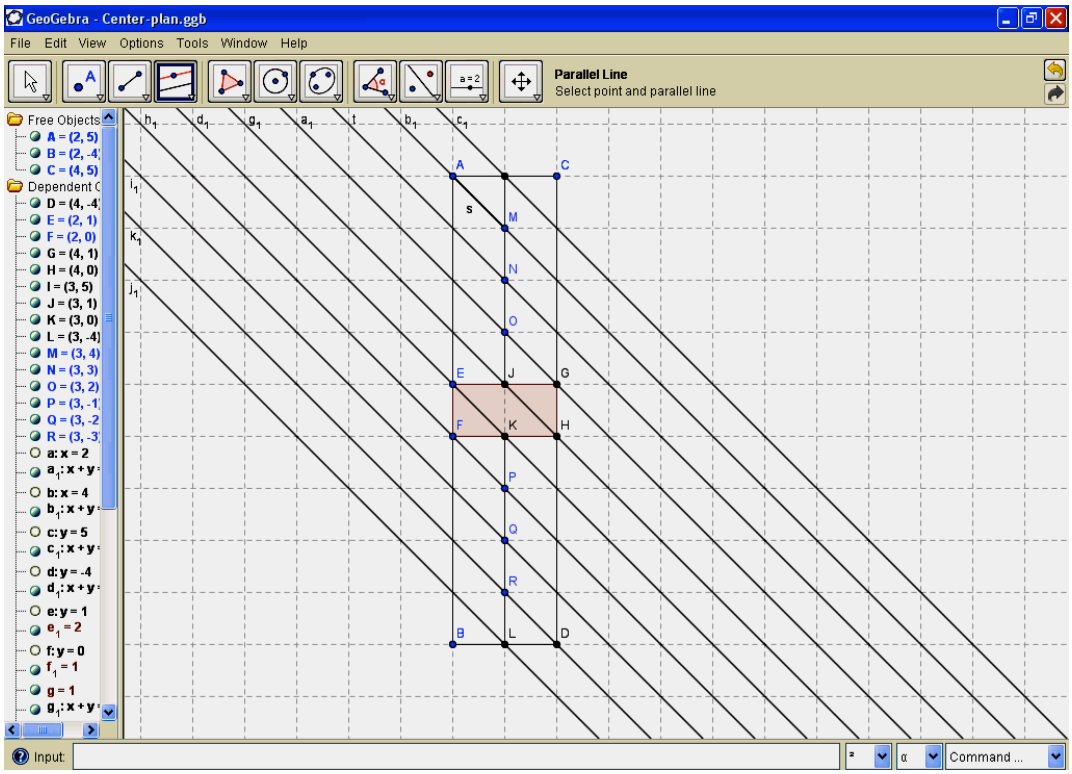
Create a line parallel to segment AM by using the Parallel line tool (4th button from left). Click on point N and then click on segment AM.



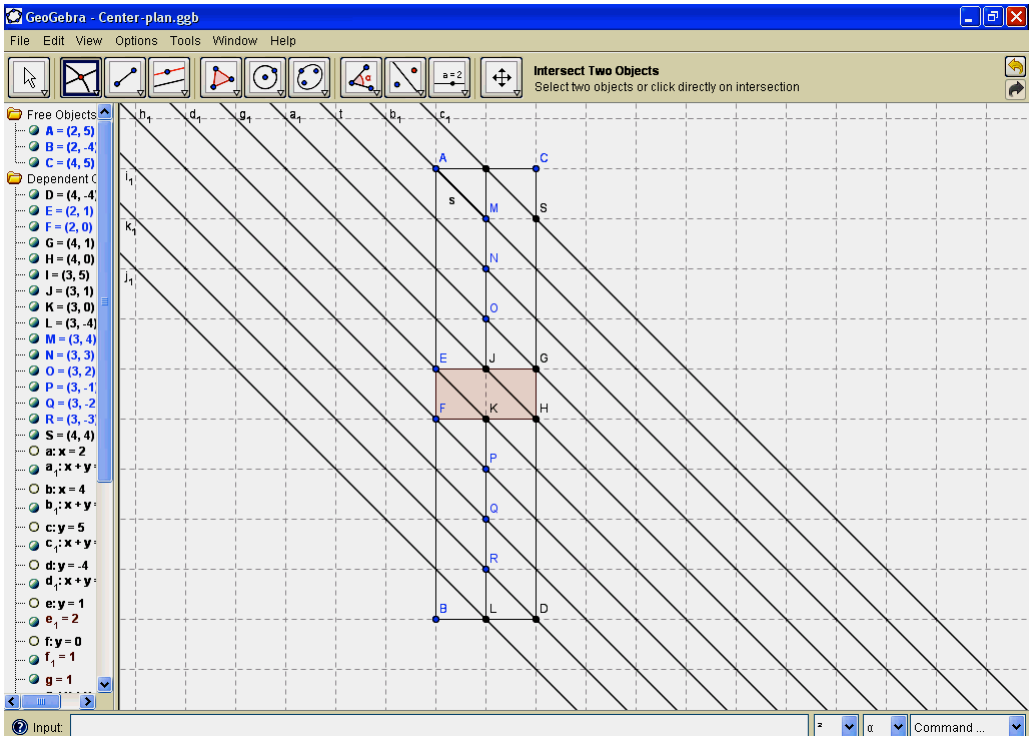
Using the parallel line button, create a line parallel to line t by clicking on point O and line t.



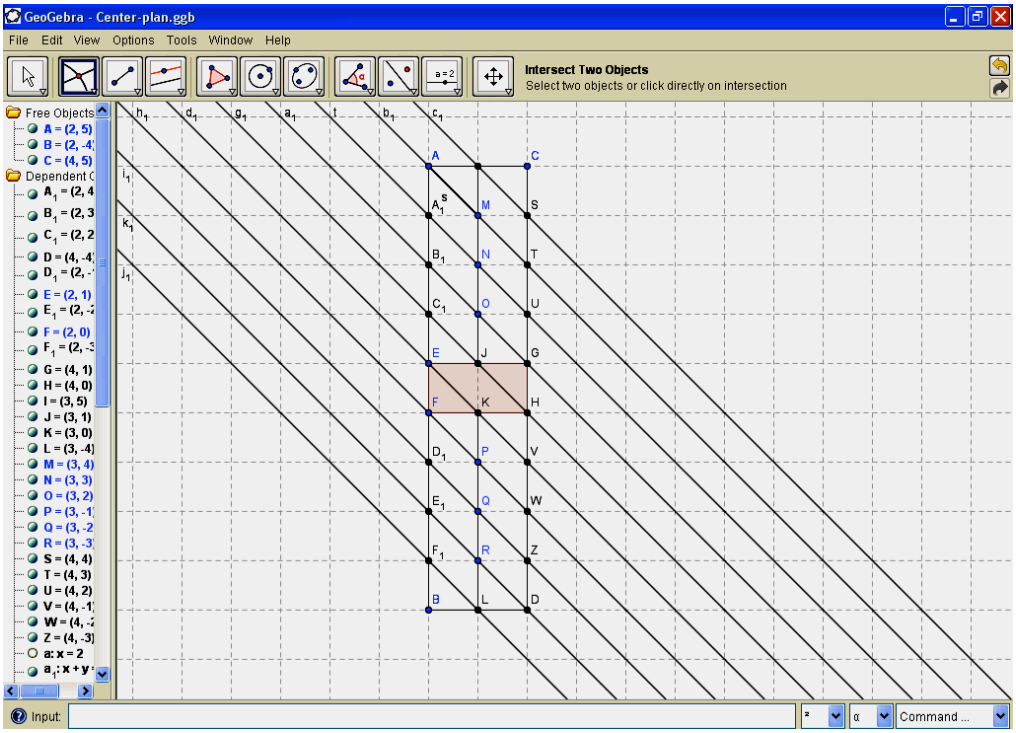
Continue the process of creating parallel lines.



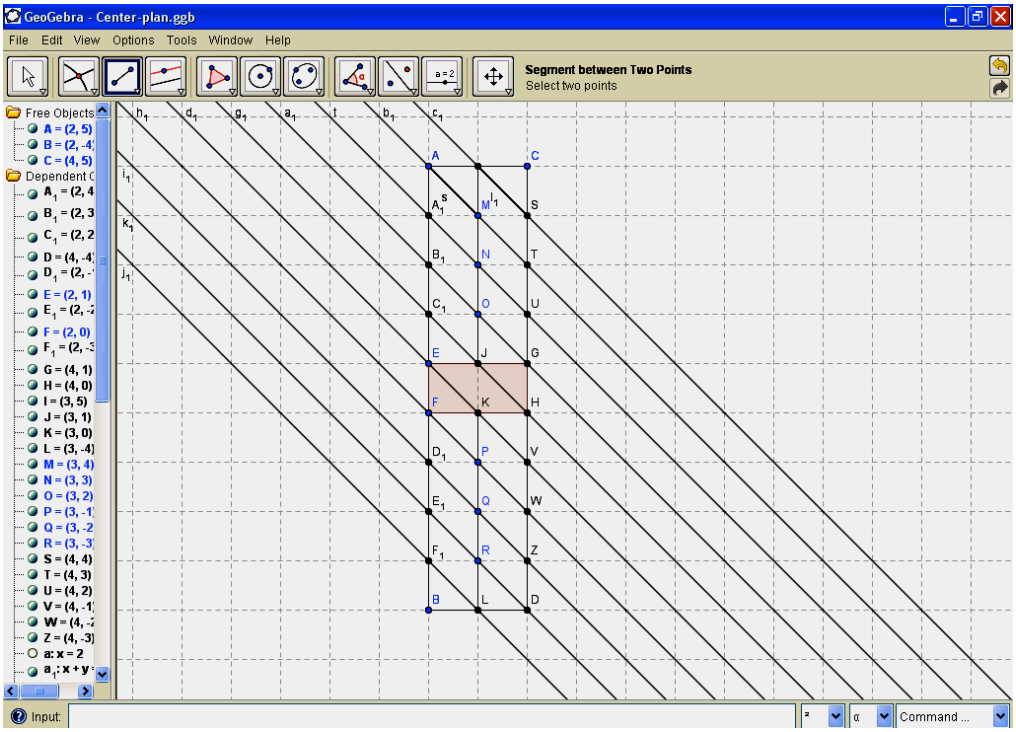
Using the intersection tool, find the intersection of segment CG and line c_1.



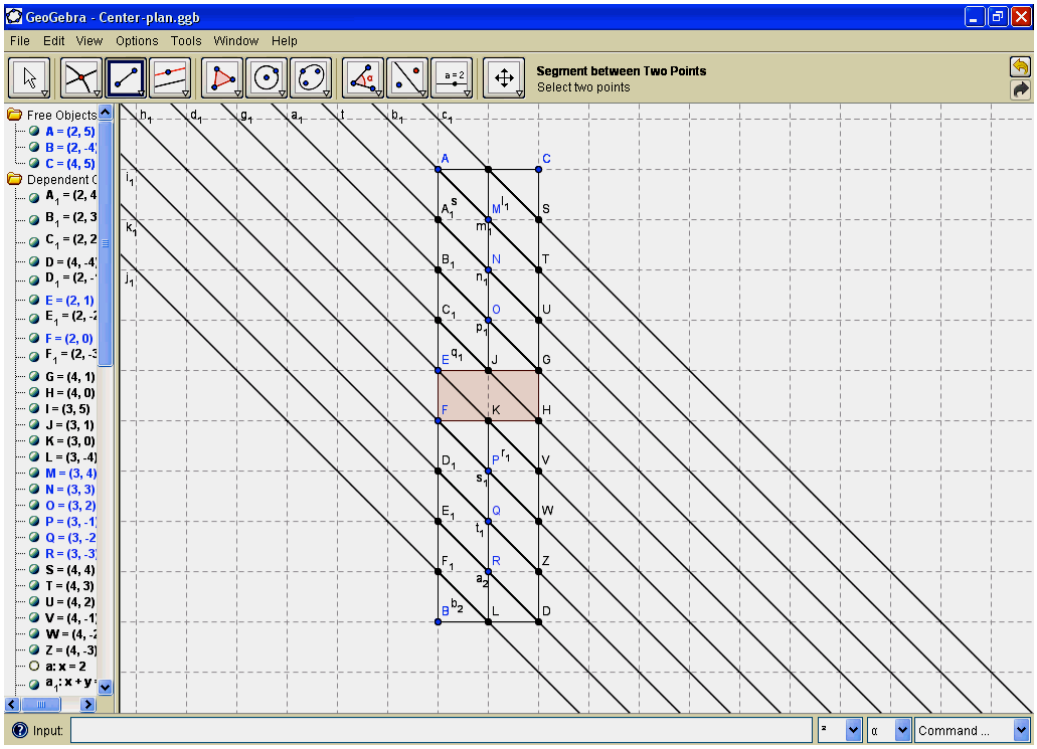
Notice the intersection has been labeled as point S. Continue the process until all points along segment CD have been labeled.



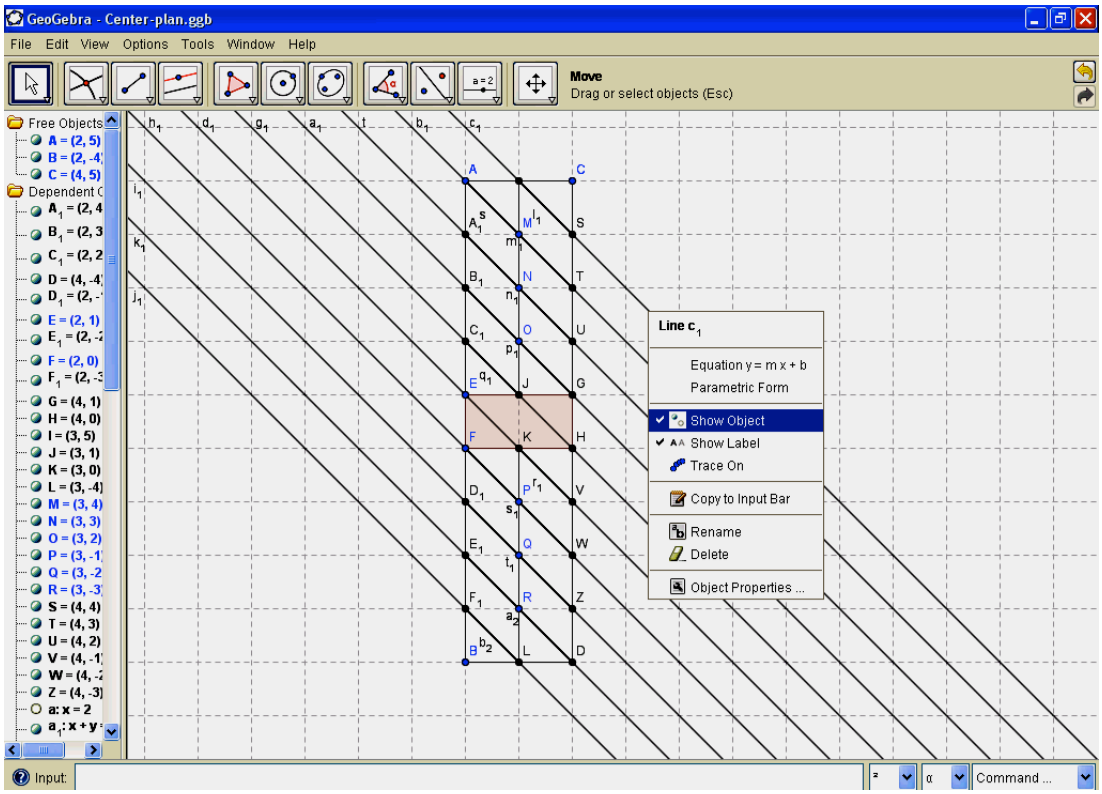
Create a segment from the midpoint of AC to point S. This will allow us to see only the segment and hid the rest of the line.



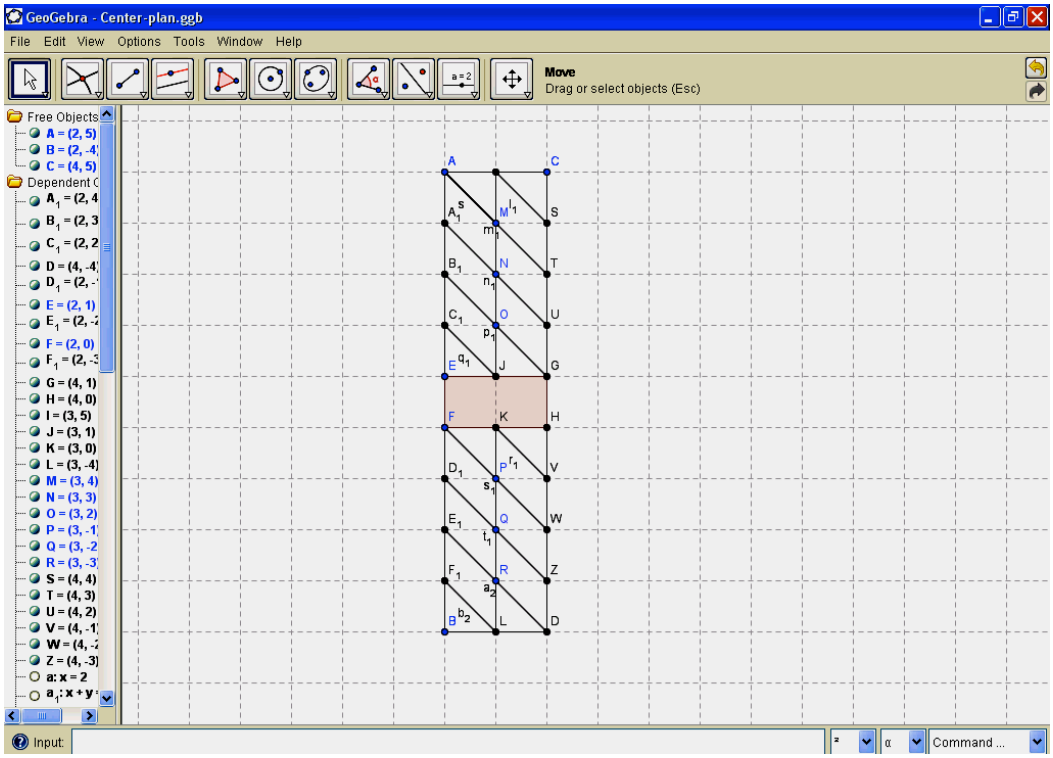
Continue segments AT, A_3U, B_1O, etc. until all segments have been created.



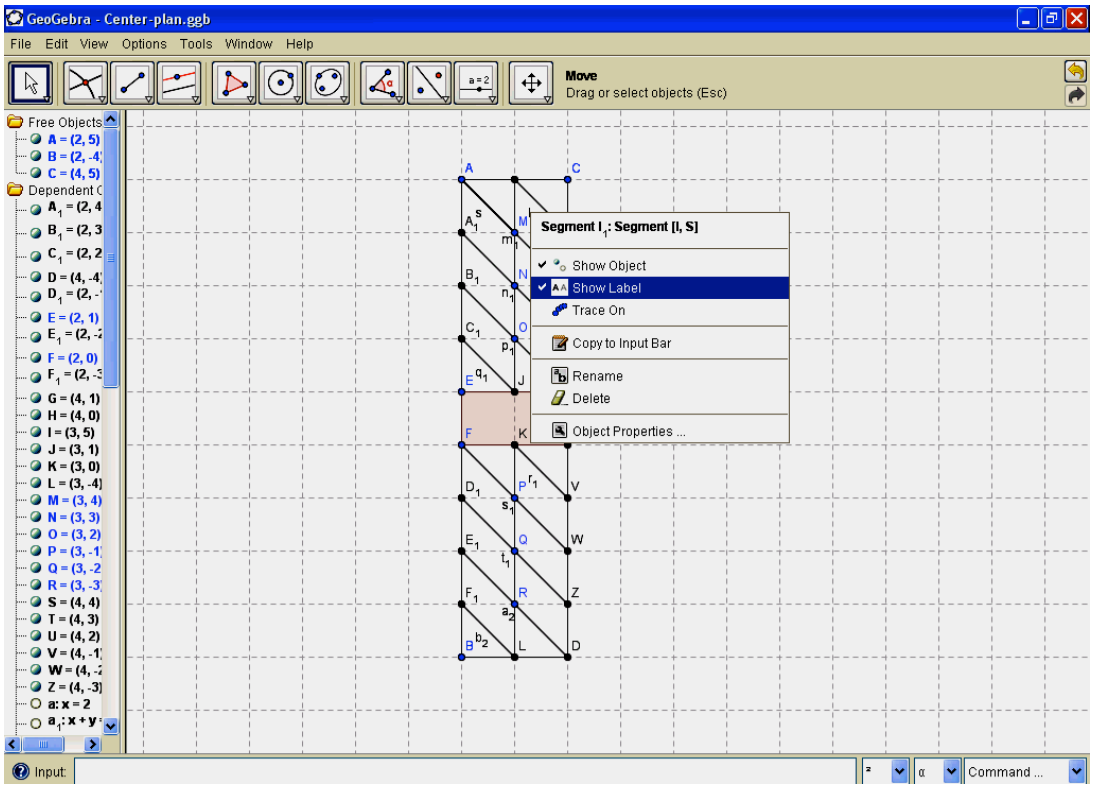
Once all of the segments have been created, hide the lines by right clicking and unclick Show Object.



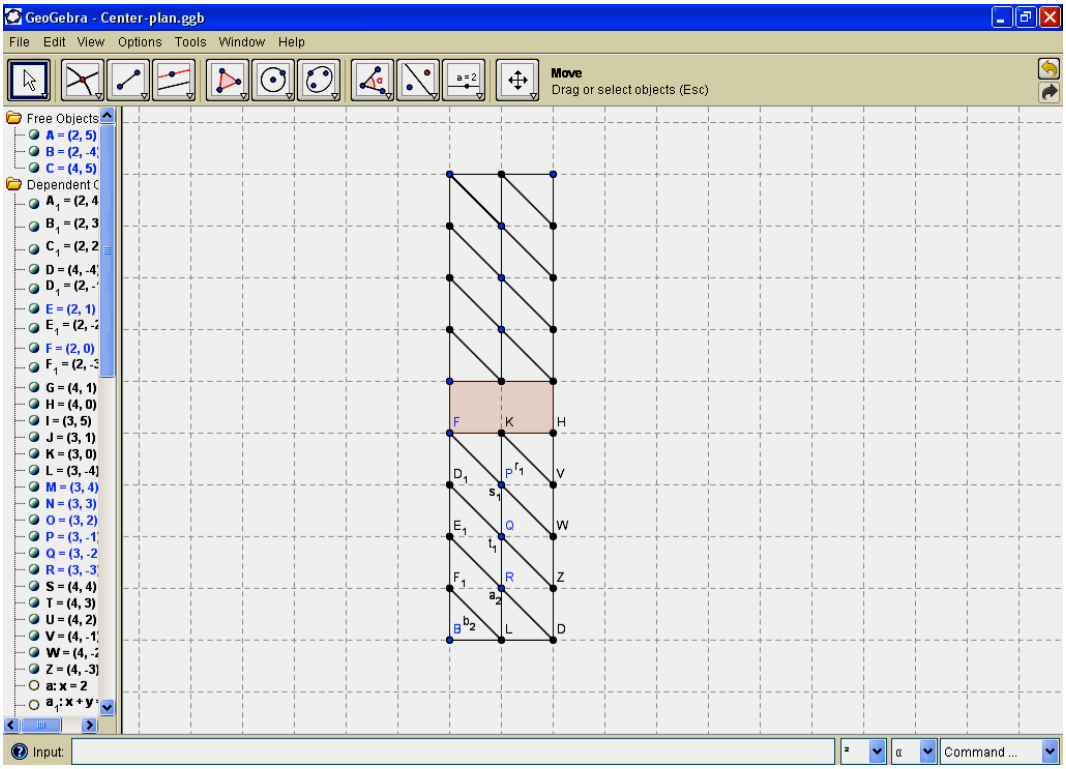
Notice that the lines extending past the diagonals have been hidden.



Now, clean up the diagram a bit by hiding the labels.

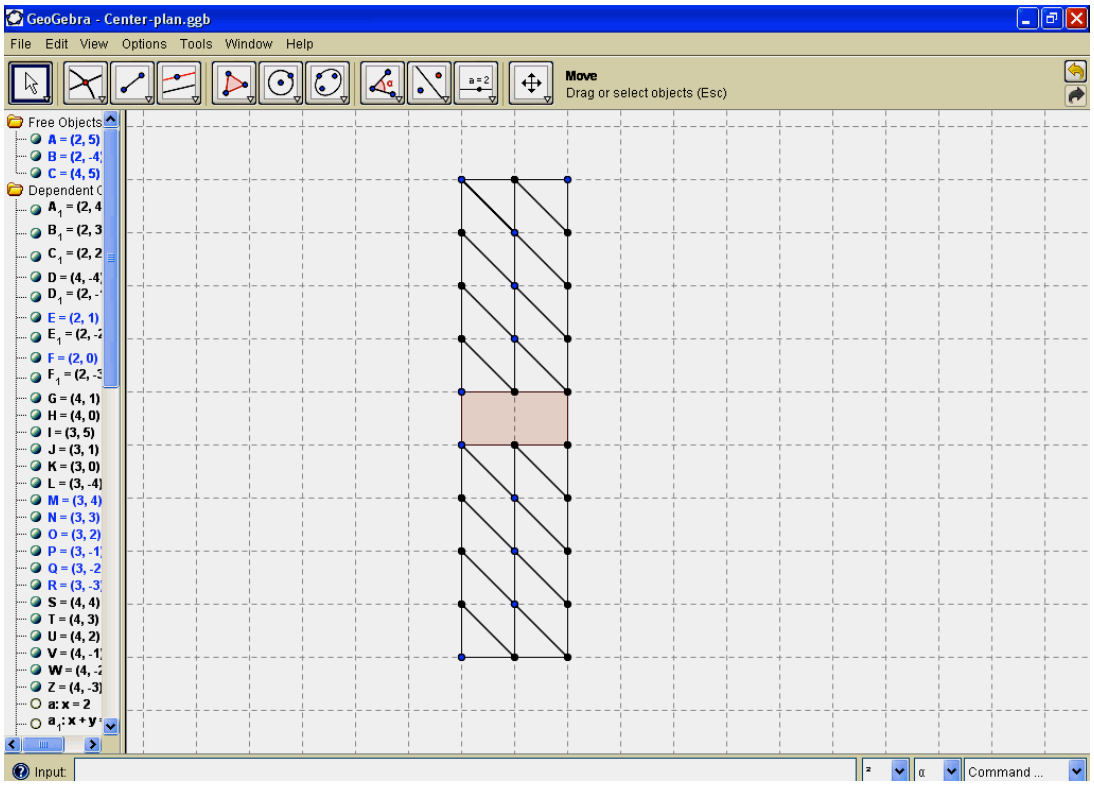


Notice that the labels in the upper portion of the diagram have been hidden. Continue the process for the bottom portion

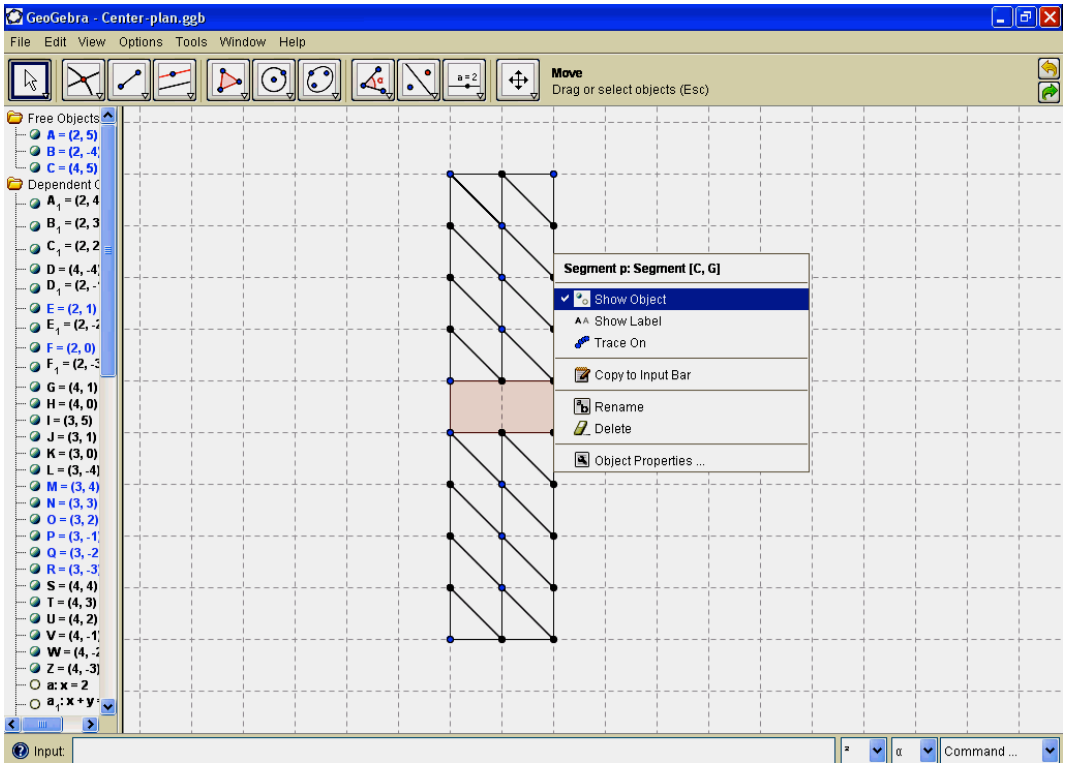


.

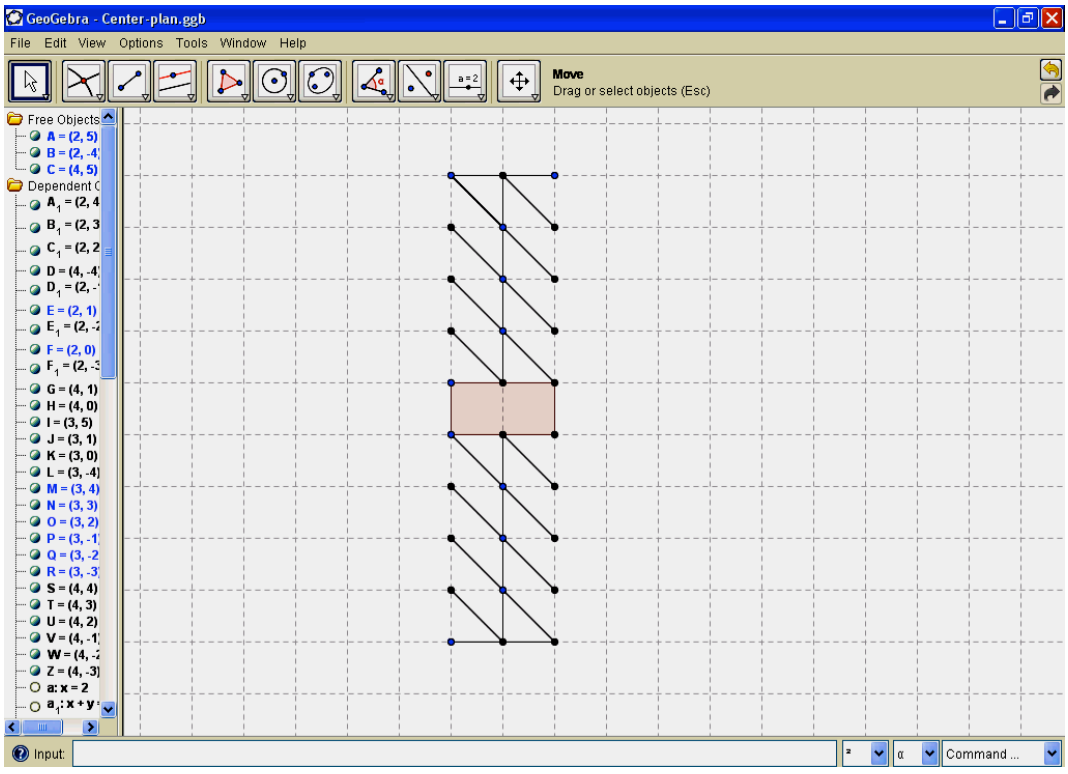
Notice all labels have been hidden.



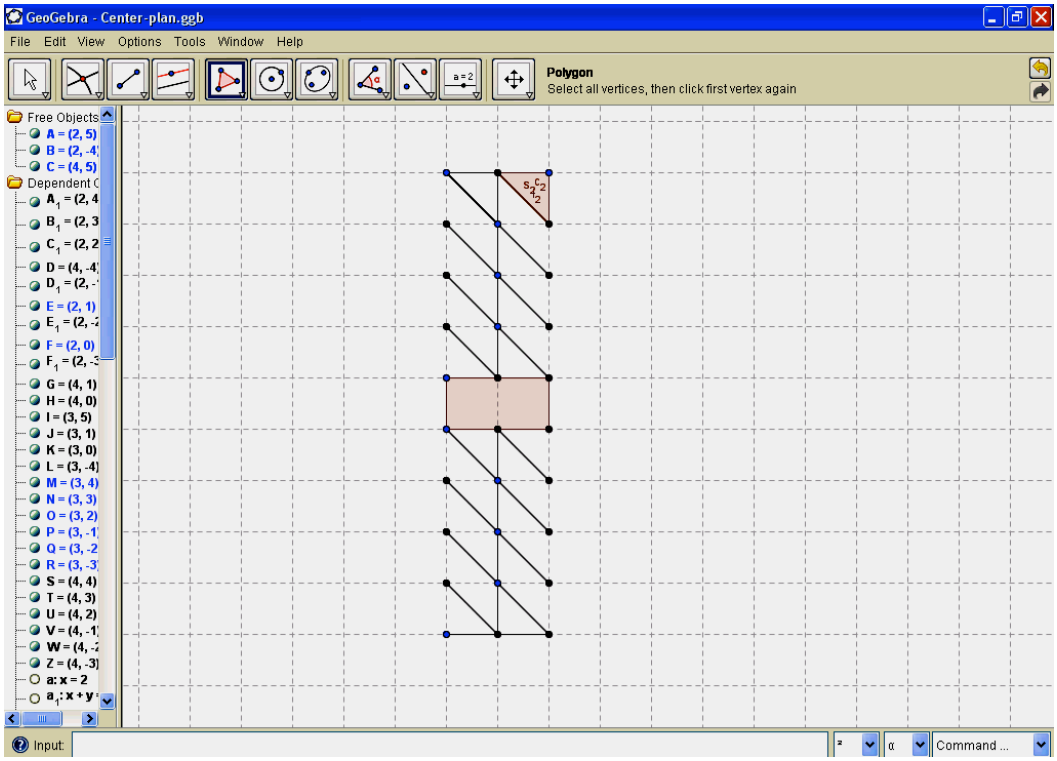
Since we are creating parking spaces, hide the segment that passes through all of the points.



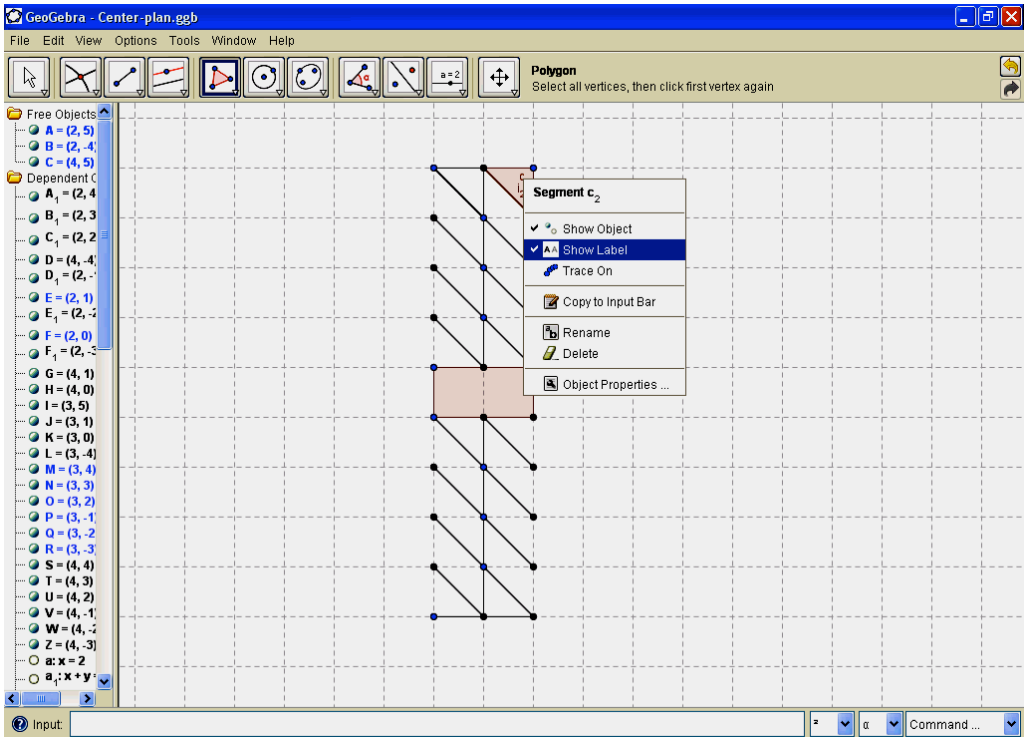
Notice that the segments have been hidden.



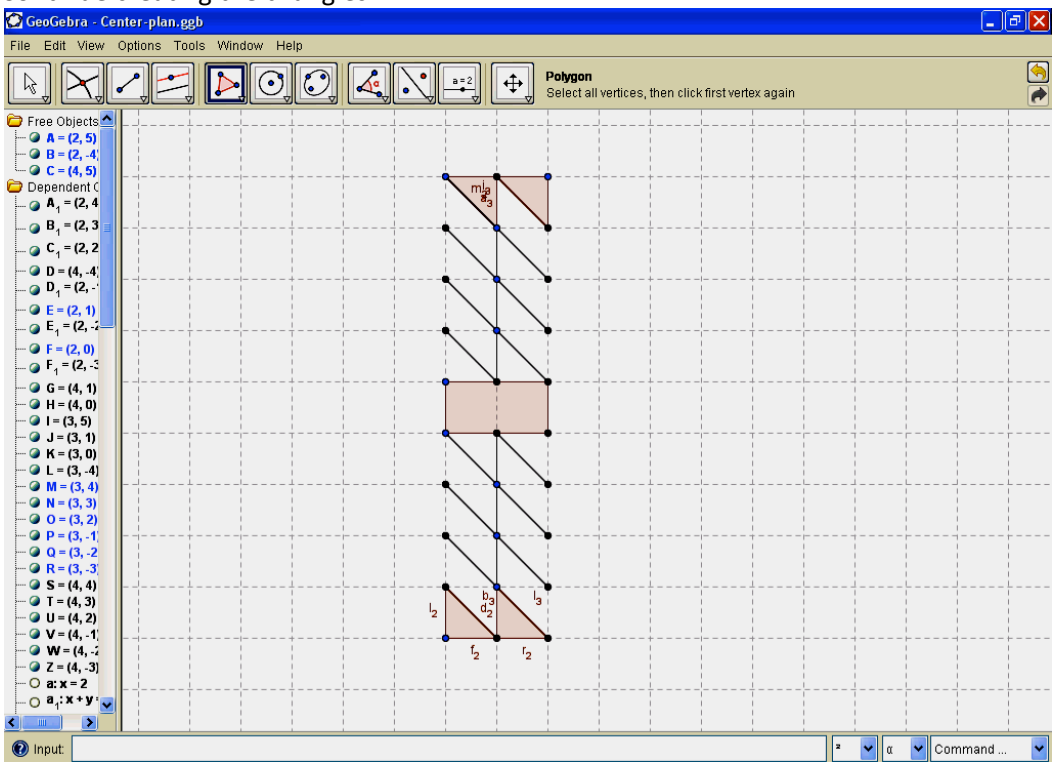
Create polygons (triangles) at the corners since these spots would not be large enough for a car to park.



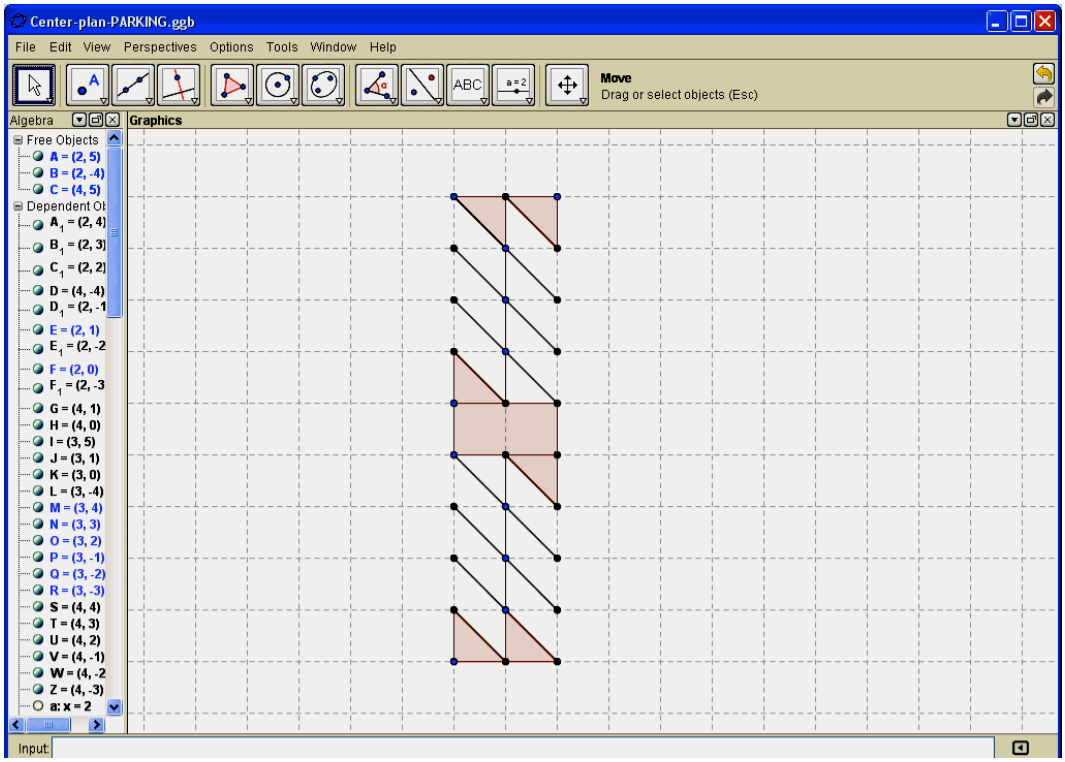
Hide the labels.



Continue creating the triangles.

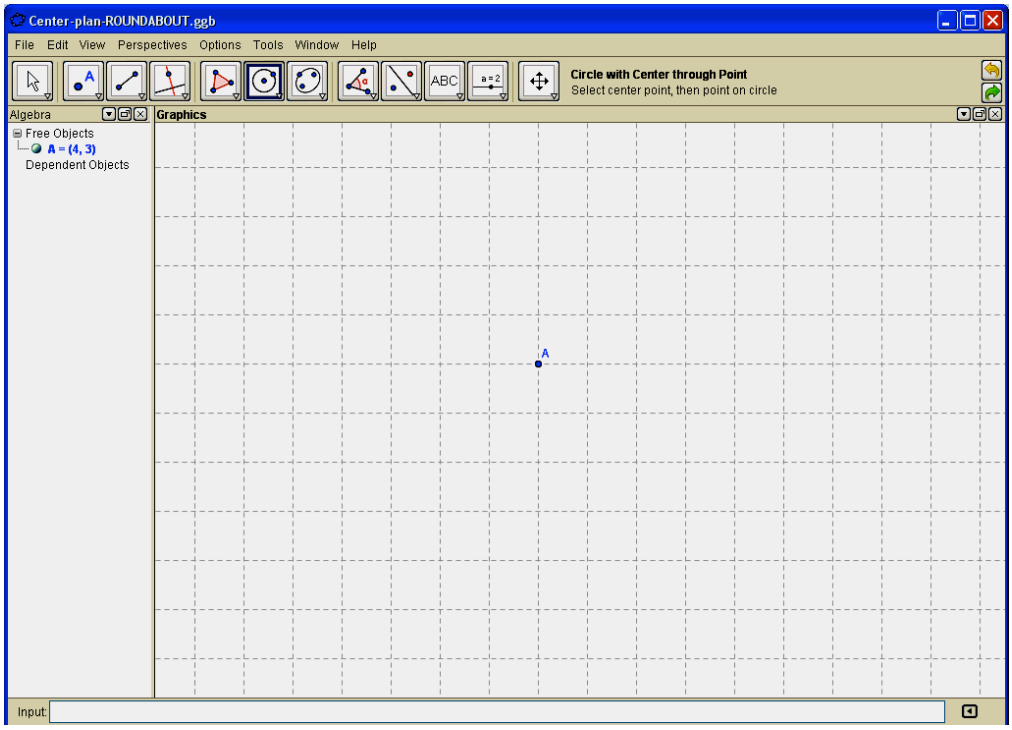


Hide the labels. This final diagram represents our parking area. The shaded areas represent "green space" where we will plant trees, shrubs, or flowers.

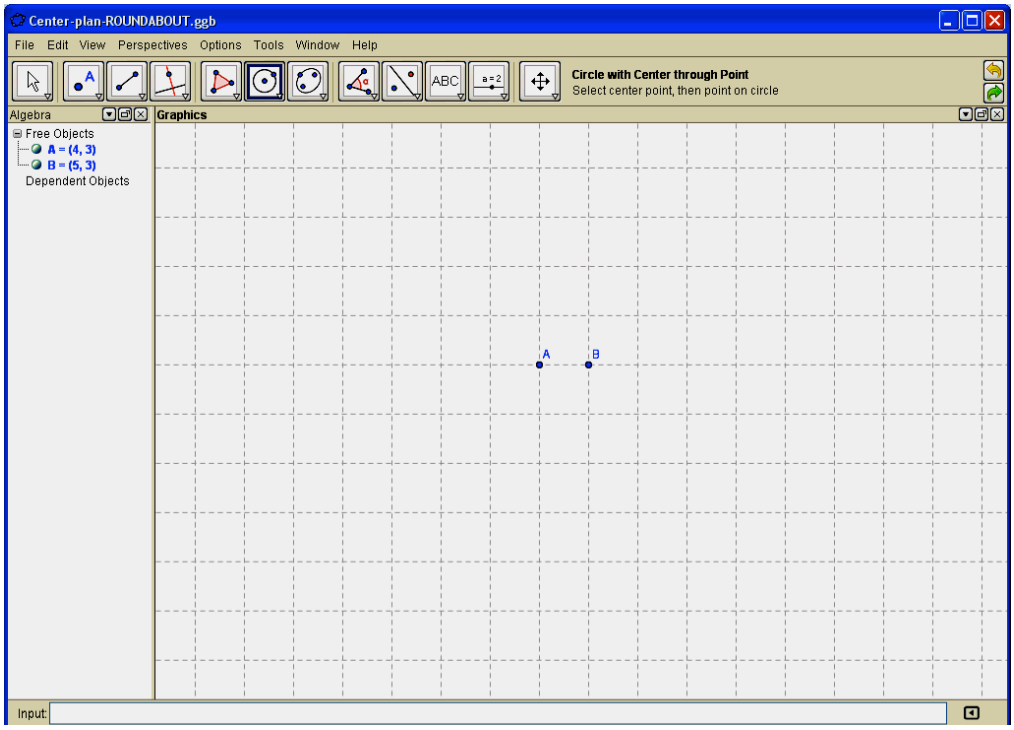


Instructions for creating the highlighted traffic circle in Geogebra

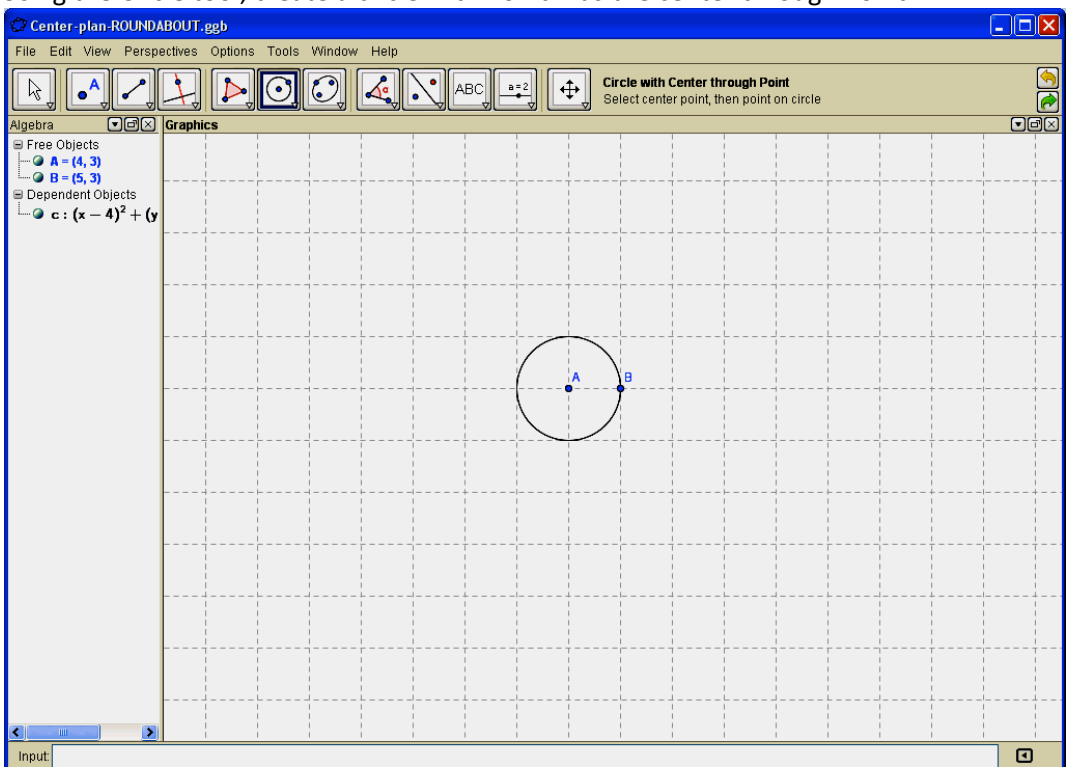
Begin creating a Traffic Circle by constructing a circle. We begin with creating a point A.



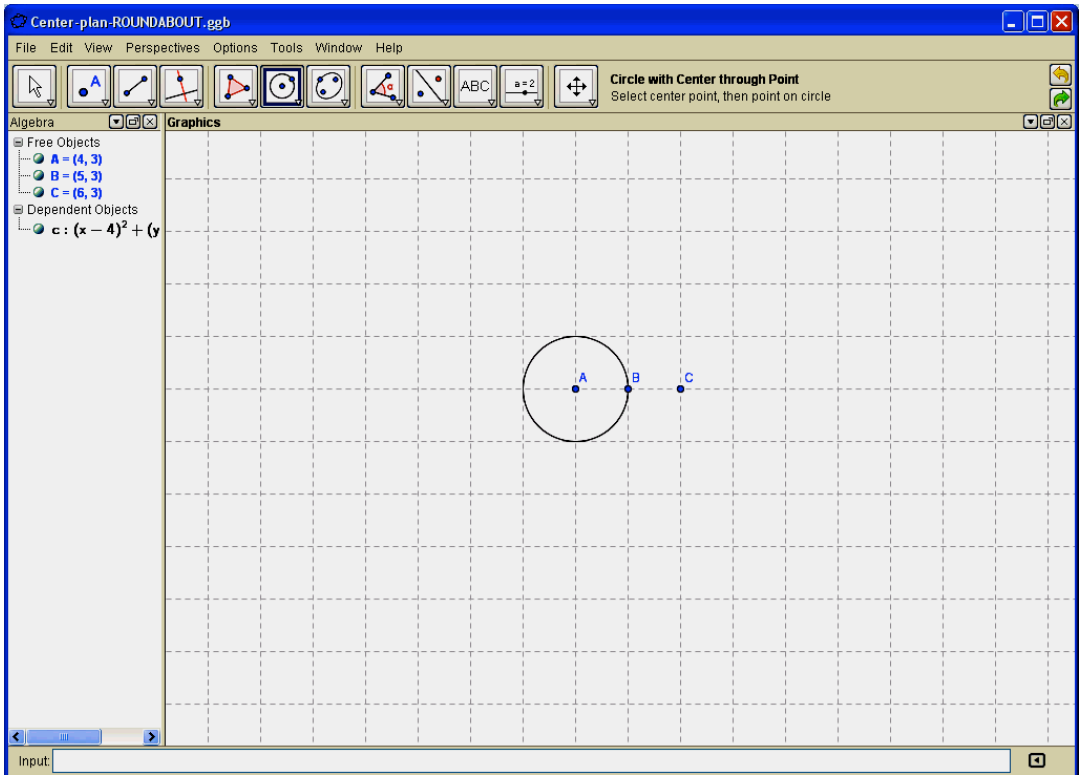
Create a second point B.



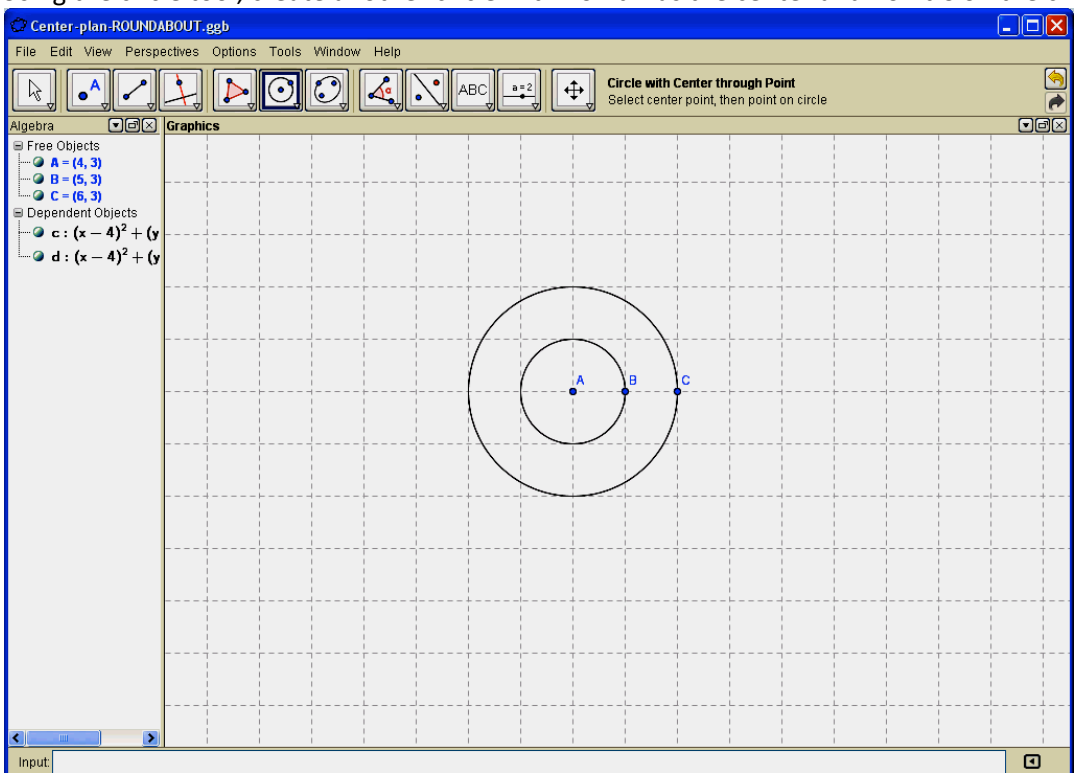
Using the Circle tool, create a circle with Point A as the center through Point B.



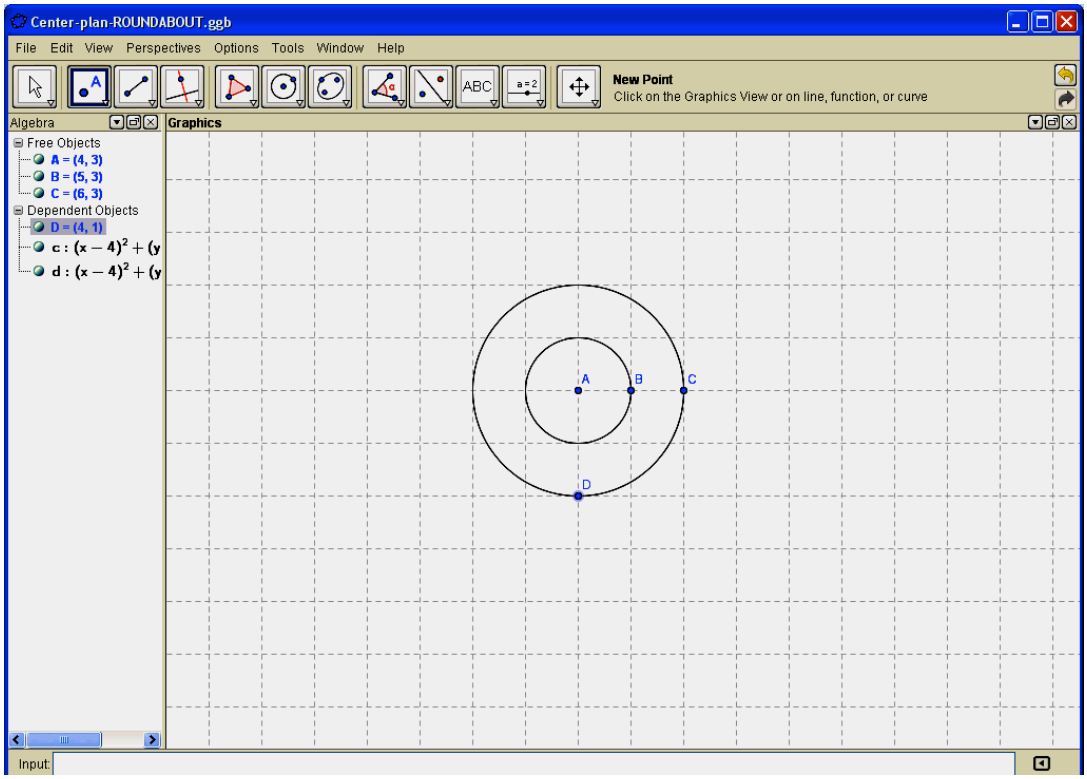
Create another Point C.



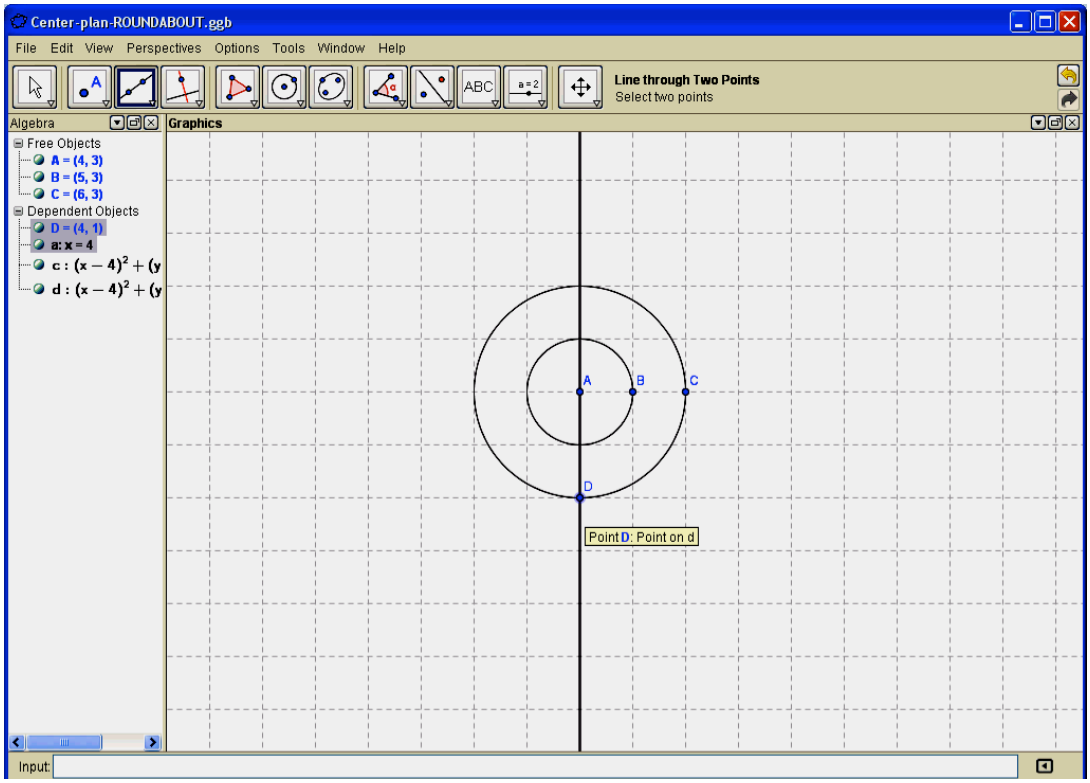
Using the circle tool, create another circle with Point A as the center and Point C on the circle.



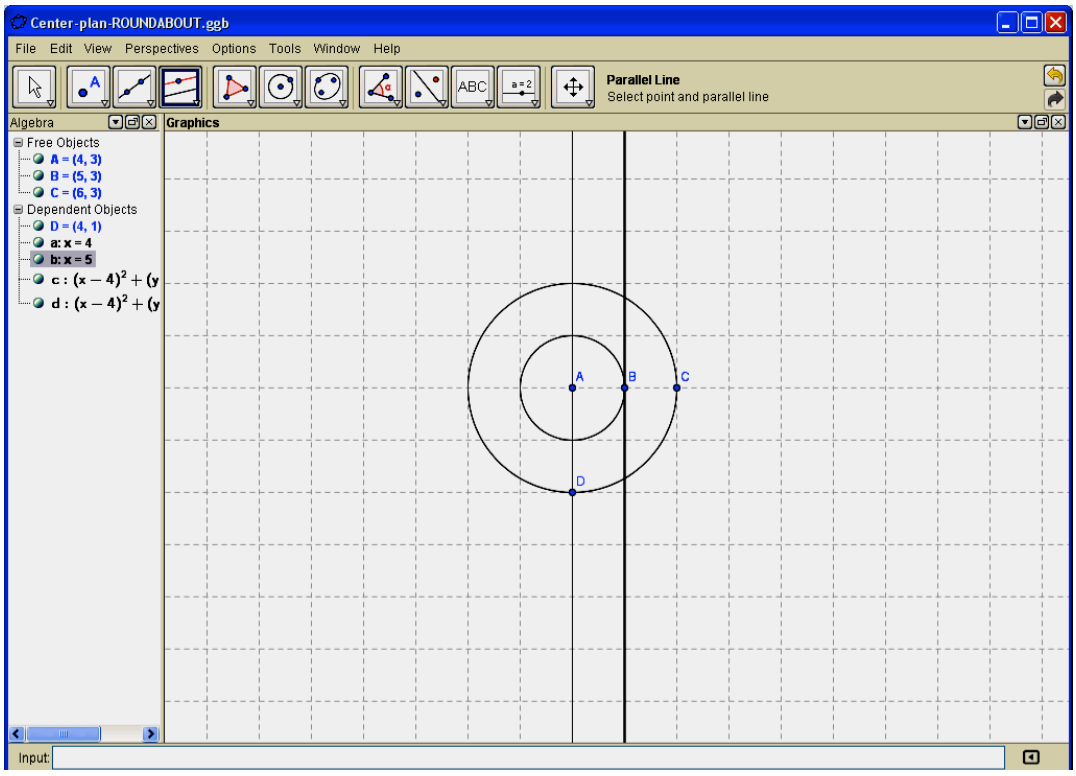
Create point D on the circle with point C.



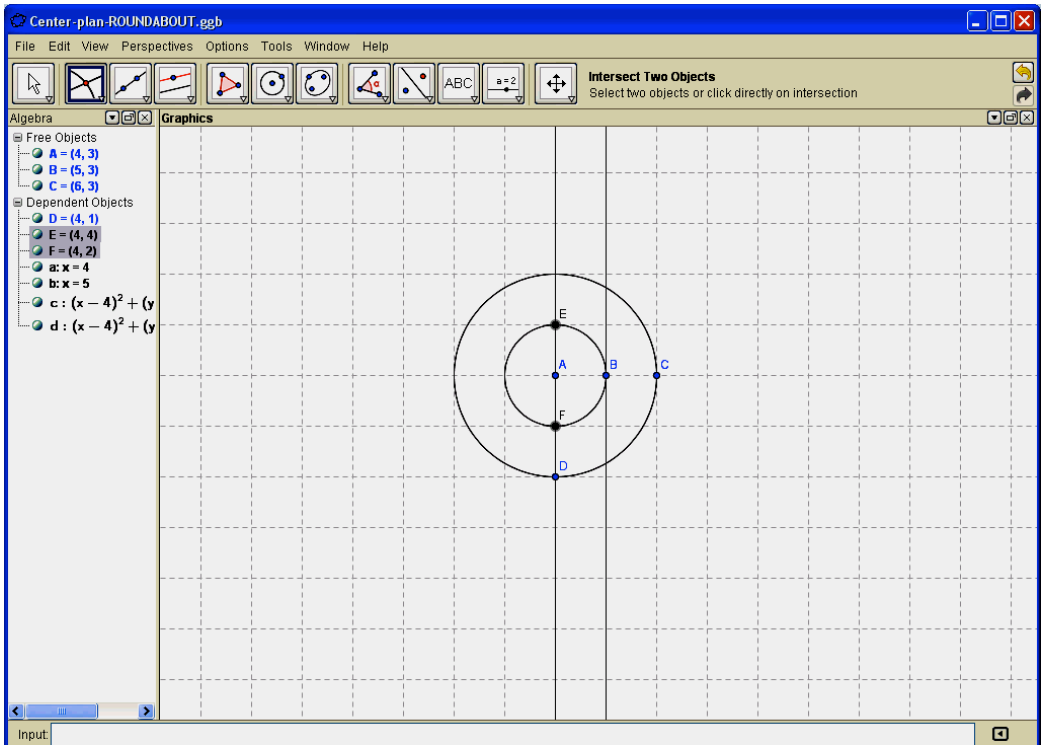
Using the Line Tool, create a line through points A and D.



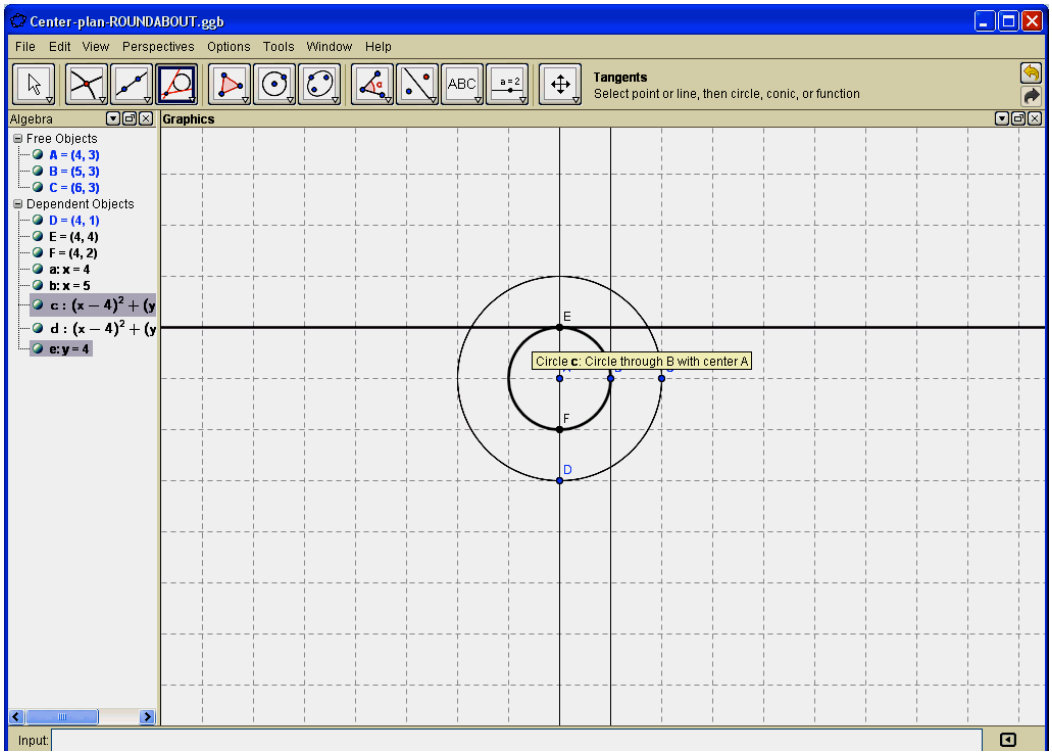
Using the Parallel Line tool, create a line through Point B parallel to the line through points A and D.



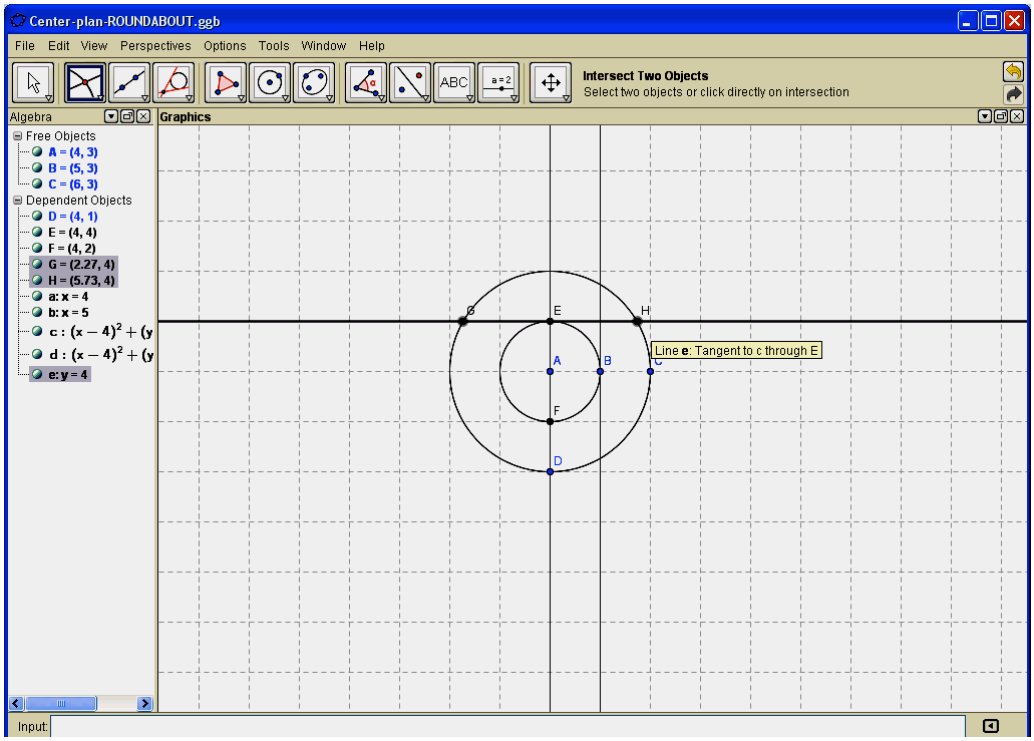
Using the Intersection Tool, create Point E by finding the intersection of the segment through Points AF and the Circle through Point B.



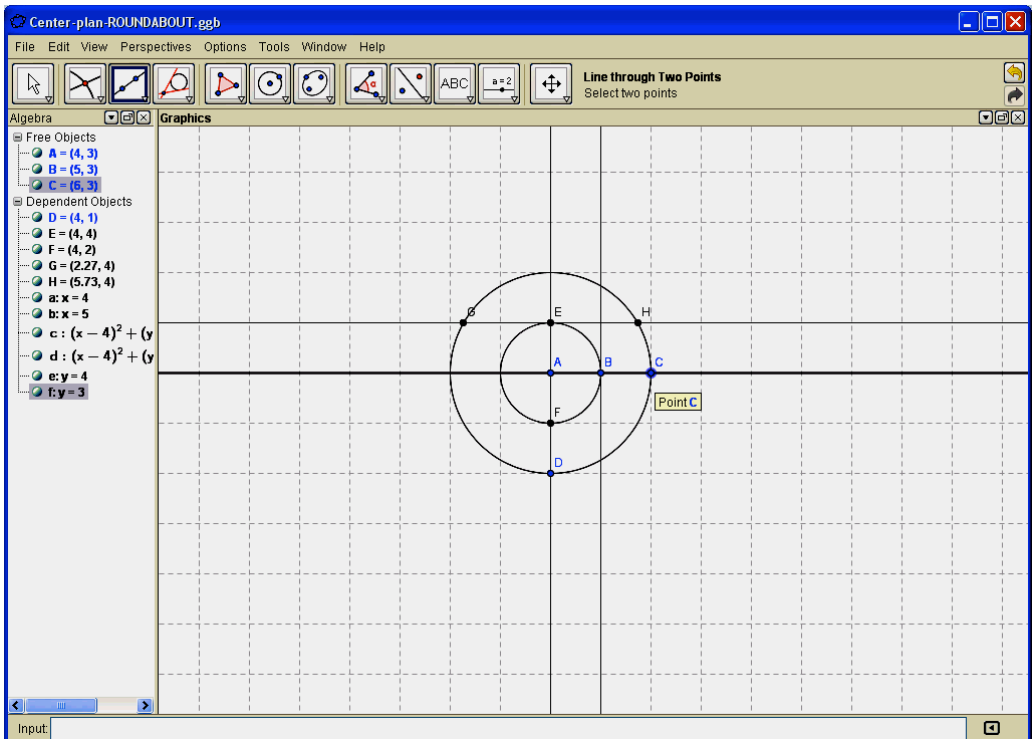
Using the Tangent tool, create a line tangent to the circle through Point B at Point E.



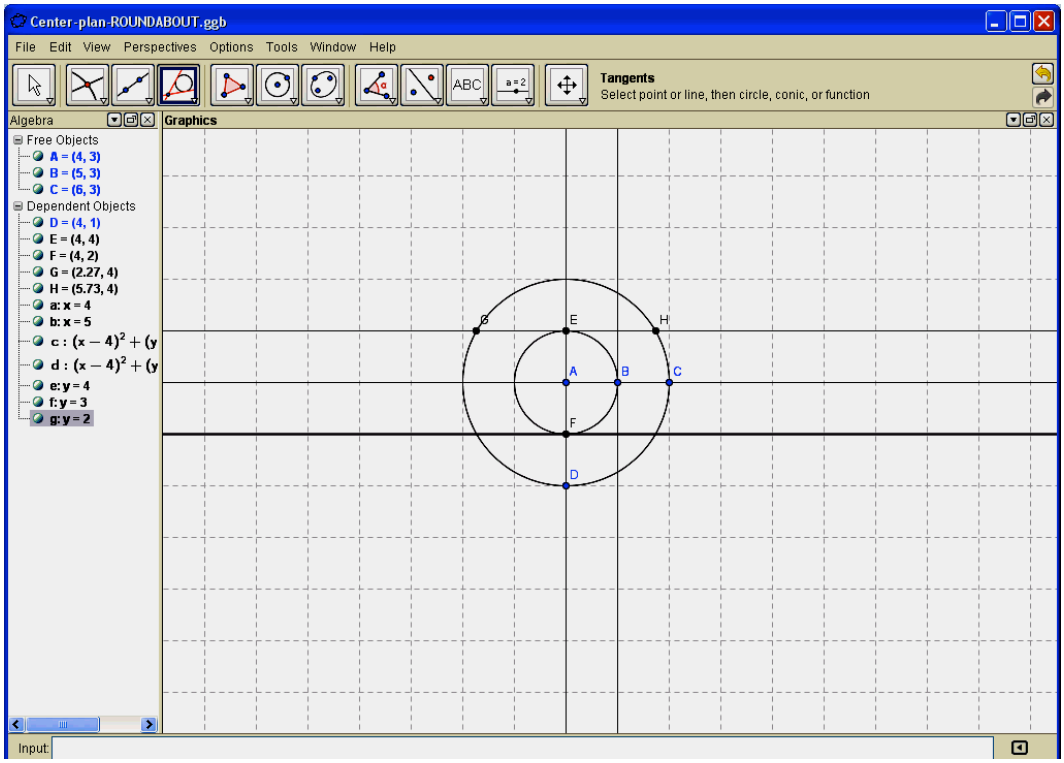
Using the Intersection tool, find the intersection of the tangent line through E and the circle through C. Notice that two points of intersection were found.



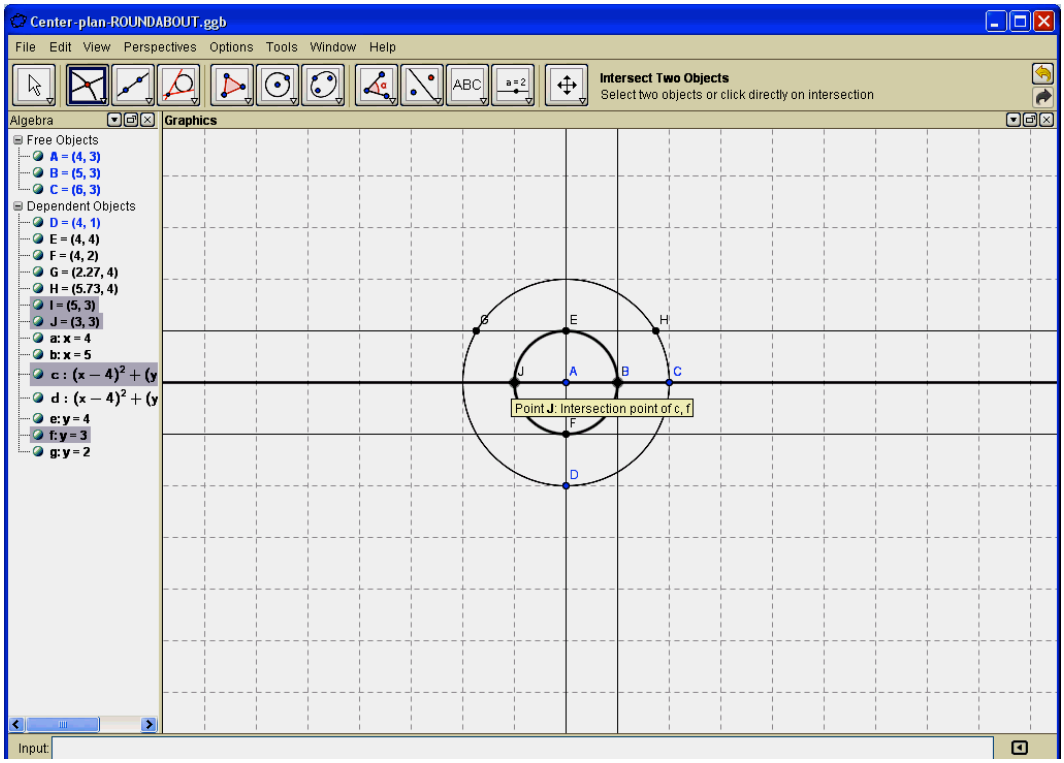
Using the Line tool, create a line through points B and C.



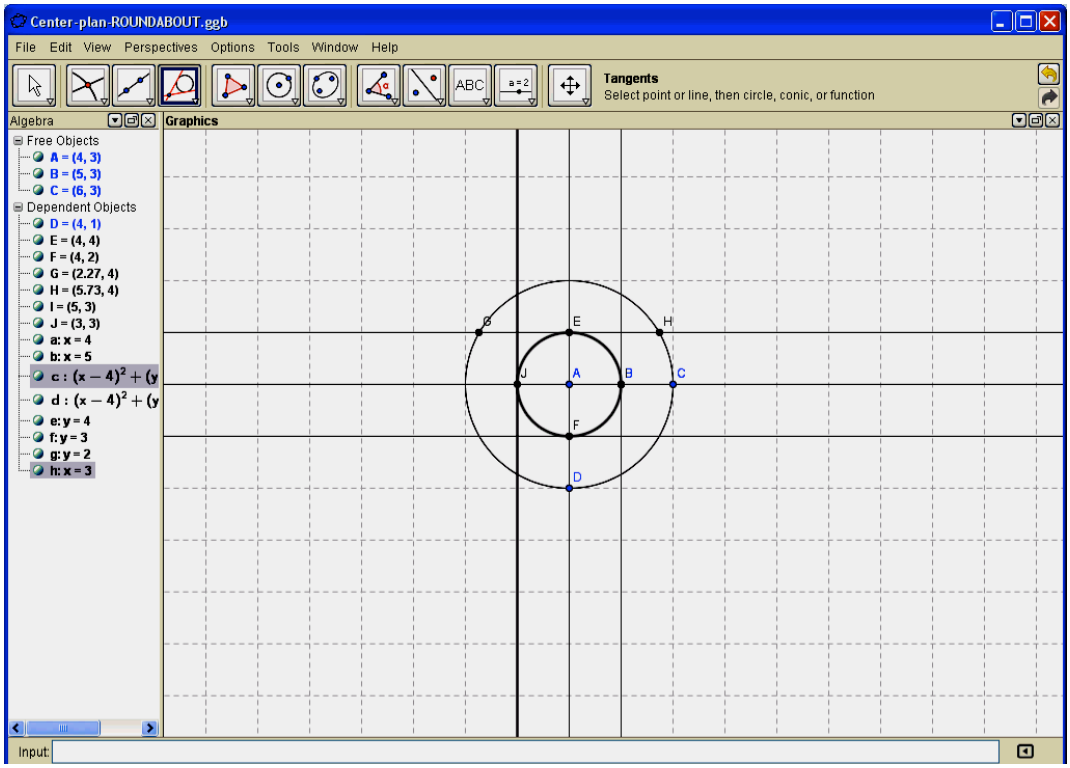
Using the Tangent tool, create a line tangent to the circle that passes through Point B at point F.



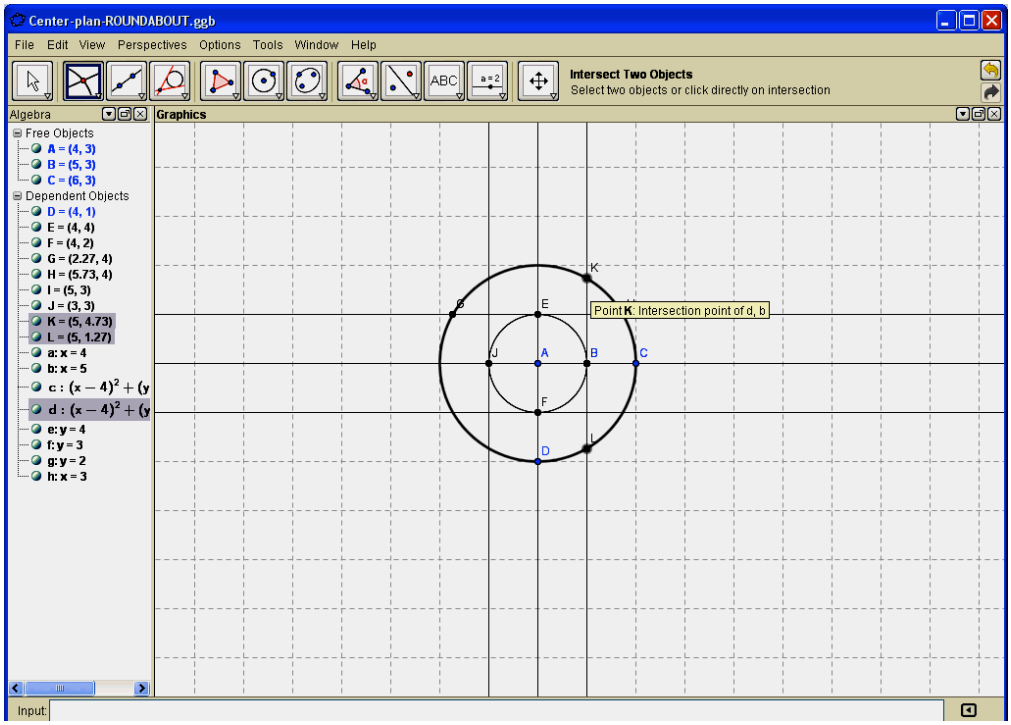
Using the Intersection tool, find the intersection of the circle that passes through Point B and the line passing through Points B and C.



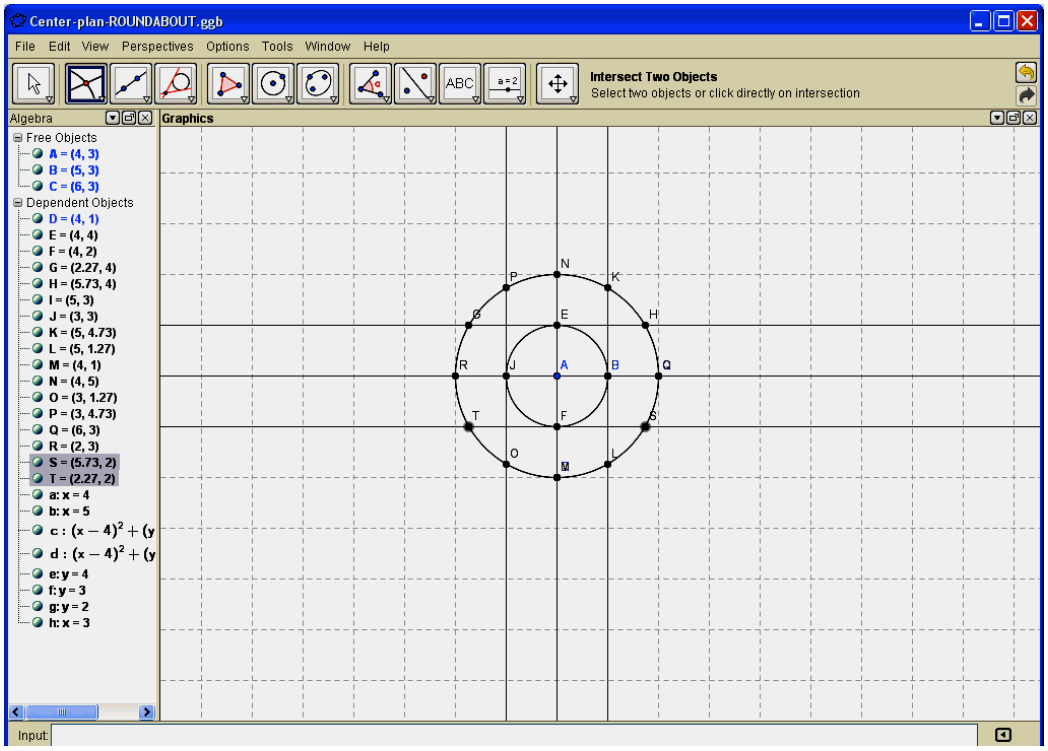
Using the Tangent tool, create a line tangent to the circle that passes through Point B at Point J.



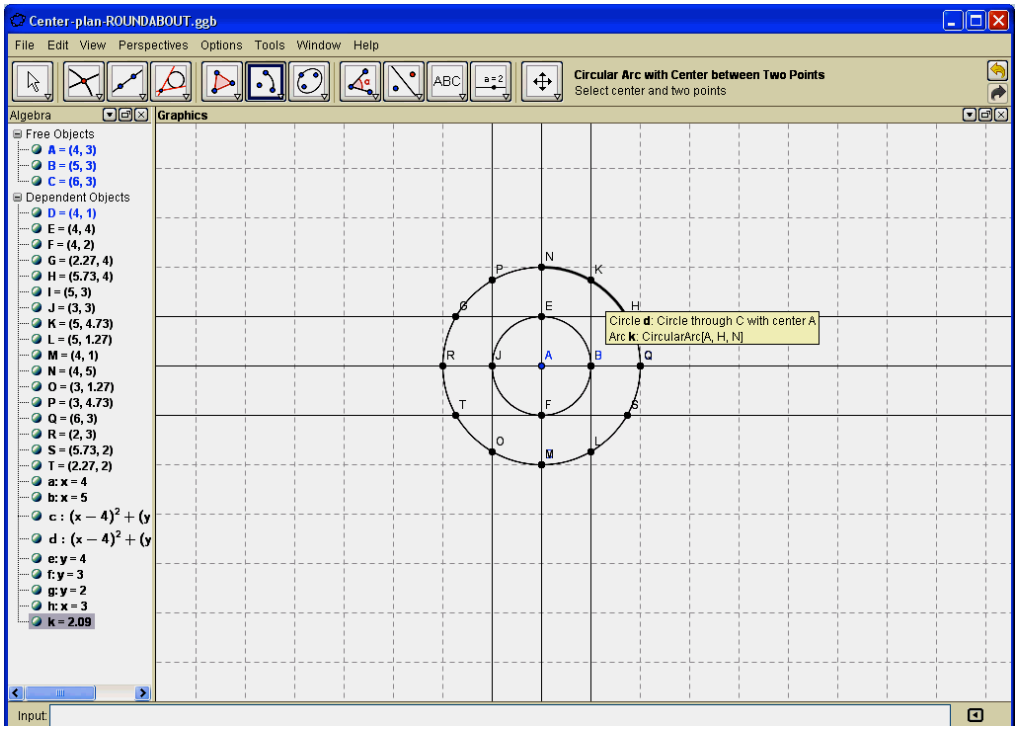
Using the Intersection Tool, find the points of intersection of the six lines passing through the Circle with Points C, H, and D.



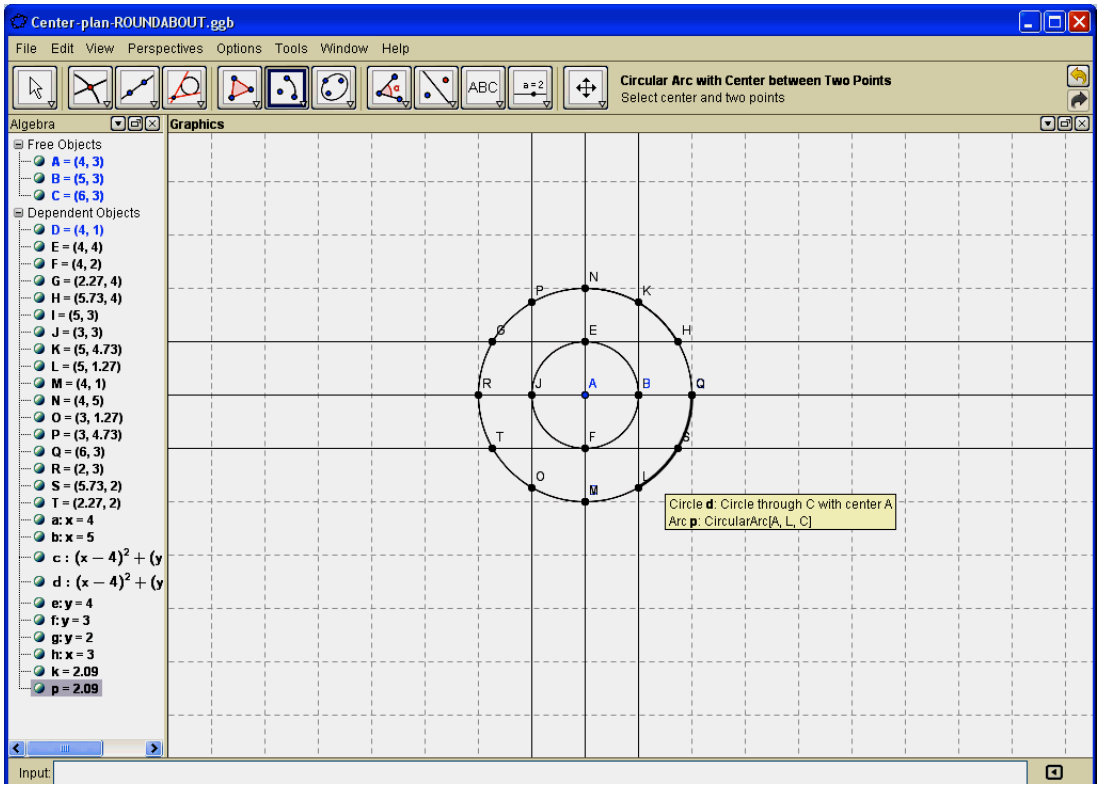
Using the Intersection tool, continue finding the points of intersection. Notice that Point C is now Point Q.

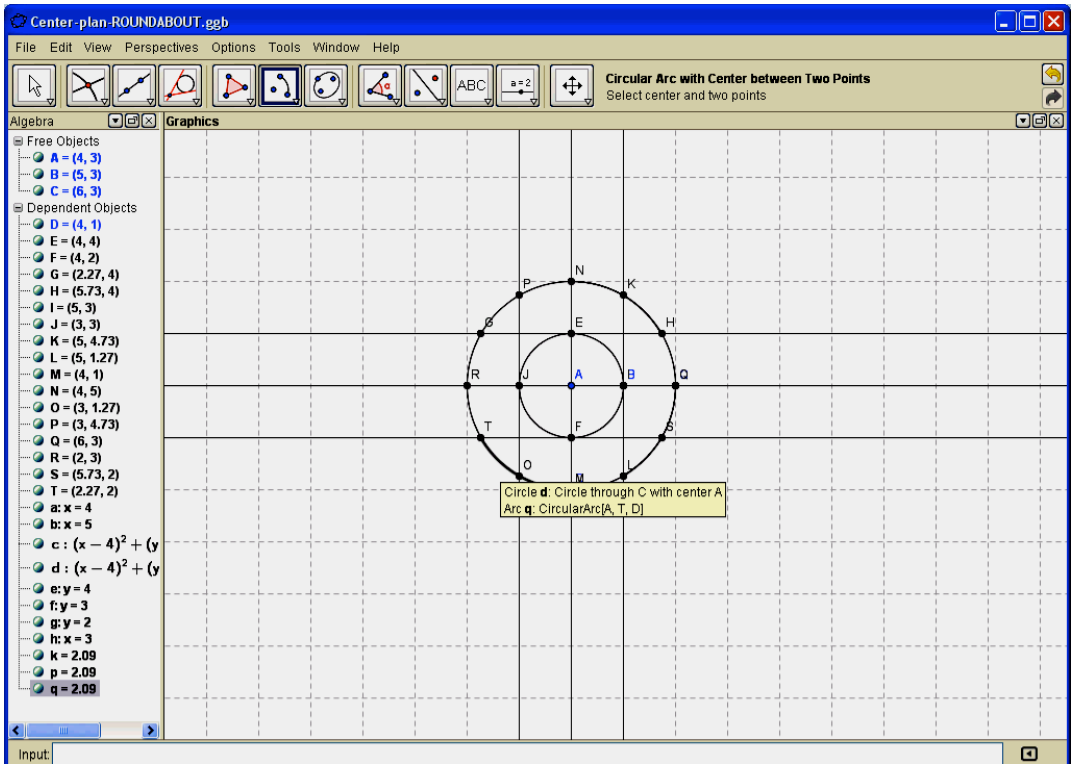


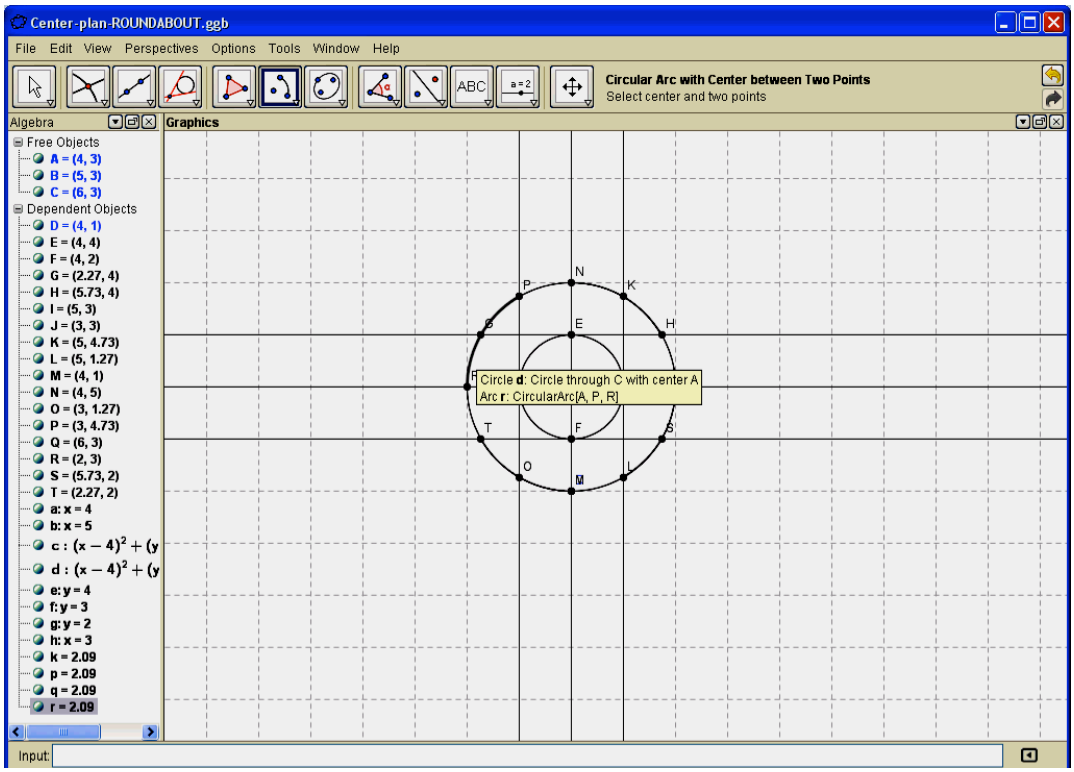
Using the Circular Arc tool, create minor arcs using Point A as the center and clicking on the two points that will be the endpoints of the arc. We created minor arc NH. Be sure the minor arc is darkened and not the major arc. If the major arc is created simply use the UNDO feature and click the endpoints of the arc in a different order.



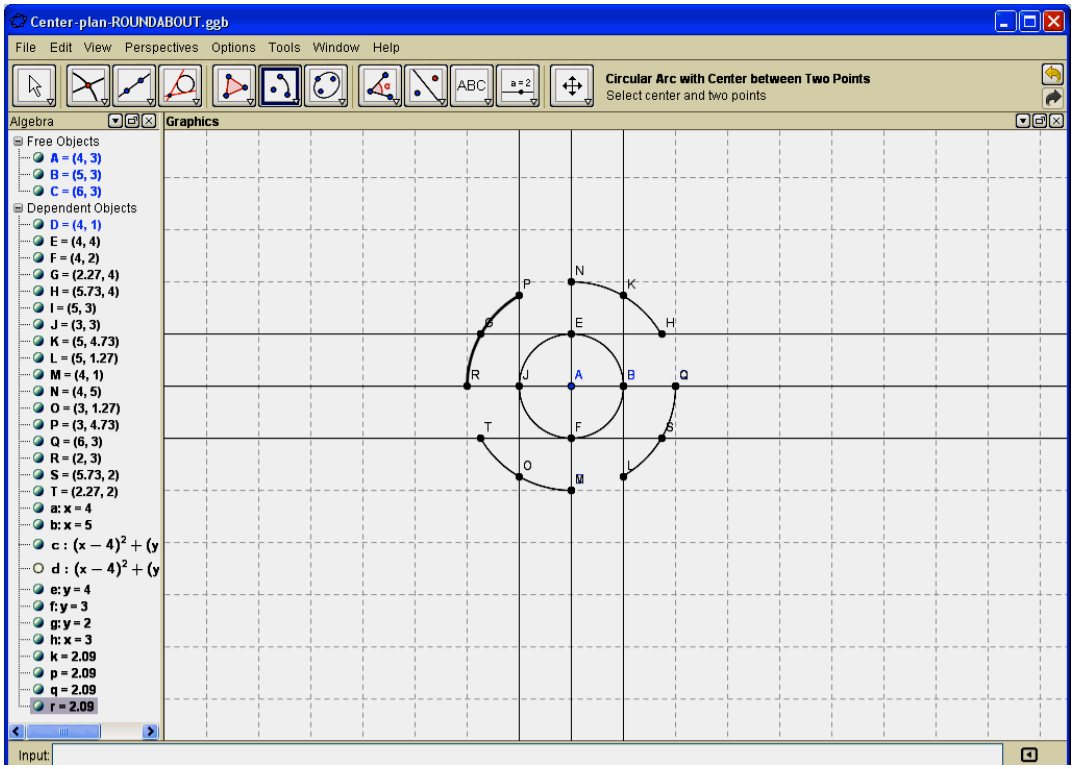
Using the Circular Arc tool, continue creating minor arcs.



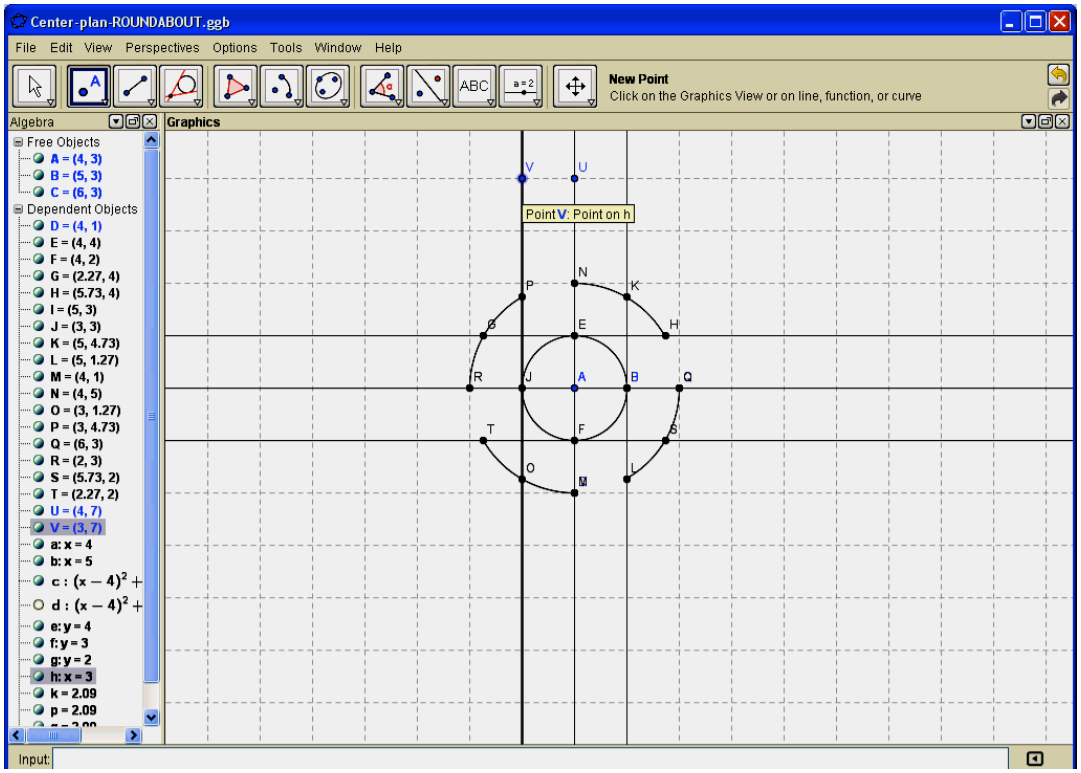




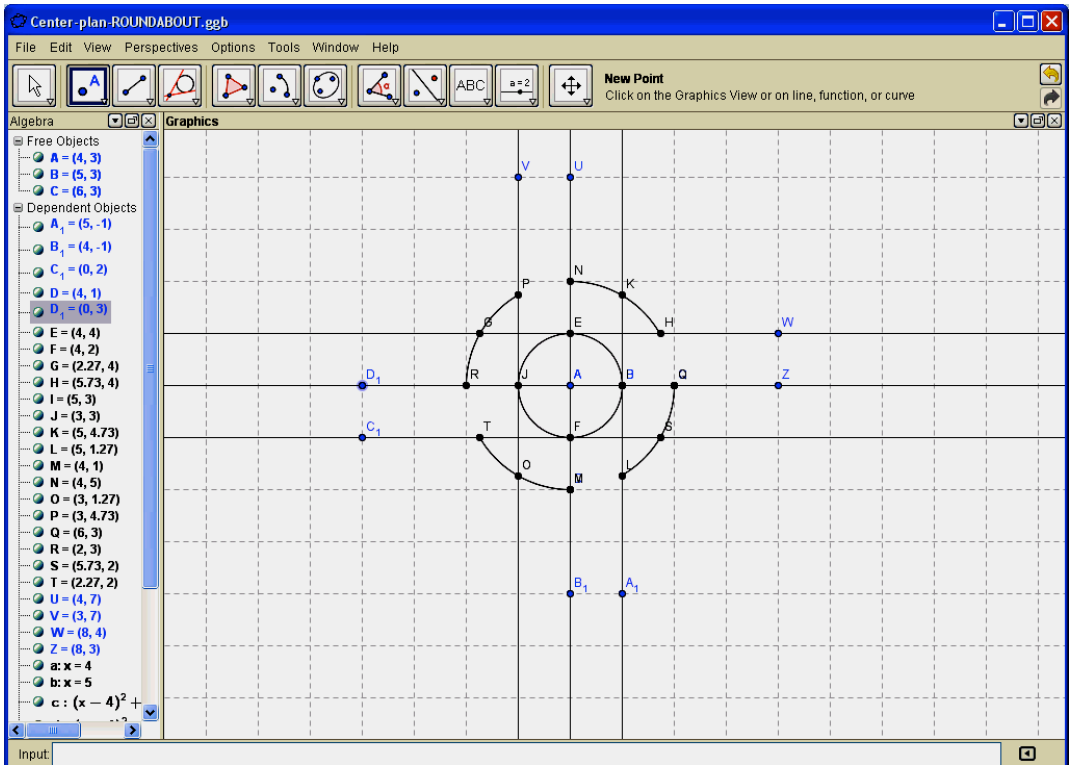
Once all of the minor arcs have been created, right click on the outside circle, away from any points, and unclick show object to hide the circle. This creates openings so that we can create openings for our road.



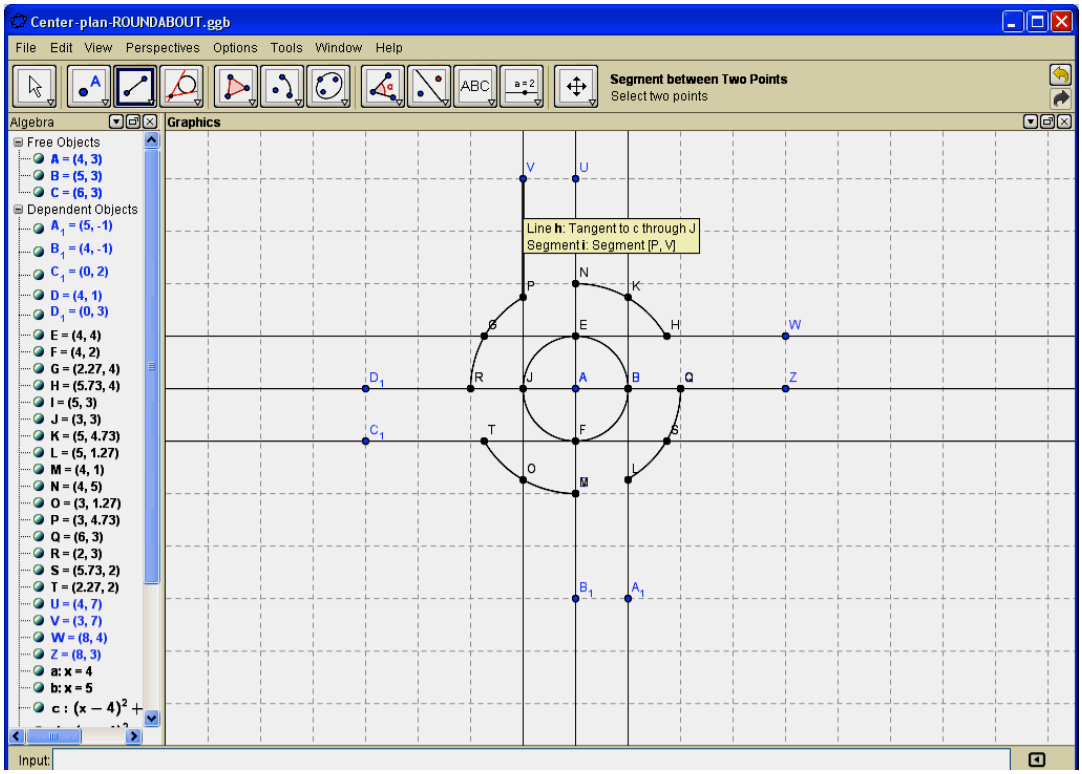
Using the Points tool, create points U and V.



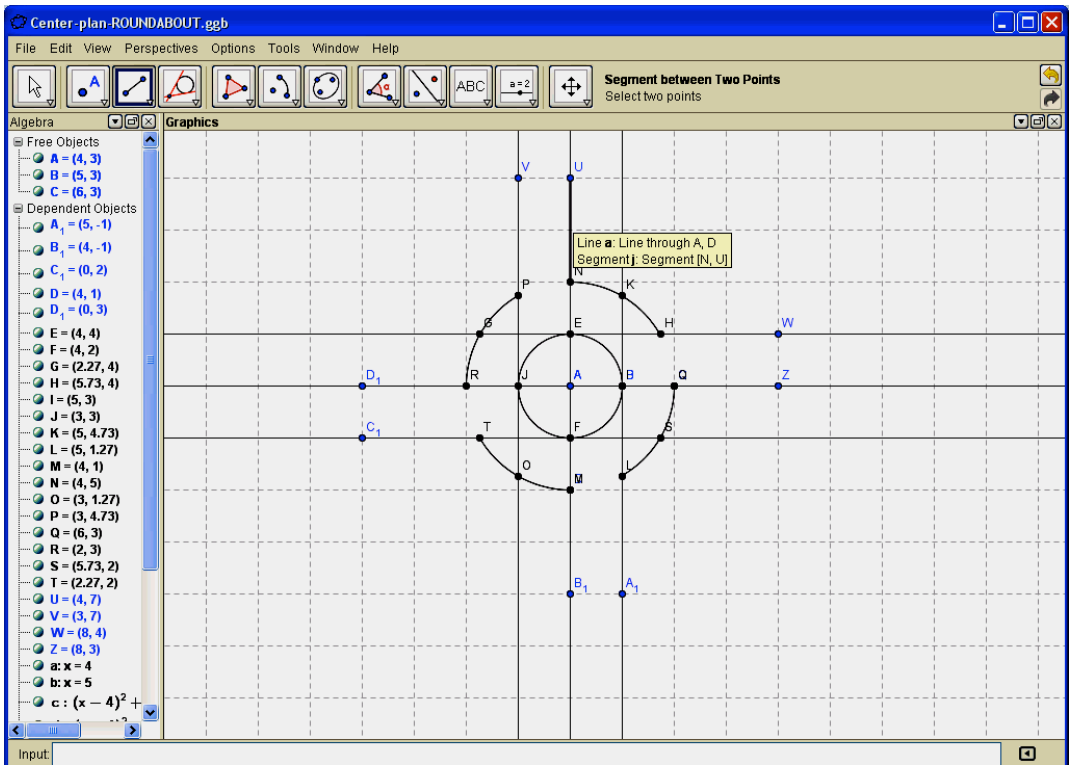
Continue creating points.



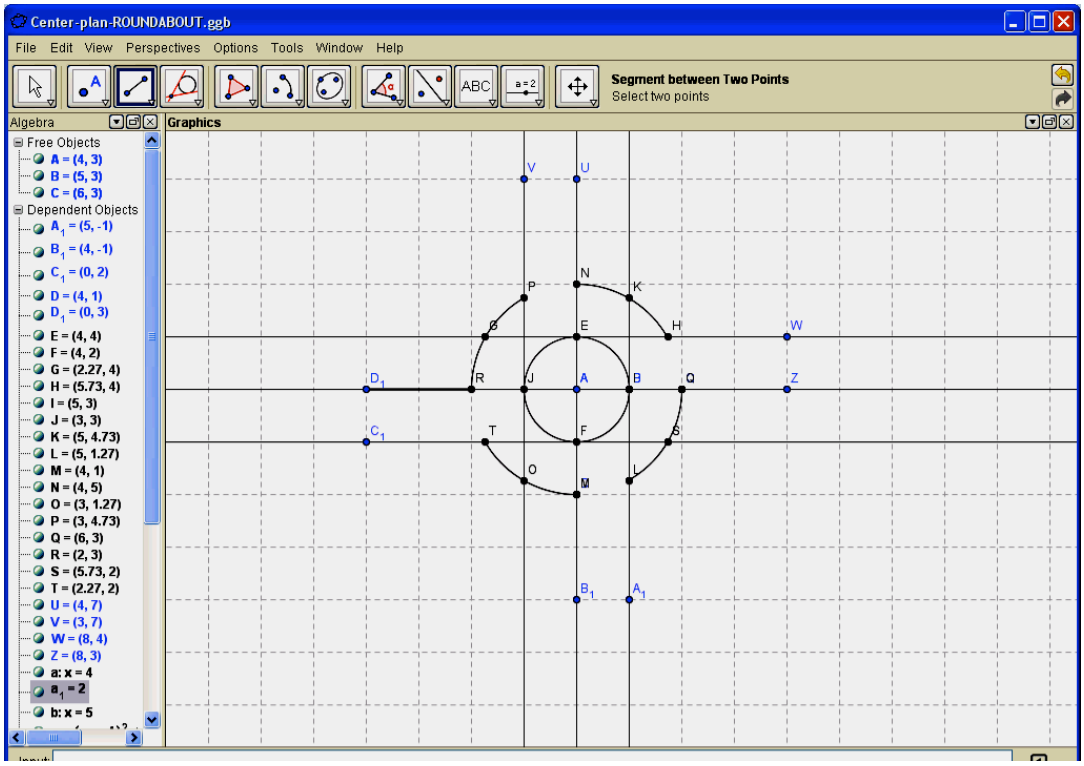
Using the segment tool, create segments using the recently created points and the openings of our circle.



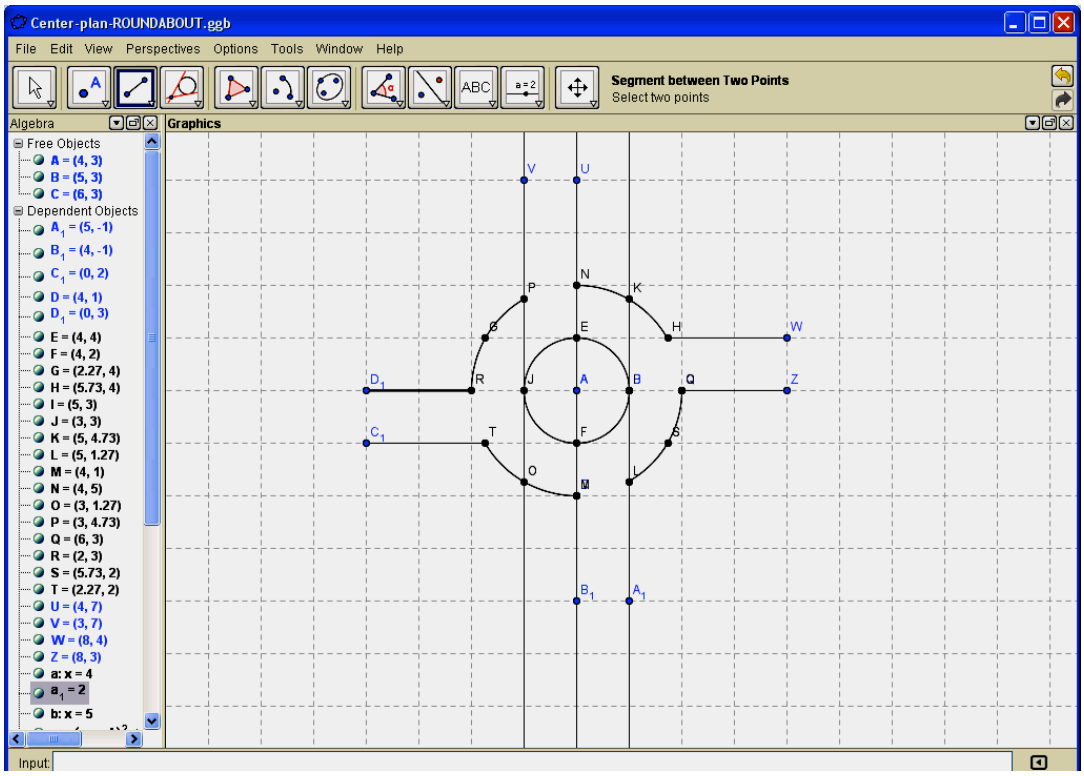
Create segment UN.



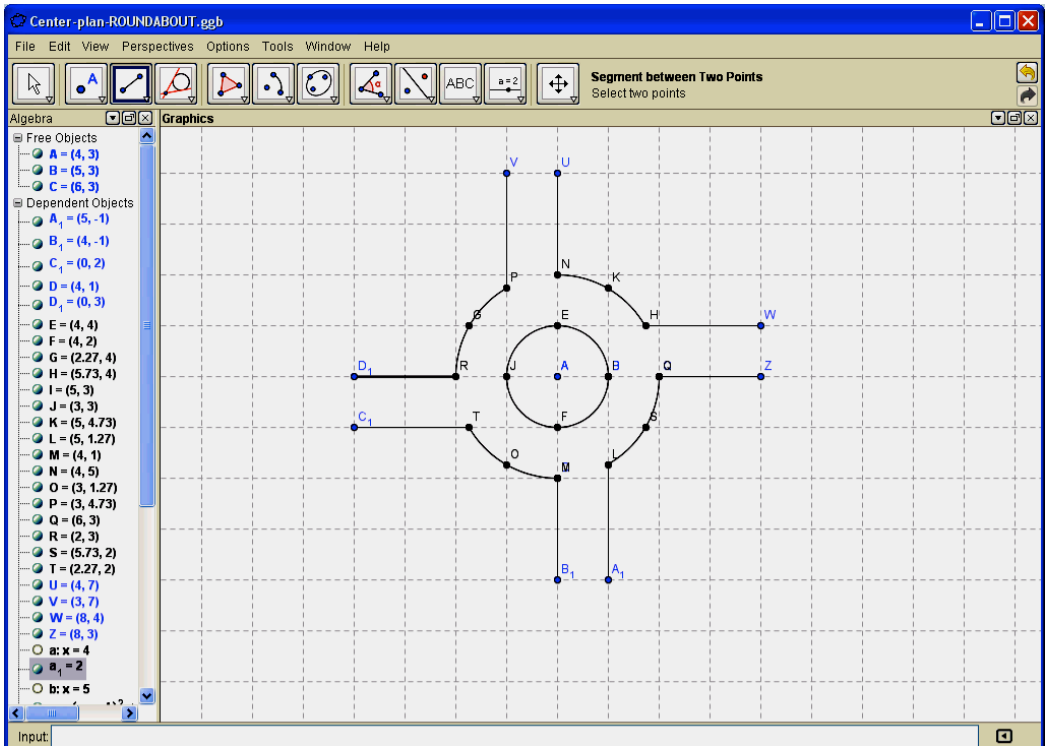
Using the Segment tool, create segments HW, QZ, LA_1, MB_1, TC_1, and RD_1.



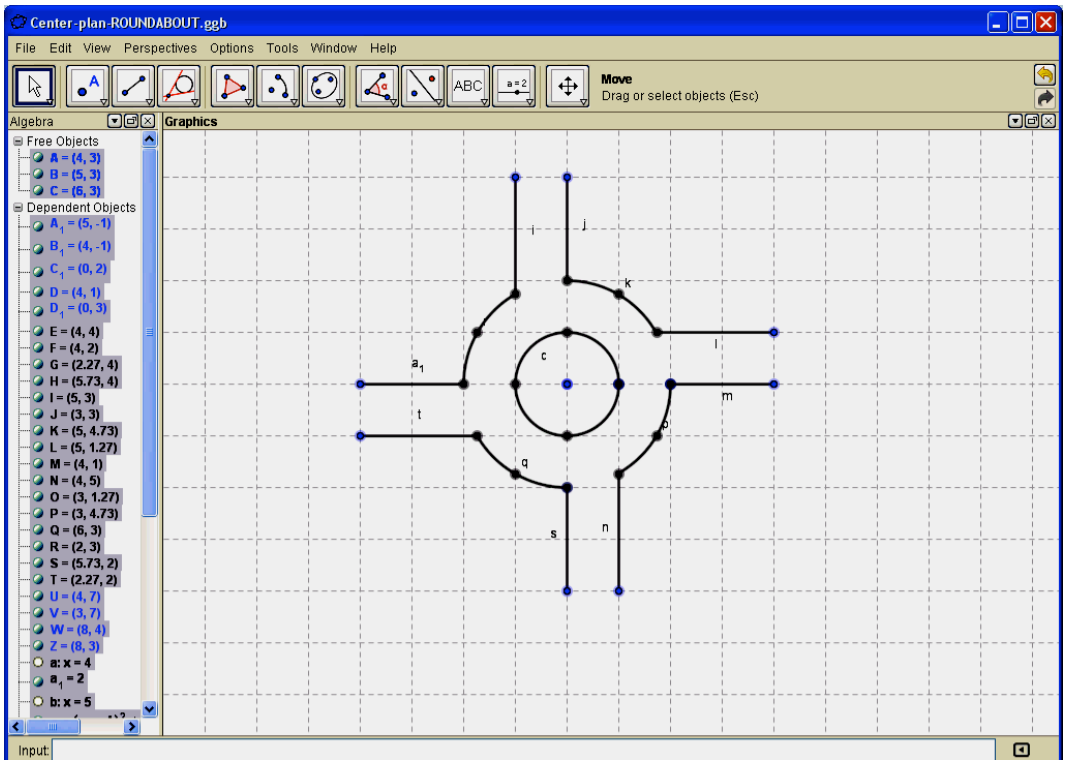
Right Click on the lines passing through the recently created segments to hide the line.



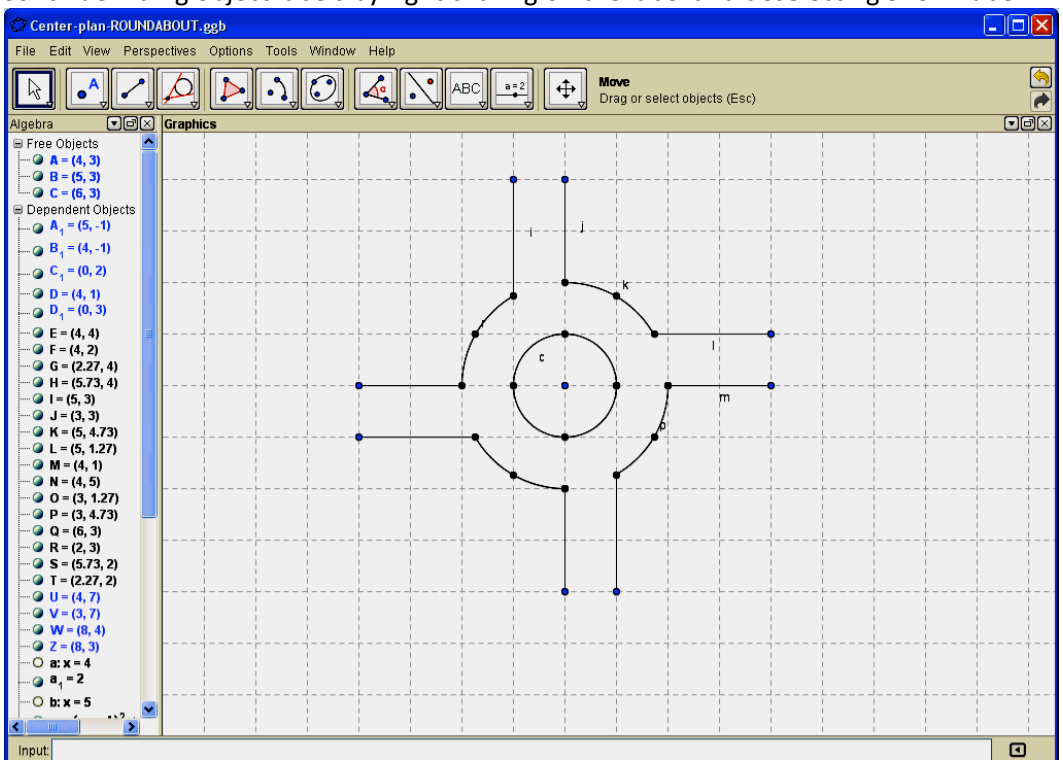
Continue hiding lines.



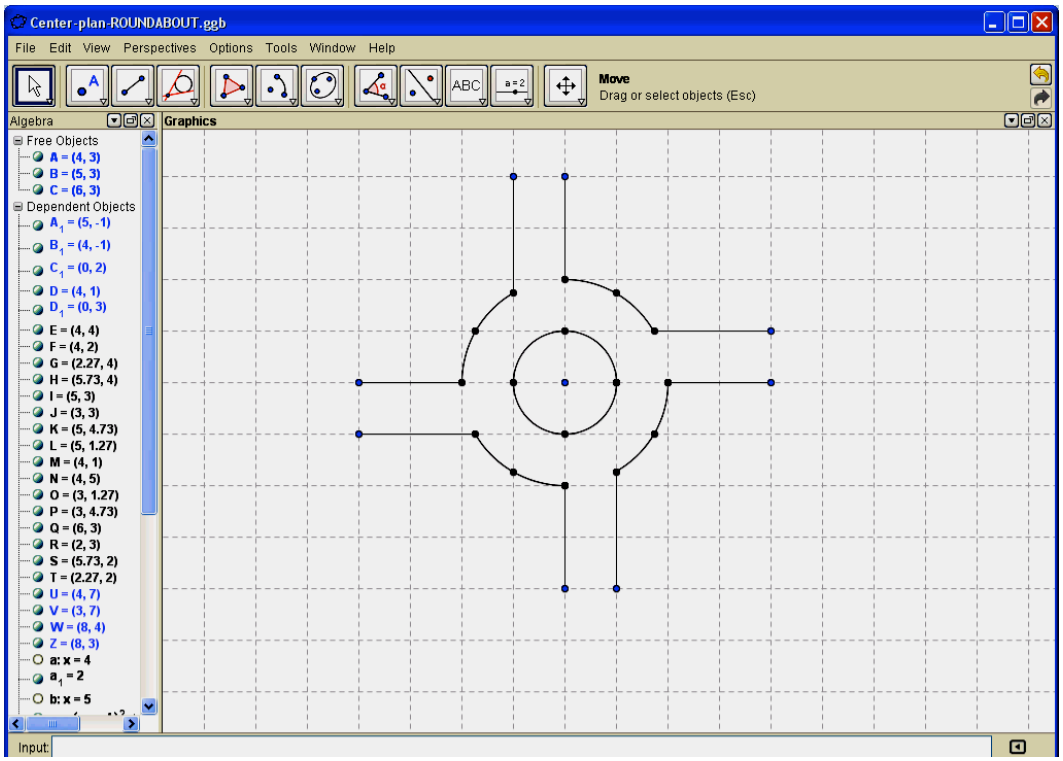
Now we can hide the labels. If you go to Edit, Select All, then Right Click and unclick Show Label, most of the labels will be hidden.



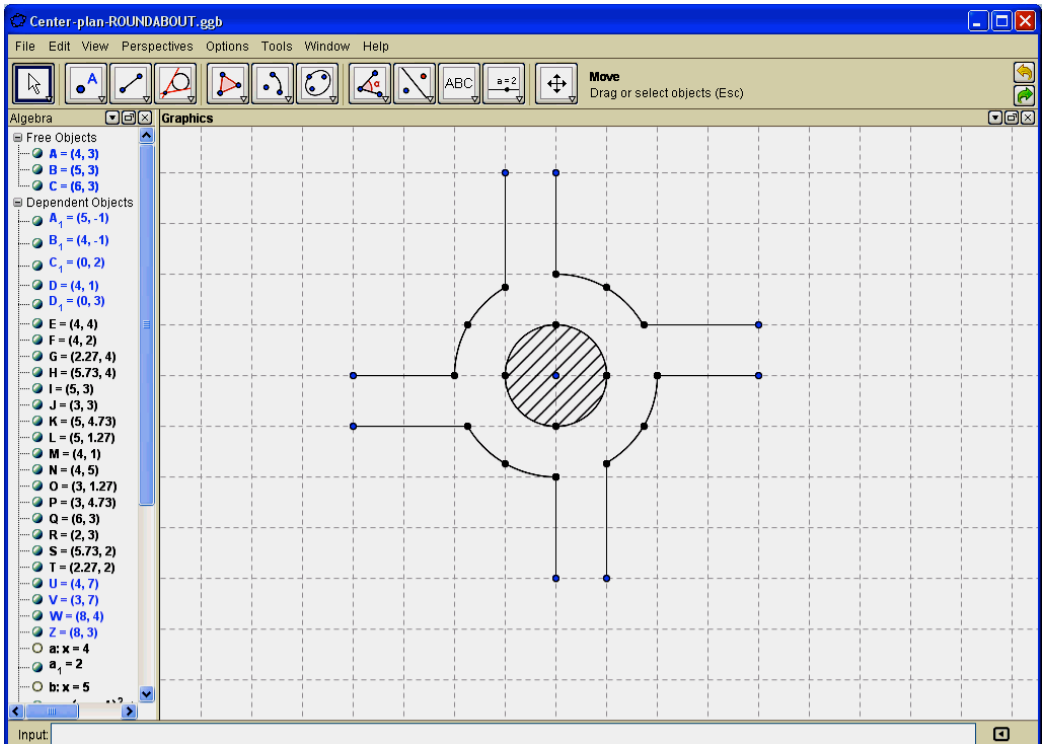
Continue hiding object labels by right clicking on the label and deselecting Show Label.



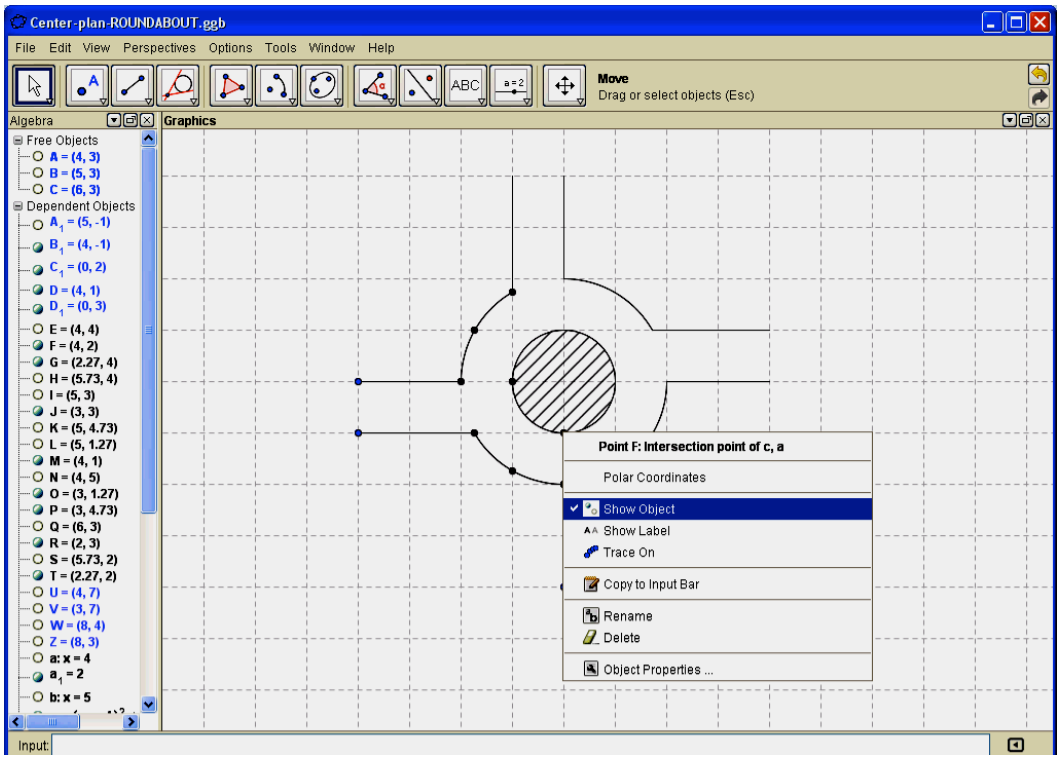
Continue hiding labels.



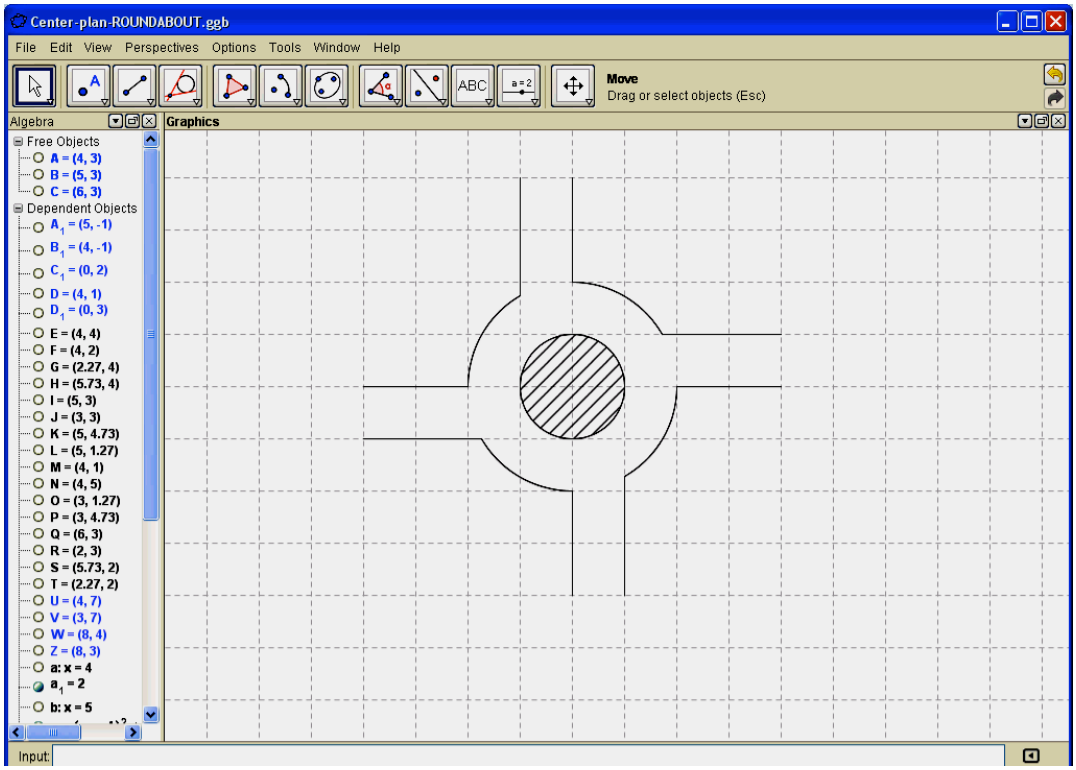
Right click on the small, inner circle and go to object properties. There you can add some filling to represent "green space".



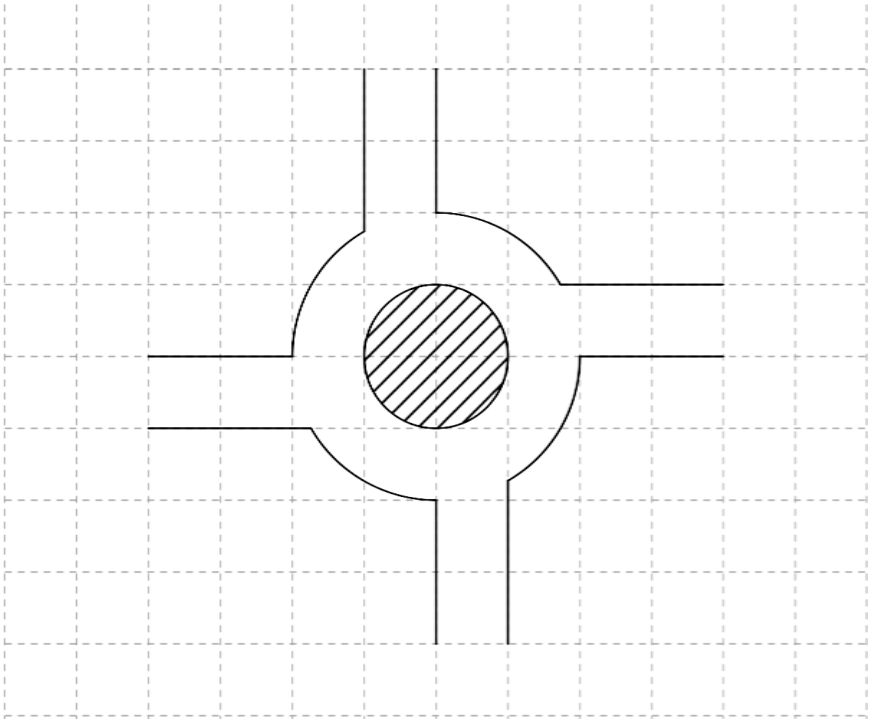
If you want, you can hide each of the points by right clicking on each point and deselecting Show Object.



Our final Traffic Circle with all objects and labels hidden.



You can go to File, Export, Graphics View to Clipboard and have a picture to insert into another document. Or, you can take a screenshot and paste into your document. Then crop.



Lesson 4

Strand

Geometry

Mathematical Objective(s)

The student will explore angle relationships formed when lines are cut by a transversal in city planning models, specifically bridge building.

Mathematics Performance Expectation(s)

Students will apply inductive and deductive reasoning skills to make and test parallel lines and the relationship between transversals and angles. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid.

MPE. 32 Use the relationships between angles formed by two lines cut by a transversal to

a) determine whether two lines are parallel;
b) verify the parallelism, using algebraic and coordinate methods as well as deductive proofs; and
c) solve real-world problems involving angles formed when parallel lines are cut by a transversal.

Related SOL

G.2 The student will use the relationships between angles formed by two lines cut by a transversal to

a) determine whether two lines are parallel;
b) verify the parallelism, using algebraic and coordinate methods as well as deductive proofs; and
c) solve real-world problems involving angles formed when parallel lines are cut by a transversal.

NCTM Standards List all applicable NCTM standards related to each lesson. Example:

Grades 9–12 Expectations: In grades 9–12 all students should–

Apply appropriate techniques, tools, and formulas to determine measurements.

Measurement

Understand measurable attributes of objects and the units, systems, and processes of measurement

- make decisions about units and scales that are appropriate for problem situations involving measurement
- **GEOMETRY**

Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships

- analyze properties and determine attributes of two- and three-dimensional objects
- establish the validity of geometric conjectures using deduction, prove theorems, and critique arguments made by others

Use visualization, spatial reasoning, and geometric modeling to solve problems

- use geometric models to gain insights into, and answer questions in, other areas of mathematics;
- use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture

Additional Objectives for Student Learning (include if relevant; may not be math-related):
World Geography
WG.1 The student will use maps, globes, satellite images, photographs, or diagrams to
b) apply the concepts of location, scale, map projection, or orientation;

WG.2 The student will analyze how selected physical and ecological processes shape the Earth's surface by
a) identifying regional climatic patterns and weather phenomena and their effects on people and places;
b) describing how humans influence the environment and are influenced by it;
c) explaining how technology affects one's ability to modify the environment and adapt to it.

WG.11 The student will analyze the patterns of urban development by
b) explaining how the functions of towns and cities have changed over time;
c) describing the unique influence of urban areas and some challenges they face.

Materials/Resources

- Handout – Student Research worksheet
- Rubric 1 – Collaborative Work Rubric
- Rubric 2 – Building Rubric
- Popsicle sticks & hot glue or straws & tape
- Classroom set of graphing calculators
- Rulers
- Computer lab

What are preparation considerations of the materials for the lesson? Attach copies of all supplemental materials. The teacher will need to purchase materials prior to the lesson. Supplemental materials are included in this document.

Assumption of Prior Knowledge

- Students at this point are well aware of the mathematics involved with parallel lines cut by a transversal. Students have had opportunities to sketch and create parallel lines and discuss the angles formed when those lines are cut by a transversal.
- Students are able to not only recognize angle relationships formed by parallel lines and transversals, but are now capable of arguing why lines are parallel or not. Students are above level 2 (Analysis) on the Van Hiele scale model. They are ready to test their knowledge for bridge building now on the Van Hiele abstraction and deductive reasoning levels.
- Students will notice that all types of bridges have some type of parallel lines. After researching bridges, students will notice certain types of bridges use the perpendicular transversal theorem.

- Students may have trouble at first figuring out which types of bridges are best in which situations.
- Alternate Interior angles, Alternate Exterior angles, Corresponding angles, and Consecutive Interior angles relationships have been studied.
- This lesson builds on Van Hiele level 2 (Analysis) and level 3 (Abstraction). Students will research bridges and have to make connections between bridge building and parallel lines cut by a transversal.

Introduction: Setting Up the Mathematical Task

- The goal of this lesson is to for students to investigate various types of bridges, and their geometry, so that they will be able to choose a correct bridge type when completing their island project in lesson 5.
- The time necessary will vary, depending on the depth at which the teacher wants to cover the material. The lesson could take one 45 minute class if the teacher chooses to have students learn the basics of the different types of bridges and their uses. The lesson can be extended using online videos, online bridge-building games, and a bridge-building activity.
- Students will research different types of bridges and their uses. Some bridges are better than others in particular locations and climate conditions. Students will eventually decide which types of bridge will be best for the island in their island project.
- Students will begin thinking about the task by brainstorming about bridges they are familiar with. Students could be asked to bring in pictures of bridges as a homework assignment preceding this lesson.
- What bridges do you see on a daily basis? What famous bridges do you know about? Have you visited any famous bridges? What are some different types of bridges? Why are bridges useful? What is the most important trait of a bridge? What happens to a bridge during an earthquake? What happens to a bridge during a hurricane? What makes a bridge strong? Can you think of any other concerns we should have about bridges?
- Once these questions have been discussed the teacher will have the students brainstorm the following statement: How does bridge-building relate to the geometry we have been studying (Parallel lines cut by a transversal)? After a few minutes of thinking/discussing with a partner, students will share their ideas verbally while the teacher compiles the list on the board.
- The beginning activity will be a think/pair/share discussion as we brainstorm about bridges. Students were placed in groups at the beginning of the unit and will continue in those groups for the duration of the unit.
- Activities that may move students toward stated objectives can include watching online video about bridge building, researching bridge building using the internet, researching bridges and natural disasters using the internet, and finally, choosing a bridge to build and test their new knowledge.
- Students will draw upon their previous knowledge during the think/pair/share activity while we are brainstorming bridges.
- The teacher will help students understand the task by researching internet sites and online videos to find safe websites that give age-appropriate information and safe, interesting, and informative online videos.
- Students will make their mathematical thinking and understanding public during the think/pair/share activities and during their bridge-building experience.

Student Exploration 1:

This exploration will have students investigate types of bridges, and their geometry, to find a bridge that will lead visitors from the mainland to the island, in their island project that culminates in lesson 5, and across a shorter body of water on the island.

Individual Work

Students may be asked to bring in pictures of bridges, from the internet or magazines, as a homework assignment preceding this lesson.

Small Group Work

Within their previously assigned small groups students will decide which bridges to research, how natural disasters affect bridges, and which bridges are best for particular types of crossings. The teacher can have all students completing the same research at one time or let each group decide how they would like to complete their necessary research.

Students may want to use the following websites to help with their research. Teachers are going to need to check to make sure that students can access these sites from school.

http://pics.tech4learning.com/details.php?img=portland_or_bridges.jpg
http://pics.tech4learning.com/details.php?img=dsc00330.jpg
http://www.pbs.org/wgbh/nova/tech/build-bridge-p1.html
http://www.eduweb.com/portfolio/bridgetoclassroom/engineeringfor.html

Student Research Worksheet Here

Student/Teacher Actions:

- Students should be researching, with guidance from their Student Research Worksheet, and the teacher should be making sure students are staying on task and finding their information.

Monitoring Student Responses

- Describe how you expect:
    - Each student will complete a Student Research Worksheet that will serve as a guide for their research and a way to communicate their thinking and new knowledge.
    - Students will use their Research Worksheet to help guide communication as they research. Once the research is completed, students will discuss their findings and submit one research worksheet for grading (all worksheets will be turned in, but only one will be graded).
    - When students have difficulties with their research the teacher will help guide the student to one of the recommended websites.
    - For groups that finish early, the teacher may recommend for students to spend time playing one, or several, of the interactive games that can be found on the included websites.
- How do you plan to summarize your lesson?
    - The lesson will be summarized using the small-group discussion of the Research Worksheet and through a journal entry.
    - Student knowledge will be collected using verbal, whole-class discussions, small group discussions, the Student Research Worksheet, and the journal entry.

Assessment

- Describe and attach the assessments for each lesson objective.
    - One assessment for student exploration 1 is the Student Research Worksheet
    - A second assessment is the Collaborative Work Rubric that each student will complete after working with their groups.

Collaborative Work Rubric Here

    - **Journal/writing prompts**

- Discuss the connection between building strong bridges and geometry. In particular, discuss the use of parallel lines and transversals. You can create a diagram to help with your explanation.

Extensions and Connections

The lesson can be extended by students actually building a bridge using Popsicle sticks and hot glue or drinking straws and clear tape. By having students physically build a bridge, the students experience the excitement and frustrations that come with creating a working object from scratch. The following materials can help if the teacher decides to extend the lesson.

http://www.wonderhowto.com/how-to-make-bridge-out-popsicle-sticks-4429/

Bridge Building Rubric Here

Strategies for Differentiation

- List ideas for addressing needs of a diverse population of students such as:
  - The activities in this lesson are designed with many learner types in mind. Kinesthetic students will have the ability to manipulate the geometric tools, auditory and visual learners will be able to hear and see instructions, demonstrations, music, and video clips.
  - English Language Learners have the ability to change the language on their internet search, if they desire.
  - Students with processing, memory, or motor issues will benefit from the group work throughout the activities.

- Each group can assign particular jobs to group members. Each group member can create a checklist of their required responsibilities.

Parallel Lines, Transversals, and Angles: What's the connection?

Student Research Worksheet

Please complete each question. Write down the address of any websites you use.

Name of student completing worksheet:

Names of other group members:

1. Identify several types of bridges. Include a basic sketch of each type of bridge.

2. Identify the best location for different types of bridges. For example, what bridge design is best to use for long spans over water? Explain why the particular bridge is best.

3. Identify several things that can weaken the strength of a bridge.

4. Identify several ways geometry is used to strengthen a bridge.

5. Identify several weather-related disasters that may weaken or destroy a bridge.

6. What types of people research, plan, and build bridges?

7. What affects the cost of a bridge?

8. How long does it take (on average) to build a bridge?

9. Are there any "Green", environmentally friendly, methods to bridge building?

10. How does building a bridge affect the eco-system of the area where the bridge is being located?

Collaborative Work Rubric

Please fill out one of these rubrics for each person in your group. Remember to be honest. Include your group name to ensure proper credit is given for completion.

Group Name: _____

This is a collaborative work review for : _____

CATEGORY	4	3	2	1	0	Score
Contributions	Routinely provides useful ideas when participating in the group.	Usually provides useful ideas when participating in the group.	Sometimes provides useful ideas when participating in the group.	Rarely provides useful ideas when participating in the group.	Does not participate in the group.	
Attitude	Attitude about the plan and construction is primarily positive.	Attitude about the group plan and construction is positive.	Attitude about the group, plan and construction is generally negative.	Attitude is negative regarding the group, plan and/ or construction.	Attitude is very negative regarding all aspects of the building process.	
Working with Others	Almost always listens to, shares with, and supports the efforts of others.	Usually listens to, shares, with, and supports the efforts of others.	Often listens to, shares with, and supports the efforts of others, but may not express best team member.	Rarely listens to, shares with, and supports the efforts of others. Often is not a good team player.	Student does not respect other's ideas or tries undermines the group mentality.	
Focus on the task	Consistently stays focused on the task and what needs to be done. Does not need any reminders to stay on track with the assignment.	Focuses on the task and what needs to be done most of the time. May need 1-2 reminders to stay on track with the assignment.	Focuses on the task and what needs to be done some of the time. May need 3-4 reminders to stay on track with the assignment.	Has difficulty focusing on the task and may need multiple reminders from the teacher or other group members to stay on track with the assignment.	Did not stay on track with the assignment and prevented others from focusing on the task.	
Preparedness	Always ready to work by bringing any needed materials.	Almost always brings needed materials to class and is ready to work.	Almost always brings needed materials but sometimes needs to settle down and get to work. May have to be reminded to work.	Often forgets needed materials or is rarely ready to get to work. Has to borrow materials.	Does not have required materials and makes no effort to obtain them.	
Total						_____/25

Provide at least one comment regarding working with this group member.

Would you want to work with this group member in the future? Why or why not?

Bridge Building Rubric

Student Name: _____

CATEGORY	4	3	2	1	0	Score
Plan	Plan is neat with clear measurements and labeling for all components.	Plan is neat with clear measurements and labeling for most components.	Plan provides clear measurements and labeling for most components.	Plan does not show measurements clearly or is otherwise inadequately labeled.	No plans were created or made available with the completed bridge.	
Bridge Building Materials	Materials were selected and modified appropriately.	Materials were selected and there was an attempt to modify the materials appropriately.	Appropriate materials were selected.	Inappropriate materials were selected and contributed to a product that performed poorly.	The materials that were selected contributed to a completely non-functional bridge.	
Construction - Care Taken	Great care was taken in the construction process so that the bridge is neat, attractive and follows the submitted plans accurately.	Construction accurately followed the submitted plans careful and accurate for the most part, but 1-2 details could have been refined for a better bridge.	Construction followed the submitted plans, but 3-4 details could have been refined for a better bridge.	Construction appears careless or haphazard. Many details need refinement for a strong or attractive product.	Construction appears careless, untidy or incomplete.	
Function	Structure functions extraordinarily well, holding up under atypical stresses.	Structure functions well, holding up under typical stresses.	Structure functions pretty well, but deteriorates under typical stresses.	Fatal flaws in function with almost complete failure under typical stresses.	Structure does not stand on its own power.	
Quality of Work	Provides work of the highest quality that the group could produce.	Provides high quality work.	Provides average quality work.	Provides work that usually needs to be redone by others to ensure quality.		
Total						_____/25

Comments:

Lesson 5

Strand
Geometry
Mathematical Objective(s)
The student will explore angle relationships formed when lines are cut by a transversal in city planning models, specifically bridge building.
Mathematics Performance Expectation(s)

Students will apply inductive and deductive reasoning skills to make and test parallel lines and the relationship between transversals and angles. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid.

MPE. 32 Use the relationships between angles formed by two lines cut by a transversal to
a) determine whether two lines are parallel;
b) verify the parallelism, using algebraic and coordinate methods as well as deductive proofs; and
c) solve real-world problems involving angles formed when parallel lines are cut by a transversal.

Related SOL
G.2 The student will use the relationships between angles formed by two lines cut by a transversal to
a) determine whether two lines are parallel;
b) verify the parallelism, using algebraic and coordinate methods as well as deductive proofs; and
c) solve real-world problems involving angles formed when parallel lines are cut by a transversal.

NCTM Standards List all applicable NCTM standards related to each lesson. Example:
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Apply appropriate techniques, tools, and formulas to determine measurements.
Measurement
Understand measurable attributes of objects and the units, systems, and processes of measurement
- make decisions about units and scales that are appropriate for problem situations involving measurement
- **GEOMETRY**

Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships
- analyze properties and determine attributes of two- and three-dimensional objects
- establish the validity of geometric conjectures using deduction, prove theorems, and critique arguments made by others

Use visualization, spatial reasoning, and geometric modeling to solve problems
· use geometric models to gain insights into, and answer questions in, other areas of mathematics;

· use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such
as art and architecture

Additional Objectives for Student Learning (include if relevant; may not be math-related):
World Geography
WG.1 The student will use maps, globes, satellite images, photographs, or diagrams to
b) apply the concepts of location, scale, map projection, or orientation;
d) create and compare political, physical, and thematic maps;

WG.2 The student will analyze how selected physical and ecological processes shape the Earth's surface by
a) identifying regional climatic patterns and weather phenomena and their effects on people and places;
b) describing how humans influence the environment and are influenced by it;
c) explaining how technology affects one's ability to modify the environment and adapt to it.

WG.11 The student will analyze the patterns of urban development by
b) explaining how the functions of towns and cities have changed over time;
c) describing the unique influence of urban areas and some challenges they face.

Materials/Resources
- Rubric 1 – Collaborative Work Rubric
- Classroom set of graphing calculators
- Rulers
- Computer lab

Supplemental materials are included in this document.

Assumption of Prior Knowledge
- Students at this point are well aware of the mathematics involved with parallel lines cut by a transversal. Students have had opportunities to sketch and create parallel lines and discuss the angles formed when those lines are cut by a transversal.
- Students are able to not only recognize angle relationships formed by parallel lines and transversals, but are now capable of arguing why lines are parallel or not. Students are above level 2 (Analysis) on the Van Hiele scale model. They are ready to test their knowledge for bridge building now on the Van Hiele abstraction and deductive reasoning levels.
- Students will notice that all types of bridges have some type of parallel lines. After researching bridges, students will notice certain types of bridges use the perpendicular transversal theorem.
- Students may have trouble at first figuring out which types of bridges are best in which situations.
- Alternate Interior angles, Alternate Exterior angles, Corresponding angles, and Consecutive Interior angles relationships have been studied.
- This lesson builds on Van Hiele level 2 (Analysis) and level 3 (Abstraction). Students will research bridges and have to make connections between bridge building and parallel lines cut by a transversal.

Introduction: Setting Up the Mathematical Task

- In this lesson, all of the previous concepts learned from lessons 1 – 4 are brought together. Students will need to use their knowledge of geometry, parallel lines, transversals, angles, maps, and bridges to design the plan for the city found on the island.
- The timeline for this lesson can range from one 45 minute lesson to several 45 minute lessons.
- Students will be given their island on a sheet of paper and written instructions that will direct their small group to organize and plan the important aspects of their design.
- Students will be given the basic introduction (from lesson 2) and then students will think/pair/share ideas.
- What is important to you to make sure you have on your island (in your city)? Why is this important to you? What else do you need on your island? How will visitors get to your island? How will goods be transported on your island? Where will people live on your island? How will you insure the safety of the people on your island? What is the name of your island? What language do the people of your island speak? What geometric shapes/patterns/concepts will be used when designing and implementing your city plan?
- During discussions students will use think/pair/share. But, otherwise students will be working in their small groups.
- Students will complete the Parallel Line Map project that will help them synthesize previously learned information.
- Students will make their understanding public by presenting their island to the class.

Student Exploration 1:

Small Group Work

Students will be working in their small groups and complete the parallel line island map activity.

Whole Class Sharing/Discussion

Groups will share their projects by presenting to the class.

Student/Teacher Actions:

- Students will complete the Parallel line island map activity.
- Teachers should circulate and make sure that students are staying on task and following instructions correctly.
- Students could potentially create these projects using Geogebra or Geometer's Sketchpad.

Parallel Line Island Map Project Here

Monitoring Student Responses

- Describe how you expect:
    - Students will discuss in small groups and share ideas with the class.
    - When students have difficulties the teacher will need to help guide them through that particular portion.

- For groups that finish early they may want to either create a second island or add optional objects to their original island. For example, add an airport or a park.

- How do you plan to summarize your lesson? Describe it here.
  - Once students have completed their projects and presented their islands they will complete a journal entry and have a class discussion to summarize their learning.
  - Student knowledge will be collected using the parallel line island map project, group discussions, and journal entry.

Assessment

- Describe and attach the assessments for each lesson objective.

  Parallel line island map project rubric

  Collaborative Work Rubric

Journal/writing prompts

- Think about the various bridges you have researched. What similarities and differences do these bridges have in relation to parallel lines cut by a transversal? Be as specific as possible.

Extensions and Connections (for all students)

The lesson can be extended by students actually building a bridge using Popsicle sticks and hot glue or drinking straws and clear tape. By having students physically build a bridge, the students experience the excitement and frustrations that come with creating a working object from scratch. The following materials can help if the teacher decides to extend the lesson.

http://www.wonderhowto.com/how-to-make-bridge-out-popsicle-sticks-4429/

Bridge Building Rubric Here

Strategies for Differentiation

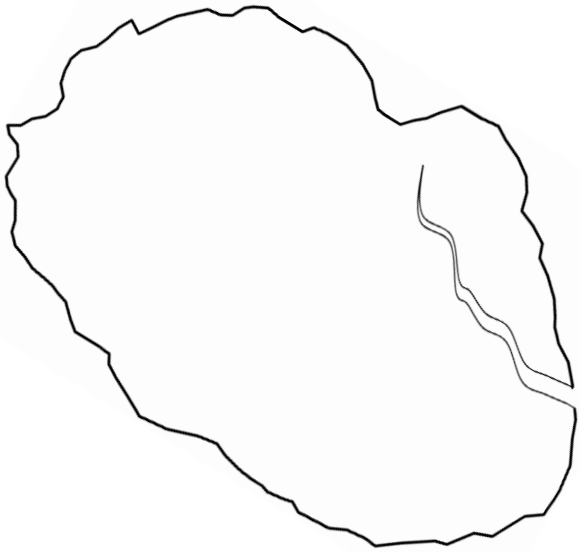
- List ideas for addressing needs of a diverse population of students such as:
  - The activities in this lesson are designed with many learner types in mind. Kinesthetic students will have the ability to manipulate the geometric tools, auditory and visual learners will be able to hear and see instructions, demonstrations, music, and video clips.
  - English Language Learners have the ability to change the language on their internet search, if they desire.
  - Students with processing, memory, or motor issues will benefit from the group work throughout the activities.

- Each group can assign particular jobs to group members. Each group member can create a checklist of their required responsibilities.

Map Project
Parallel Lines and Angles

Name: _____ Group Members: _____

You have just been hired as the city planner of a remote, deserted island. Your first task is to plan and design the first community on this island. While you have free rein to include any structures you wish, in any orientation you wish, you may want to consider about a layout that will work with the traffic, emergency situations and daily life on the island. Emergency situation may include natural disasters as well as unnatural incidents. You must include the items listed below but also include places that people need to live their lives in a healthy, meaningful manner.



I. At least two sets of streets that are parallel (4 streets)

II. At least two sets of streets that are perpendicular (4 streets)

III. At least one street that intersects other streets to form an obtuse angle

IV. At least one street that intersects other streets to form an acute angle

V. One street that is a line segment

VI. One street that is a ray

VII. A restaurant in the shape of a scalene triangle

VIII. An elementary school that is in the shape of an equilateral triangle

IX. A weather station in the shape of an isosceles triangle

X. Include a compass rose

XI. A bridge to the island.

XII. A bridge on the island to cross a smaller body of water.

Maps should fit on 8 ½ by 14 paper or larger.

Label each of the above places with the appropriate Roman numeral. Remember when creating your city consider all the places that have not been included. Think about including places that are **necessary** to the lives of your new inhabitants. You may want to refer to a map of real cities, towns and villages to assist you with what to include and their placement. Label and list all additional places with alphabets.

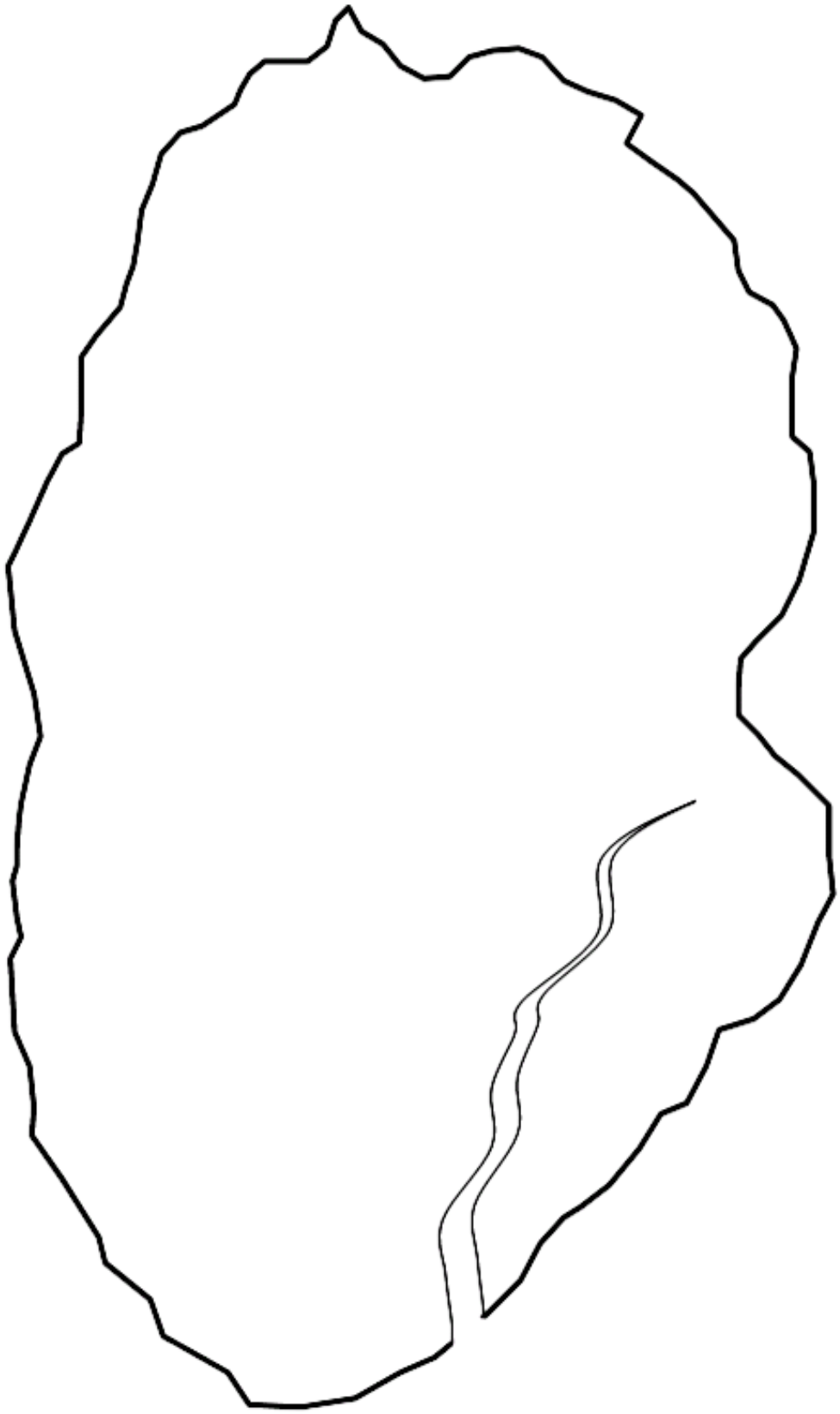
Brain storming ideas
What types of disasters might my inhabitants need to prepare?

How will traffic flow in and out of my city?

Additional places I will need to include:

Parallel Lines, Transversals, and Angles: What's the connection?

Name _____ Members of Group: _____



Map Project
Parallel Lines and Angles Grading Rubric

Name: _____ Group Members: _____

Below is the grading rubric which will be used for this extended learning opportunity. Make sure you check your work against it to see what you may be missing prior to submitting your final sketch.

I. At least two sets of streets that are parallel (4 streets total)	II. At least two sets of streets that are perpendicular (4 streets total)	III. At least one street that intersects other streets to form an obtuse angle	IV. At least one street intersects another to form an acute angle
V. One street that is a line segment	VI. One street that is a ray	VII. An restaurant in the shape of a scalene triangle	VIII. An elementary school in the shape of an equilateral triangle
IX. A weather station that is in the shape of an isosceles triangle	X. Compass rose	XI. Bridge to island	XII. Bridge on island.

Each item 4 points each.

Total _____