Next Top Model

I. UNIT OVERVIEW & PURPOSE:

This unit will allow students to use their knowledge of proportional reasoning and similar geometric figures to explore how 2-dimensional models are used to represent 3-dimensional objects and spaces. They will then apply their knowledge of similarity to create their own 2-dimensional models. Students will then explore how 3-dimensional models are created to represent large objects, and how such models are interpreted and used to design and plan projects. Students will then explore how changes in dimension affect similar objects, and the models which represent them.

II. UNIT AUTHOR:

Stephanie Brady, Patrick Henry High School, Hanover County Public Schools
Christine Belcher, Patrick Henry High School, Hanover County Public Schools
Hedy Keller, Powhatan High School, Powhatan County Public Schools

III. COURSE:

Mathematical Modeling: Capstone Course

IV. CONTENT STRAND:

Geometry, Measurement

V. OBJECTIVES:

The student will use standard measurement tools to determine the size of three dimensional objects, use the properties of similar figures to create 2-dimensional scale drawings of 3-dimensional objects, use the properties of similar figures to create 3-dimensioal scale models, use scale models to analyze similar figures by comparing measures and determine how changes in one dimension affect area and volume of 3-dimensional figures.

VI. MATHEMATICS PERFORMANCE EXPECTATION(s):

MPE 6: The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

MPE 7: The student will use similar geometric objects in two- or three-dimensions to

a) compare ratios between side lengths, perimeters, areas, and volumes;

b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;

c) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and

d) solve real-world problems about similar geometric objects.

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VII. CONTENT:

Students will use mathematics to interpret and analyze scale models such as those found in the fields of architecture, construction and other building trades in 2 and 3 dimensions. Students will learn to use the library and internet as resources to find plans, blue prints or other scale models to help them purchase or build offices or homes.

VIII. REFERENCE/RESOURCE MATERIALS:

- http://www.visitingdc.com/white-house/white-house-floor-plans.htm
- http://www.cnn.com/2009/POLITICS/02/02/inside.thewhitehouse/
- http://www.whitehouse.gov
- http://www.whitehousemuseum.org
- http://www.youtube.com/watch?v=q4SLBdrwzzE
- http://www.youtube.com/watch?v=RJLrkbBvK3Y

IX. PRIMARY ASSESSMENT STRATEGIES:

Assessment will be through scoring of students' floor plans, blue prints, models and final report.

X. EVALUATION CRITERIA:

Explorations products and journal prompts with rubrics attached

XI. INSTRUCTIONAL TIME:

Nine and a half 90-minute blocks.

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Lesson 1: Prints for a Prince (or Princess)

Strand
Geometry, Measurement

Mathematical Objective(s)

- Solving proportions
- Measuring with a measuring tool.
- Calculating a scale factor.
- Students will be able to measure a room and objects in a room and use this information to develop a floor plan of the space and items in the room

College and Career Ready Mathematics Performance Expectation

MPE 7: Use similar geometric objects in two- or three- dimensions to

a) compare rations between side lengths, perimeters, areas, and volumes;
b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;
c) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and
d) solve real-world problems about similar geometric objects.

RELATED SOL

G.14 A AND G.14 D

NCTM Standards

- analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships;
- apply appropriate techniques, tools, and formulas to determine measurements.
- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems;
- monitor and reflect on the process of mathematical problem solving.
- communicate mathematical thinking coherently and clearly to peers, teachers, and others

Additional Objectives for Student Learning

Read, analyze, apply and create floor plans such as those found in tourism and building trades course and careers.

Materials/Resources

- Classroom set of straight edges
- Classroom set of graphing calculators
- ¼ " graph paper
- Display sized graph paper (optional)
- Computer access
- White House floor plan handout

Assumption of Prior Knowledge

- It is assumed that students have studied similar figures and solved proportions in previous math classes.
- Students should begin talking about finding the scale factor by computing the ratio of corresponding parts of two figures. Additionally, students should be beginning to use the scale factor as a multiplier instead of repeatedly setting up and solving proportions.
- This lesson builds on Van Hiele levels 2 and 3: analysis and abstraction.
- Relevant contexts that are drawn on in relation to this concept include interior design, landscape design, and architecture.

Introduction: Setting Up the Mathematical Task

- Goal of the lesson.

  "In this lesson, you will investigate the use of floor plans, and how the concept of similarity helps you to read and understand, and then create, a 2-dimensional drawing of a 3-dimensional space."
- Warm up: solve proportions (at end of lesson)
- Timeline for lesson: Two 90 minute blocks

What	Who	Amount of Time
Warm up	Students complete; compare answers with partner; share strategies when prompted by teacher	10 min
A Tour of the White House Exploration	Whole class and individually	60 min
Mini Project description Work on mini project	Teacher	10 minutes 90 -minutes
Journal Prompt	Students	10 minutes

- A Tour of the White House
  - Provide students with a copy of the floor plan for the White House. Printable copies are available at http://www.visitingdc.com/white-house/white-house-floor-plans.htm and other sites on the internet.
  - Discuss with students how floor plans are created, as well as how they are interpreted. Discussion questions:
    - What information does this floor plan give you about the White House?
    - Can you tell how big the White House is from this floor plan?
    - Could this floor plan be used to build a copy of the White House? Why or Why not?
    - Is this floor plan a very accurate scale model of the White House?
  - On their own computers, or as a class with a video display, allow students to explore an interactive floor plan of the White House: http://www.cnn.com/2009/POLITICS/02/02/inside.thewhitehouse/ And discuss:
    - Are these floor plans more or less accurate than the handout?
    - What different information did you learn from this website?

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        - Why do you think that these different floor plans were created? Are they meant to be truly similar to tell you about the size of the rooms, or is there a different purpose?
        - Do you think you have enough information to decide how big the rooms in the White House are?
        - How much information do you need to figure out how big all of the rooms are?

            - Time permitting, you may also explore the official interactive tour at http://www.whitehouse.gov/about/interactive-tour
            - Various other historical floor plans and photos at http://www.whitehousemuseum.org/residence.htm

- According to the White House Museum, the State Dining Room is 36 X 49 feet. http://www.whitehousemuseum.org/floor1/state-dining-room.htm
    - Using the measurements of the State Dining Room, use your knowledge of similar figures to determine the actual size of each room on the State Floor. *As the Blue Room is elliptical, students may not be able to determine the dimensions. You may choose to provide this information, or walk the class through this portion of the activity. It is important to include, as students need to be aware that not all spaces are rectangular.
    - Create your own floor plan.
        - Have students create an accurate floor plan of the State Floor of the White House on graph paper. Depending on your students, you may choose to have them:
            - Work individually on 8.5 X 11 graph paper
            - Work in pairs or groups on presentation sized graph paper
        - In giving directions ask students to consider:
            - What is an appropriate scale to be sure that the floor plan fits on the graph paper?
            - Will every one choose the same scale?
            - How will I represent walls on the graph? Based on my scale, should I consider the thickness of walls, or not?
            - How do I label my floor plan to be sure that other people can use it to find the size of the White House?

- **Mini-Project**
  - Prints for a Prince (or Princess) mini-project. Attached.
  - Accommodations: have loaner cameras and measuring tapes available for students who may not have access to these items at home.
- For students who may not wish to share photographs of their living accommodations, allow them to make a blueprint of a classroom or an office in the school building

Extensions and Connections (for all students)

- If all students need more time to understand the process of creating a floor plan, you may choose to have students create a floor plan of your classroom in groups. Provide demonstration sized graph paper, and in small or whole groups work, create a floor plan for the classroom, including the foot prints of furnishings.
- Connections to other mathematics content: This connects to students previous learning of similar figures (Math 7) and solving proportions (Math 7, Prealgebra, and Algebra I)
- Connections to content in other subject areas: landscaping, architectural drawing, building trades, art

Strategies for Differentiation

- List ideas for addressing needs of a diverse population of students such as:
  - This activity should address the needs of kinesthetic, auditory and visual learners.
  - Working with carefully chosen partners helps meet the needs of students with processing, memory, motor issues;
  - English language learners (ELLs); ELL students may need additional time to translate the directions and outcomes.
  - high-ability students: Could have students compare ratios of areas and/or volumes
- Provide graph paper and units for floor plans and tables.

Name____ Date______________

Prints for a Prince (or Princess) Mini-Project

I. Take Pictures

Using your phone or a camera, take pictures of your room. Stand at the doorway and take one photo of the left side of the room, a photo of the middle of the room, and a photo of the right side of the room. If you have not captured all of the contents of the room, take additional photographs.

II. Measure

- Measure the floor of your room. If your room has recessed areas or areas that jut out, include the dimensions of these areas.
- Measure the dimensions of all objects that sit on the floor of your room
- Measure the dimensions of all windows and doors.
- Make tables or charts that include all measurements.

III. Floor Plan

Make a floor plan of your room.

The floor plan should include the scale you are using, markings indicating all doors and window, and show all objects that sit on the floor of your room. Attached to the floor plan include a chart or table including the dimensions of the actual objects and the scaled dimensions. Extend the table or chart used for your measurements in part II to include this information.

IV. Timeline:

Complete parts I and II before coming to class next time. Time will be provided in class to work on part III.

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Rubric for Prints for a Prince (or Princess) mini project

	Excellent – 4 points	Good – 3 points	Average –2 points	Poor – 1 points
Pictures	Minimum of 3 pictures. All contents of room that are on blueprints are shown on pictures	Minimum of 3 pictures. Most contents of the room that are shown on blueprints are shown on pictures.	Minimum of 3 pictures. Some contents of the room that are shown on blueprints are shown on pictures.	Fewer than 3 pictures. Pictures do not represent the contents of the room that are shown on the blueprints.
Table or chart of measurements.	Table or chart includes accurate measurements of dimensions of 1. floor, 2. all objects that sit on the floor, (3 dimensions) and 3. windows and doors (3 dimensions).	Table or chart includes accurate measurements of dimensions of 1. floor, 2. most objects that sit on the floor, (3 dimensions) and 3. windows and doors (3 dimensions).	Table or chart includes accurate measurements of dimensions of two of the following: 1. floor, 2. most objects that sit on the floor, (3 dimensions) and 3. windows and doors (3 dimensions).	Table or chart includes accurate measurements of dimensions of one of the following: 1. floor, 2. most objects that sit on the floor, (3 dimensions) and 3. windows and doors (3 dimensions).
Scaled values	Table or chart includes accurate measurements of dimensions of 1. floor, 2. all objects that sit on the floor, (3 dimensions) and 3. windows and doors (2 dimensions).	Table or chart includes accurate measurements of dimensions of 1. floor, 2. most objects that sit on the floor, (3 dimensions) and 3. windows and doors (2 dimensions).	Table or chart includes accurate measurements of dimensions of two of the following: 1. floor, 2. most objects that sit on the floor, (3 dimensions) and 3. windows and doors (2 dimensions).	Table or chart includes accurate measurements of dimensions of one of the following: 1. floor, 2. most objects that sit on the floor, (3 dimensions) 3. windows and doors (2 dimensions).
Floor Plan	Floor plan accurately shows scaled models of 1. floor, 2. all objects that sit on the floor, (2 dimensions) and 3. windows and doors (width)	Floor plan accurately shows scaled models of 1. floor, 2. most objects that sit on the floor, (2 dimensions) and 3. windows and doors (width)	Floor plan accurately shows scaled models of two of the following: 1. floor, 2. most objects that sit on the floor, (2 dimensions) and 3. windows and doors (width).	Floor plan accurately shows scaled models of one of the following: 1. floor, 2. most objects that sit on the floor, (2 dimensions) and 3. windows and doors (width).

Warm up: Solve each of the following proportions. Show appropriate work.

1. $\frac{4}{15} = \frac{w}{75}$

2. $\frac{6}{10} = \frac{15}{n}$

3. $\frac{p}{36} = \frac{9}{8}$

4. $\frac{2}{x} = \frac{x}{8}$

Solutions to warm up:

1. w = 20
2. n = 25
3. p = 40.5
4. x = 4 or -4

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Lesson 2: You're in My Space!

Strand

Geometry, Number and Operations

Mathematical Objective(s)

- Using dynamic geometry software to explore and compare the area of dilated polygons—convex (regular and non-regular) and concave
- Constructions of polygons using dynamic geometry software
- Transformations of polygons using dynamic geometry software
- Calculating the factor an area will change when a figure is dilated
- Students will be able to discover that because area uses two dimensions, one can square the dilation factor to calculate the amount of change in the area of the dilated shape
- Students will construct regular polygons using transformation commands within the dynamic geometry software.
- Students will find area through measurement commands within the dynamic geometry software.
- Students will find the amount of change in the area using measurement/calculate commands within the dynamic geometry software.

Virginia College and Career Ready Mathematics Performance Expectation(s)

MPE 7: Use similar geometric objects in two- or three- dimensions to

a) compare ratios between side lengths, perimeters, areas, and volumes;

b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;

c) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and

d) solve real-world problems about similar geometric objects.

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Related SOL

G.3d, G.4a,d,g, G.9, G.14a,b,c,d

NCTM Standards

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
- Understand meanings of operations and how they relate to one another.
- Understand measurable attributes of objects and the units, systems, and processes of measurement.
- Make decisions about units and scales that are appropriate for problem situations involving measurement.
- Apply appropriate techniques, tools, and formulas to determine measurements.
- Understand and use formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders.
- Build new mathematical knowledge through problem solving. Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.
- Select and use various types of reasoning and methods of proof.
- Organize and consolidate their mathematical thinking through communication.
- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognize and apply mathematics in contexts outside of mathematics.
- Create and use representations to organize, record, and communicate mathematical ideas.
- Select, apply, and translate among mathematical representations to solve problems.
- Use representations to model and interpret physical, social, and mathematical phenomena.

Materials/Resources

- Class set of laptops or computer lab access; LCD projector for demonstration
- Geometer's Sketchpad, Geogebra, or any dynamic geometry software (instructor should be comfortable using the software chosen)
- Exploration handouts
- Student's floor plan from Lesson 1: Prints for a Prince (or Princess)

Assumption of Prior Knowledge

- Area of polygons (regular, non-regular, irregular).
- Knowledge of similar polygons.
- Transformations of polygons.
- This lesson assumes the student has calculated squares of rational numbers.

- A student should be at the Level 2/3 on the Van Hiele scale concerning polygons
- Familiarity with dynamic geometry software.
- The relevant concepts of architecture and interior design are drawn on in relation to the concepts presented in this lesson.

Introduction: Setting Up the Mathematical Task

- Goal of the lesson:

  "In this lesson, you will investigate the relationship between the areas of dilated regular, non-regular, and irregular polygons—both convex and concave."

- Timeline for lesson— Two and a half 90-minute blocks:

What	**Who**	**Amount of Time**
Warm up	Students complete with a partner. Pairs will share strategies with rest of class when prompted by teacher.	15 minutes*
Demonstration of Dynamic Geometry Software	Teacher demonstrates basics of dynamic geometry software using LCD projector. Students will follow teacher lead on their own computer or laptop.	20 minutes*
Student Exploration I (Construction of regular polygon, area, and dilation)	All students work independently on their own laptop or computer to discover areas of dilated polygons using dynamic geometry software	45 minutes*
Student Exploration II (Construction of non-regular convex polygon, area, and dilation)	All students work independently on their own laptop or computer to discover areas of dilated polygons using dynamic geometry software	30 minutes ^
Student Exploration III (Construction of concave polygon, area, and dilation)	Students that have completed Explorations I and II before the end of the allotted time for the explorations.	15 minutes^
Discussion of Student Explorations	Students will e-mail saved files to teacher; teacher can then open files on a laptop and discuss individual student's work	15 minutes*^
Extension: Real-world application (Bathroom Project)	Teacher will describe the bathroom project. Students will use their individual floor plans to add a bathroom 1/3 the size of and similar to their room.	Description—10 minutes** Student work: dynamic geometry software—20 minutes; internet research—20 minutes; bathroom design—homework
Reflective writing	Students will complete in a Word document	10 minutes each class of this lesson (20 minutes total)^

*Block 1; ^Block 2; **Block 3

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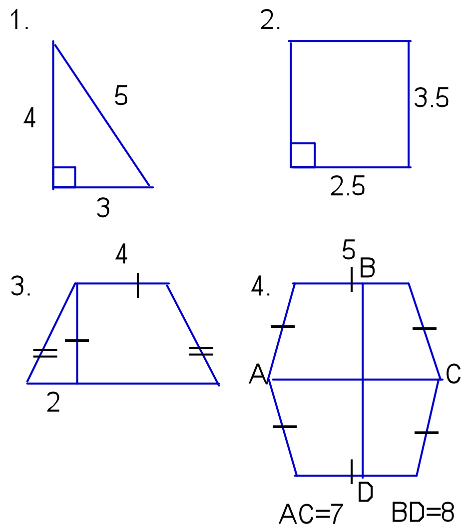
Warm-up:

Area Formulas:

Triangle—A=1/2bh

Parallelogram—A=bh

Using the formulas given, calculate the area of each figure below with a partner. Discuss with your partner how to achieve the calculation(s). Be ready to share your strategies with the class.



Dynamic Geometry Software Demonstration

Teacher should be comfortable with the dynamic geometry software chosen. Preferably, the teacher will demonstrate from a computer/laptop connected to an LCD projector. Teacher should demonstrate the toolbar and what each one does. Teacher should demonstrate how to highlight items in preparation for commands within the software. Teacher should demonstrate how to construct a polygon so that it can be dragged around by a vertex while maintaining its characteristics.

After the Demonstration. . .

Students will then be given Student Exploration I and II to complete. Those students that are proficient with the dynamic geometry software and finish both explorations accurately and faster than the majority of the rest of the class can then be given Student Exploration III to complete.

As the students are working on the Student Exploration activities, the teacher will be walking around the computer lab or classroom examining what students are doing, assisting those that need help with the use of the dynamic geometry software, and leading students to be able to answer the questions posed on each Student Exploration. Students that are proficient with the dynamic geometry software will be encouraged by the teacher to assist their peers with the use of the software if the teacher is helping another student(s).

Assessing the Explorations. . .

The teacher will grade Explorations I and II using the attached rubric. Exploration III was not required of each student, so it will only be assessed for the purpose of comments to the student.

Students will also be required to create a Word document reflecting about what they learned from each exploration. They will also be asked to reflect on how they can extend this thinking to a real-world situation and to describe the real-world situation.

Name____ Date_________

Student Exploration I—Exploration of area of regular polygon and dilation of same

1. Construct the following regular polygons using the dynamic geometry software demonstrated: triangle, square, pentagon
2. Construct the polygon interior.
3. Calculate the area of the polygon interior using commands.
4. Dilate the regular polygons as follows: triangle—dilation factor 1/2; square—dilation factor 1.5; pentagon—dilation factor 3.
5. Did the dilation factor cause the original polygon to decrease or increase in size?
   Triangle? Square? Pentagon?
6. Construct the polygon interior of the dilated polygon. Change the color of the dilated polygon. What geometry term describes the original and its dilated shape? (similar polygons)
7. Calculate the area of the dilated polygon.
8. Use a command to calculate the amount of change between the original and the dilated shape if one is available in the software. If not, then calculate it on your own. Using a text box briefly answer the following questions about each pair of similar shapes.
   a. Describe what happened with the areas of the similar shapes (When the dilation factor was a fraction less than 1, the dilated shape and its area was smaller than the original. When the dilation factor was larger than 1, the dilated shape and its area was larger than the original.)
   b. Did the area increase the same amount as the dilation factor? (No) Why or why not? (The area increased more than the dilation factor when the dilation factor was larger than one and decreased less than the dilation factor when the dilation factor was less than one.)
   c. How can you explain the amount of change in area just using the dilation factor and the dimensions? (square the dilation factor to get the change in the area)
9. Grab a vertex on each regular polygon and drag it. Observe the areas change as you drag the polygons. Did the amount of change in area between the similar shapes change? (No). Briefly explain why. (The similarity ratios held, so the areas remained proportional.)
10. Save your document as follows into your student folder:
    Regular polygons_first initial last name (ex. Regular polygons_hkeller)

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Name____ Date_______

Student Exploration II—Exploration of area of non-regular convex polygon and dilation of same

1. Construct the following non-regular convex polygons using the dynamic geometry software demonstrated: right triangle, rectangle, and pentagon.
2. Construct the polygon interior.
3. Calculate the area of the polygon interior using commands.
4. Dilate the figures as follows: right triangle—dilation factor 4/3; rectangle—dilation factor 2; pentagon—dilation factor 1/3.
5. Did the dilation factor cause the original polygon to decrease or increase in size? Triangle? Square? Pentagon?
6. Construct the polygon interior of the dilated polygon. Change the color of the dilated polygon. What geometry term describes the original and its dilated shape? (similar polygons)
7. Calculate the area of the dilated polygon.
8. Use a command to calculate the amount of change between the original and the dilated shape if one is available in the software. If not, then calculate it on your own. Using a text box briefly answer the following questions about each pair of similar shapes.
   - Describe what happened with the areas of the similar shapes (When the dilation factor was a fraction less than 1, the dilated shape and its area was smaller than the original. When the dilation factor was larger than 1, the dilated shape and its area was larger than the original.)
   - Did the area increase the same amount as the dilation factor? (No) Why or why not? (The area increased more than the dilation factor when the dilation factor was larger than one and decreased less than the dilation factor when the dilation factor was less than one.)
   - How can you explain the amount of change in area just using the dilation factor and the dimensions? (square the dilation factor to get the change in the area)
9. Grab a vertex on each regular polygon and drag it. Observe the areas change as you drag the polygons. Did the amount of change in area between the similar shapes change? (No). Briefly explain why. (The similarity ratios held, so the areas remained proportional.)
10. Save your document as follows into your student folder:
    Non-regular convex polygons_first initial last name
    (ex. Non-regular polygons_hkeller)

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Name____ Date_______

Student Exploration III—Exploration of area of concave polygon and dilation of same

1. Construct a concave nonagon using the dynamic geometry software demonstrated.
2. Construct the polygon interior.
3. Calculate the area of the polygon interior using commands.
4. Predict what you think will happen if you dilate the concave nonagon using a dilation factor of 2. (Each segment of polygon will double in length. Thus, similar shape will be larger.)
5. Dilate the nonagon using a dilation factor of 2.
6. Was your prediction right?
7. Predict what you think will happen to the area of the dilated shape. (It should increase by 4.)
8. Calculate the area of the dilated polygon. Was your prediction right?
9. Use a command to calculate the amount of change between the original and the dilated shape if one is available in the software. If not, then calculate it on your own. Using a text box briefly answer the following questions about each pair of similar shapes.
   - Describe what happened with the areas of the similar shapes (When the dilation factor was a fraction less than 1, the dilated shape and its area was smaller than the original. When the dilation factor was larger than 1, the dilated shape and its area was larger than the original.)
   - Did the area increase the same amount as the dilation factor? (No) Why or why not? (The area increased more than the dilation factor when the dilation factor was larger than one and decreased less than the dilation factor when the dilation factor was less than one.)
   - How can you explain the amount of change in area just using the dilation factor and the dimensions? (square the dilation factor to get the change in the area)
10. Grab a vertex on each regular polygon and drag it. Observe the areas change as you drag the polygons. Did the amount of change in area between the similar shapes change? (No). Briefly explain why. (The similarity ratios held, so the areas remained proportional.)
11. Save your document as follows into your student folder:
    concave polygons_first initial last name (ex. concave polygons_hkeller)

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Discussion of Student Explorations

Teacher should have a few students e-mail their documents for each exploration.

Teacher can pull these up on her computer and project them through the LCD projector.

Since no specific measurements were given to construct the polygons, there should be different pictures to view from student to student.

Teacher should drag a vertex so students can see as a class that the similarity ratio holds so the areas of the similar figures remain proportional.

Teacher should lead class to see that the area changes by the dilation factor squared.

The teacher should emphasize this on all three explorations.

Reflection Writing

- Please create a Word document titled *Reflection Writing—Area of Similar Shapes*
- Please write a paragraph of at least 5 good sentences explaining what you learned from each Student Exploration. Also, please include a real-world situation where you think area of similar shapes might be used or applied.
- Please save your document into your student folder as reflection writing_first initial last name (example: reflection writing_hkeller)

Assessment of Student Explorations I and II (Use a separate rubric for each exploration)

	Construction of triangle	Construction of square/rectangle	Construction of pentagon	Questions answered	Evidence of knowledge of Dilation factor and dimensions	Used time wisely and appropriately, stayed on task, etc.
4	Triangle is regular (exploration I) or right (exploration II)	Polygon is a square (exploration I) or rectangle (exploration II)	Polygon is a regular pentagon (exploration I) and a non-regular pentagon (exploration II)	All questions answered as directed with thoughtful answers	Proper terminology and/or vocab. Used to describe the change in areas of similar shapes for each shape	Student did not have to be redirected
3	Polygon is a constructed triangle; however, transformation not 100% accurate	Polygon is a constructed square/rectangle; however, transformation not 100% accurate	Polygon is a constructed pentagon; however, transformation is not 100% accurate	95% of the questions answered as directed with thoughtful answers	Text box answers approach correct conclusion but proper terminology and/or vocab. Not used	Student had to be redirected once
2	Polygon is drawn, not constructed	Polygon is drawn, not constructed	Polygon is drawn, not constructed	90% of the questions answered as directed with thoughtful answers	Student shows some calculation involving areas but has not made connection	Student had to be redirected twice
1	Polygon is not an equilateral/right triangle	Polygon is not a square/rectangle	Polygon is not a pentagon	85% of the questions answered as directed with thoughtful answers	Student has not made connection but did complete the work as directed	Student had to be redirected more than twice
0	Student actively chose to not participate in this activity	Student actively chose to not participate in this activity	Student actively chose to not participate in this activity	Student actively chose to not participate in this activity	Student actively chose to not participate in this activity	Student actively chose to not participate in the activity

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Extensions and Connections (for all students)

Real-World Application—Bathroom Project

You have been selected to be on a home improvement reality show for television. You will get your very own bathroom, decorated for your personality! You get to design it, furnish it, paint it, and accessorize it any way you want. The only stipulation is that it must be 1/3 the size of your bedroom.

Using the scale drawing that you made of your bedroom, use a dilation factor of 1/3.

- Create a new sketch using the dynamic geometry software of your bedroom and your new bath. In a new sketch, choose to have the grid appear on your sketch.
- Now recreate the blueprint of your bedroom on the sketch.
- Draw a blueprint of your new bathroom.
- Calculate the area of your bedroom.
- Now calculate the area of your new bathroom. Did the change in the area behave the same way it did in the Student Explorations?
- Go to appropriate sites on the internet and research the size of a standard bathtub, toilet, and single- vanity sink. Record those in a textbox on your sketch.
- You will need to determine whether these items will fit into your new bathroom. You may have to make some concessions at this point and take a shower stall instead of a bathtub. Please indicate on your sketch whether you are going with the bathtub or the shower stall.
- Indicate on your blueprint where the bathtub/shower stall, toilet, and single-vanity sink will go. Further extensions may have the students price accessories for their bathroom, as well as wallpaper and/or paint.

Strategies for Differentiation

- The Student Explorations I and II will address the needs of a diverse population of students such as kinesthetic, auditory, and visual learners.
- There is an opportunity for peer tutoring of the dynamic geometry software for those students who are more proficient with it.
- English language learners (ELLs)—the actual working with the software should be acceptable; however, some of the vocabulary in the commands may take further explanation
- High-ability students will have a chance to further explore the concepts from the lesson in Student Exploration III. They will also have an opportunity to peer tutor.
- Websites may be provided to aide in the internet research portion of the project.

Lesson 3: Honey, you shrunk the candy bar.

Strand
Geometry, Measurement

Mathematical Objective(s)

- Solving proportions
- Measuring with a measuring tool.
- Calculating a scale factor.
- Students will be able to measure a room and objects in a room and use this information to develop a floor plan of the space and items in the room

College and Career Ready Mathematics Performance Expectation

MPE 7: Use similar geometric objects in two- or three- dimensions to

e) compare rations between side lengths, perimeters, areas, and volumes;
f) determine how changes in one or more dimensions of an object affect area and/or volume of the object;
g) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and
h) solve real-world problems about similar geometric objects.

RELATED SOL

G.14 A AND G.14 D

NCTM Standards

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
- Understand measurable attributes of objects and the units, systems, and processes of measurement.
- Make decisions about units and scales that are appropriate for problem situations involving measurement.
- Apply appropriate techniques, tools, and formulas to determine measurements.
- Understand and use formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders.
- Build new mathematical knowledge through problem solving. Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.

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- Monitor and reflect on the process of mathematical problem solving.
- Select and use various types of reasoning and methods of proof.
- Organize and consolidate their mathematical thinking through communication.
- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognize and apply mathematics in contexts outside of mathematics.
- Create and use representations to organize, record, and communicate mathematical ideas.
- Select, apply, and translate among mathematical representations to solve problems.
- Use representations to model and interpret physical, social, and mathematical phenomena.
- Communication Standard: Pre-K through 12: Instructional programs should enable all students to communicate mathematical thinking coherently and clearly to peers, teachers, and others

Materials/Resources

- Classroom set of straight edges
- Measuring tapes (preferably 25')
- Classroom set of graphing calculators
- ¼ " graph paper
- For each group of two students provide a regular size Hershey bar and a fun-sized Hershey bar.
- Napkins or paper towels
- Class set of laptops or computer lab access
- A matchbox car

Introduction: Setting Up the Mathematical Task

- Goal of the lesson.

  "In this lesson, you will investigate how people use 3-dimensional models to represent very large objects, and how these models compare to the original object."
- Warm up: identify similar geometric objects

Time Line for the lesson: One 90-minute block

What	**Who**	**Amount of Time**
Warm up	Students, individually followed by whole group discussion	15 minutes
Student Exploration I Hershey Bar Comparison	Students, in groups of 2	30 minutes
Student Exploration II Eiffel Tower Comparison	Students, in groups of 2	15 minutes
Matchbox Car reflection	Students, individually	15 minutes

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Warm Up:

Determine if the following rectangular prisms are similar:

1. prism a: has a volume of 64 cubic inches
   length of 4”
   width of 4”

   prism b: has a volume of 64 cubic inches
   length of 8”
   width of 2”

2. prism a: has a volume of 8 cubic inches
   length of 1”
   width of 2”

   prism b: has a volume of 16 cubic inches
   length of 2”
   width of 4”

3. prism a: has a volume of 144 cubic inches
   length of 4”
   width of 6”

   prism b: has a volume of 486cubic inches
   length of 6”
   width of 9”

Solutions to Warm Up:

1. no   2. no   3. yes

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Student Exploration I (Hershey bar comparison):

- Divide students into groups of two. Provide each group of students with a napkin, a regular sized Hershey bar and a fun-sized Hershey bar.
- Ask students to unwrap the bars and examine them to determine if they are similar.
- Ask students: Are the candy bars similar.
- Think-pair-share: What does it mean for two objects to be similar in mathematics? (draw upon prior knowledge)
- Through whole class discussion, agree upon and record a definition for the term *similar*.
- In groups, students determine if the candy bars are similar; teacher walks around and observes.
- Teacher selects a group to demonstrate how they determined if the candy bars were similar.
- If other groups used a different method, have all groups share their methods.
- Class discussion: what do the methods have in common?
- Ask students: Given the dimensions of the larger candy bar, if one of the dimensions of the smaller candy bar stayed the same, what would the other dimensions have to be in order for the candy bars to be similar?
- Students will have either the length, width, or height of the fun-sized bars remain constant and will then use proportions to find the missing dimensions.
- Pair the groups with other groups who had the same dimension remain constant. Have them share their work with each other and create a poster illustrating how they found the missing dimensions.

Student Exploration II (Eiffel Tower):

- Divide students into groups of two.
- Scenario: The owners of King's Dominion Amusement Park claim that the Eiffel Tower in the park is an exact one-third scale replica of the Eifel Tower in Paris, France. The tower at King's Dominion is 332 feet tall, with an observation tower 275 from the ground.
- Task: With a partner research the Eiffel Tower in Paris, France on the internet. Use the information you find to answer the question: Is the Eiffel Tower in the Kings Dominion Amusement Park similar to the Eiffel Tower in Paris, France. Is it, in fact, a one-third scale replica?
- Write a paragraph with your conclusion. Include any information or computations that you use to help you formulate your conclusion.

Monitoring Student Responses

- Students will communicate their thinking and new knowledge verbally and in writing during the writing prompt.
- Students will encourage each other to explain their work, especially when two students disagree as to an outcome to a question.
- The teacher will circulate during the group activities and question students about the processes they are using and how they know that these processes work.
- The teacher will assist students who have difficulties by phrasing the directions in different words and asking probing questions of students to determine where the difficulties lie. As appropriate, the teacher will provide background knowledge or model a similar problem to help students move to the next level of thought.
- Student who are ready to move forward could be asked to compare the ratios of the areas and volumes of the candy bars and Eifel tower in preparation for the next lesson in this unit.

- Summary: Ask students: Describe how you would determine if two figures were similar? Allow students to respond.
    - Use writing prompt to determine if students understand how to determine if two figures were similar.

Assessment

- **Journal/writing prompts**
  Consider this matchbox car and its real life counterpart. Assuming you had access to the actual real car, how could you determine if the matchbox car is actually similar to the car it represents?

Rubric for Journal/Writing Prompt

Consider this matchbox car and its real life counterpart. Assuming you had access to the actual real car, how could you determine if the matchbox car is actually similar to the car it represents?

Excellent – 4 points	**Good – 3 points**	**Average – 2 points**	**Poor – 1 points**	**0 points**
Student demonstrates mastery of the concept of similar figures. Examples are specific, relevant, and explained.	Student demonstrates some knowledge of the concept of similar figures. Examples are present, but may not be as specific as possible or explained as clearly as possible.	Student's knowledge of similar figures is unclear. Examples are present, but they may be vague or undeveloped.	Student's knowledge of the similar figures is limited. Examples are not present or do not relate to the concept of similar figures.	Student chose not to participate.

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Lesson 4: Living Inside the Box

Strand
Geometry, Measurement

Mathematical Objective(s)
Students will apply their knowledge of linear measure, area and volume to explore similarity in 3 dimensions. The goal of this lesson is for students to create a 3-dimensional model of an actual space with a given scale factor. Students' knowledge of similar figures, proportional reasoning, measurement skills and ability to understand how changes in measure affect area and volume are being reinforced and applied. Students will learn to choose and apply appropriate scale factors.

Mathematics Performance Expectation(s)

MPE 6: The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

MPE 7: The student will use similar geometric objects in two- or three-dimensions to

a) compare ratios between side lengths, perimeters, areas, and volumes;

b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;

c) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and

d) solve real-world problems about similar geometric objects.

Related SOL

G.13, G.14 a, b, c, d

NCTM Standards

- analyze properties and determine attributes of two- and three-dimensional objects;
- explore relationships (including congruence and similarity) among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them
- draw and construct representations of two- and three-dimensional geometric objects using a variety of tools
- visualize three-dimensional objects and spaces from different perspectives and analyze their cross sections

- use geometric models to gain insights into, and answer questions in, other areas of mathematics
- use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture.

Materials/Resources

- Projection equipment to show video
- Measuring tapes, rulers, meter sticks or other tools for measuring 3-dimensional objects.
- Calculators
- Area and Volume formula sheets
- Legos or other type for building block or modeling material for use in class
- Shoe boxes, cardboard or other similar materials for final model
- Student handouts. (attached)

Assumption of Prior Knowledge

- Ability to apply area and volume formulas.
- Understanding of similar objects in 3 dimensions.
- The student should be familiar with similarity of 2 dimensional objects and how change in dimensions affects area from prior lessons in this unit.
- The student should be familiar with the use of 2 dimensional modeling as tool in architecture/building and planning in real world situations.
- The skills in this lesson will apply to areas of architecture, building, planning, marketing and science as students learn more about the use of scale models in 3 dimensions.

Introduction: Setting Up the Mathematical Task

Goal of the lesson.

> "In this lesson, you will investigate how scale models and replicas are created to represent objects on a different scale."

Time – line for lesson: Two 90-minute blocks

<u>**What**</u>	<u>**Who**</u>	<u>**Amount of Time**</u>
Introductory videos	Whole Class	15 minutes
Student Exploration I Big Things in Small Packages	Students, in groups of 3 to 4	60-75 minutes
Student Exploration II Living Inside the Box	Students, in groups of 2	90 minutes

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- Get students to think about the task by showing the following videos:

  Lego model of a house:

  http://www.youtube.com/watch?v=q4SLBdrwzzE

  Lego model of DNA

  http://www.youtube.com/watch?v=RJLrkbBvK3Y

- Discussion Questions about videos:
  - When you see the finished Lego house, do you feel like you know what a "life sized" version of this house would look like?
  - How big do you think the real house would be? Do we think everyone has the same size in mind?
  - Could this one Lego house actually represent more than one real house, of different sizes? How can that be?
  - Could the Lego house actually be a model of a much smaller structure? What do we see in the 2nd video?
  - Why would someone make a larger model of a small object?

Student Exploration 1:

Big Things in Small Packages

- Divide students into groups of 3 or 4.
- Provide each group with copies of the handout, measuring tools, formula sheets and calculators.
- If appropriate resources are available, students may create their sketches in a CAD program, or something similar.
- Monitor groups to be sure that students choose objects that can be modeled easily and somewhat accurately with rectangular blocks and avoid very complicated or curved objects like people or round clocks.
- Monitor groups closely to ensure that measurements are taken completely and accurately and that every portion of the original object is measured, looking for parts that students might over-look, like legs of a desk.
- Engage students in conversation as they work about the nature of their model, and check for student understanding.
- Check for accuracy in computation as students work on the analysis portion of the activity.

- When all groups have completed the activity, engage in whole class discussion about the results of the analysis portion of the handouts. Ask the class questions about their results to summarize the lesson.
    - Were you models truly similar to the original object?
    - How do you account for this?
    - What was your intended scale factor?
    - Were you able to maintain the same proportion in each dimension?
    - Were the surface area/volume proportional? How do you account for differences in scale factor?

Assessment

- Student responses to the Big Things in Small Packages handout
- Possible Journal Prompts:
    - Can truly similar scale models of anything be created with blocks?
    - What do you need to keep in mind when you choose a scale factor for making a scale model?
    - What are ways that you could make a scale model more accurately than you can with Legos?

Extensions and Connections (for all students)

- Time permitting, have students select a very small object to make a large model of, and repeat exploration 1.
- Have students research various “World’s Largest” items on the internet, and compare them to standard versions of the same item. Tie this activity in with the Eiffel Tower Exploration from Lesson 1.
- Connect this lesson to the models used in science classes in your building, or to the globes and maps used in social studies classes. You can make connections to sculptures and drawings in art classes, and can perhaps use student art as representations of scale.

Strategies for Differentiation

- For students with processing, memory, motor issues;
    - Select the original object for them and provide them with a completed sketch with all measurements to allow more time for them to focus on their scale model
    - If necessary, provide an original and a scale model and ask students to complete the analysis portion of the activity only

- Provide graph paper for help with sketches
- Help students break irregular figures into smaller parts.

- English language learners (ELLs);
  - Allow students time to translate instructions if needed

- high-ability students
  - ask students how they could choose a scale that would be accurate, based on the size of a block.
  - Simply ask them to analyze the accuracy of their model without providing the handout with directions on how to do this.

Student Exploration 2:

Living Inside the Box

- Students will complete this portion of the lesson independently.
- This Exploration relies on their work in previous lessons in this unit, as they will be working directly from the floor plan and measurements they have already taken in their bedroom at home.
- All instructions are on the handout. Teacher should ensure that all directions are clear before students begin the exploration.
- Have students complete the peer evaluation rubric for one other student. You may use their accuracy in evaluating their peer as an assessment of their own understanding and ability to demonstrate skill with scale models.

Assessment

- Students' shoe box model
- Peer evaluation rubric / teacher evaluation
- Possible Journal Prompts:
  - What problems did you encounter in creating your shoebox model?
  - How accurate do you think your model is? Explain.

Extensions and Connections (for all students)

- Have students research the cost of purchasing remodeling materials for their bedroom.

- Have students create a model for their “dream bedroom” and compare to their current room, analyzing how these changes would affect the amount of carpet, paneling, or furniture needed to fill the new space.
- Have students create a model of their model on a different scale, and compare this new model to the original.

Strategies for Differentiation

- For students with processing, memory, motor issues;
  - Have a model prepared of your own room, and allow students with motor issues to analyze your model rather than create their own, or allow them to analyze a partner’s model.

- English language learners (ELLs);
  - Allow students time to translate instructions if needed

- High-ability students
  - have students explore the following questions:
    - Weigh the model of your bed. If the model of your bed were made of the exact same materials as the bed itself, how much would your real bed weigh. How do you know this?
    - Would the cost of materials to build a scale model be proportional to the cost of materials to build an actual building? Do not consider time or labor cost.

Big Things in Small Packages

Name____ Date________________

Group Members____

In your every day life, you are surrounded by stuff. Big stuff, little stuff, and stuff made to look like other stuff – just in a different size.

With your groups, brainstorm at least three things that you have seen this week that could been thought of scale models of a similar object. Record your thoughts here.

Now it's your turn to make some!

Select one large object in your classroom to model using your blocks. Choose something too large to fit on your desk that can be built out of blocks like a desk, a chair or a cabinet. Have your teacher approve your choice before you begin.

Name of object:

Sketch the object here and record all measurements on your sketch. Be sure to be accurate, and record the dimensions of every surface.

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Now you get to play with the toys! Make your model of the object.

Sketch the model here and record all measurements on your sketch. Be sure to be accurate, and record the dimensions of every surface.

Comparing the objects

Now we need to compare your model to the original. You already have the measure of each dimension of both objects, now we need to compute the total surface area and volume of each figure. Please show all of your work on separate paper, and record your results here.

	Original Object	Our Model
Total Surface Area		
Volume		
Height (at tallest point)		
Width (at widest point)		
Depth (at deepest point)		

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Analyze the Accuracy of your Model

Explain what the mathematical meaning of similarity between 3 dimensional objects. Write your ideas here.

Compare each measure of your original object to the corresponding measure of your model. Record each as a ratio in the table below.

	Ratio
Total Surface Area	
Volume	
Height (at tallest point)	
Width (at widest point)	
Depth (at deepest point)	

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With your group, think about your model as it compares to the original and answer the following questions.

1. Is your model a **perfect** scale model of the original? Explain why or why not.

2. Could you have made your model more accurate in any way? Could you make a change to your model right now to make it more accurate? Explain.

3. What scale factor were you trying to create between the height of your original object and the height of your model?

4. Based on that scale factor, what do you expect the ratio to be between the surface areas of the two objects?

5. Based on that scale factor, what do you expect the ratio to be between the volumes of the two objects?

6. Which of your measures were closest to what you expected? Which were farthest from what you expected? Why do you think this is so?

7. How accurate do you think your model is? Explain your thinking.

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Name____ Date____________

Living Inside the Box

You task will be to create an accurate 3-dimensional model of your bedroom and its contents that will fit inside a shoe box. You may use your work from other lessons as a starting point. Then you will use heavy paper, cardboard or other materials to create the objects in your room.

Fitting into the box:

Your model must be proportional to your room. You shoe box probably is not proportional to your room right now. Decide on scale factor for your room that will fit inside the box in some way. You may cut down a side of the box, or simply create your model to fit inside of it.

What scale did you choose?

Why did you think this scale was appropriate?

Is this the same as, or close to, the scale represented by your work on graph paper earlier?

Now, make your model. You may use some sort of written plan or blue print as you work, or make a series of sketches. Include all of this with your final product.

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Name____ Date______________

Evaluated by:____

Living Inside the Box Evaluation

	How well does the model fit into the show box?	Is supporting work shown to explain the chosen scale, and to support the measurements in the model?	Compared to the original floor plan, is the model an accurate scale representation of the real room?	Are the furnishings in the room created to the same scale and accurately represented?
4	The model uses the space well, and an accurate scale is apparent.	All supporting work is attached, and supports the dimensions seen in the model.	This model is proportional to the original measurements of the room and to the first floor plan.	The furnishings are shown to be proportion to the actual furnishings, and fit accurately into the model.
3	The model fits in show box with regard to length and width, but is too tall for the box.	Some supporting work is attached, and supports the dimensions seen in the model.	This model shows that most measurements of the room are proportional to the first floor plan.	Some furnishings are shown to be proportion to the actual furnishings, and fit accurately into the model.
2	The model was created to fit the length and width of the box rather than to the dimensions of the room.	Work is shown to support most measurements, but other measurements are inaccurate or unsupported.	This model shows some measurements of the room are proportional to the first floor plan.	The furnishings are shown to be proportion to the actual furnishings, but are not to the same scale as the model .
1	The model does not fit in a show box.	Work is shown to support some measurements, but most measurements are inaccurate or unsupported.	This model is not proportional to the floor plan.	Furnishings are inaccurate or there is no evidence to support their accuracy.
0	No model was presented.	No supporting work was provided.	There is no evidence of accuracy.	Furnishings are not present.

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Lesson 5: It has to be how big?!

Strand

Geometry, Number and Operations, Measurement

Mathematical Objective(s)

- Using dynamic geometry software to explore and design congruent areas of irregular polygons
- Constructions of irregular polygons using dynamic geometry software
- Transformations of polygons using dynamic geometry software
- Students will be able to discover that areas of irregular polygons can be deconstructed and reconstructed into a different design with congruent areas.
- Students will construct irregular polygons using transformation commands within the dynamic geometry software.
- Students will find area through measurement commands within the dynamic geometry software.
- Students will use conversion and scale factor to calculate actual dimensions of rooms using real blueprints.
- Students will calculate area of irregular polygons.
- Students will read actual blueprints.

Virginia College and Career Ready Mathematics Performance Expectation(s)

MPE 7: Use similar geometric objects in two- or three- dimensions to

a) compare ratios between side lengths, perimeters, areas, and volumes;

b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;

c) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and

d) solve real-world problems about similar geometric objects.

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Related SOL

G.3d, G.4c,d,g, G.9, G.13, G.14d

NCTM Standards

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
- Understand meanings of operations and how they relate to one another.
- Understand measurable attributes of objects and the units, systems, and processes of measurement.
- Make decisions about units and scales that are appropriate for problem situations involving measurement.
- Apply appropriate techniques, tools, and formulas to determine measurements.
- Understand and use formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders.
- Build new mathematical knowledge through problem solving. Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.
- Select and use various types of reasoning and methods of proof.
- Organize and consolidate their mathematical thinking through communication.
- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognize and apply mathematics in contexts outside of mathematics.
- Create and use representations to organize, record, and communicate mathematical ideas.
- Select, apply, and translate among mathematical representations to solve problems.
- Use representations to model and interpret physical, social, and mathematical phenomena.

Materials/Resources

- Class set of laptops or computer lab access; LCD projector for demonstration
- Geometer's Sketchpad, Geogebra, or any dynamic geometry software (instructor should be comfortable using the software chosen)
- Actual blueprint showing floor design of home including a living area and hallway or foyer
- Rulers

Assumption of Prior Knowledge

- Area of polygons (regular, non-regular, irregular).
- Knowledge of similar polygons.
- Transformations of polygons.
- This lesson assumes the student has calculated squares of rational numbers.

- A student should be at the Level 2/3 on the Van Hiele scale concerning polygons
- Familiarity with dynamic geometry software.
- Decomposition of complex figures
- Concepts of architecture and interior design

Introduction: Setting Up the Mathematical Task

- Goal of the lesson:

  "In this lesson, you will investigate how to read a blueprint, convert measurement according to a scale factor, and use dynamic geometry software to decompose and recompose congruent areas of irregular polygons."

- Timeline for lesson—One 90-minute block:

What	**Who**	**Amount of Time**
Warmup	Students complete with a partner. Pairs will share strategies with rest of class when prompted by teacher.	10 minutes
Student Exploration I (blueprint reading, measurement, and conversion)	Students will work in pairs or groups of 3 to complete the blueprint activity involving measurement and conversion. Teacher may need to review finding area of irregular polygons here.	20 minutes
Student Exploration II (Real-world application using dynamic geometry software)	All students work independently on their own laptop or computer to decompose and redesign congruent areas of irregular polygons	30-45 minutes
Reflective writing	Students will complete in a Word document	15 minutes

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Warm-up:

Calculate the following conversions. Please show your work.

1. ¼ of 1 foot = ___________ inches
2. 1/8 of 1 foot = __________ inches
3. 1/16 of 1 foot = __________ inches
4. 1.5 inches is what part of a foot? ______________
5. 9 inches is what part of a foot? _______________
6. 6-2/3 inches is what part of a foot? ____________

Answers to warm-up:

1. 3 inches 2. 1.5 inches 3. ¾ inches 4. 1/8 5. ¾ 6. 5/9

Assessing the Explorations. . .

The teacher can access the students' saved documents in their student e-folders. Explorations I and II will be graded using a rubric. .

Students will also be required to create a Word document reflecting about what they learned from each exploration. They will also be asked to reflect on how they can extend this thinking to a real-world situation and to describe the real-world situation.

Student Exploration I—Exploration of reading actual blueprints, measurement, and conversion using the blueprint's scale factor

- The teacher will have actual blueprints of floor designs of houses available for groups of students to use. The teacher may hang these on the wall for students to use or allow the groups to use them flat.

- Have students complete attached handout.

Student Exploration II—Decomposing and recomposing congruent areas of irregular polygons

- Have students follow directions on attached handout.

Name____ Date_____

It has to HOW BIG?! Exploration 1

1) With your group members, find the scale factor of your blueprint and record it here ________________________________.

   a) Briefly explain what this scale factor means in relation to this blueprint/design. (The scale factor is the proportional comparison to the actual dimensions.)

2) With your group members, using the rulers, measure the dimensions of the following rooms of the house and record your findings here. If the room is not rectangular, make sure to record all dimensions. You may want to sketch a quick picture of the shape of the room, also.

   a) Living area

   b) Foyer/hallway

   c) Master bedroom

   d) Master bedroom closet

   e) Kitchen

3) Using the scale factor and the dimensions you recorded in #2 above, calculate the actual dimensions of each room above and record here.

   a) Living area

   b) Foyer/hallway

   c) Master bedroom

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d) Master bedroom closet

e) Kitchen

4) Using your knowledge of how to calculate area of irregular polygons, calculate the actual area of each of the rooms from #3.

a) Living area

b) Foyer/hallway

c) Master bedroom

d) Master bedroom closet

e) Kitchen

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Name____ Date_____

It has to HOW BIG?! Exploration 2

Real-world scenario: You have been hired by a developer to design 4 different floor plans for homes to be built in a subdivision. The developer wants to keep the price of the homes within a certain range, so the square footage of each home must be comparable. The developer has decided the best way to do this is to make the square footage for the living area and foyer/hallway congruent for each of the 4 floor plans; however, he wants some interesting designs that will distinguish each model from the other. One of your designs will be the layout that was on the blueprint you used in Student Exploration I. Your assignment is to create the other 3 floor plans for the living area and foyer/hallway. Remember, the areas must be congruent to the area of the design on the blueprint.

Using dynamic geometry software, recreate the design of the living area and foyer/hallway from your blueprint into a sketch. Make sure you use the construction commands to preserve the shape. Make sure your dimensions are exactly as they are on the blueprint.

Explore decomposing and recomposing the polygon by dragging a vertex to change the design of the living area and foyer/hallway. Remember to keep the area congruent to the original design.

Save your original plus 3 new designs in your student folder using your first initial and last name. (example: floor designs_hkeller)

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Assessment of Student Exploration I

	Scale Factor	Ruler Measurements	Actual Dimension Conversion	Calculation of Actual Area	Used time wisely and appropriately, stayed on task, etc.
3	Scale factor is recorded and accurate	All measurements are accurate and recorded	All conversions are accurate and recorded	All calculations are accurate and recorded	Student did not have to be redirected
2	Scale factor is recorded but is not accurate	Measurements are 80-99% accurate	Conversions are 80-99% accurate	Calculations are 80-99% accurate	Student had to be redirected once
1	NA	Measurements are 50-79% accurate	Conversions are 50-79% accurate	Calculations are 50-79% accurate	Student had to be redirected twice
0	Scale factor is not recorded	Measurements are less than 50% accurate	Measurements are less than 50% accurate	Measurements are less than 50% accurate	Student had to be redirected more than twice

Assessment of Student Exploration II

	Sketches	Calculation of Area	Used time wisely and appropriately, stayed on task, etc.
3	All sketches are included and 100% accurate	All areas are congruent to given area	Student did not have to be redirected
2	All sketches are included and 80-99% accurate	Areas are 80-99% accurate	Student had to be redirected once
1	All sketches are included and 50-79% accurate	Areas are 50-79% accurate	Student had to be redirected twice
0	All sketches are not included or are less than 50% accurate	Areas are less than 50% accurate	Student had to be redirected more than twice

Reflection Writing

- Please create a Word document titled *Reflection Writing—Area of Congruent Irregular Polygons*
- Please write a paragraph of at least 5 good sentences explaining what you learned from each Student Exploration. Also, please include another real-world situation where you think area of congruent irregular shapes might be used or applied.
- Please save your document into your student folder as reflection 2 writing_first initial last name (example: reflection 2 writing_hkeller)

Extensions and Connections (for all students)

Strategies for Differentiation

- The Student Explorations I and II will address the needs of a diverse population of students such as kinesthetic, auditory, and visual learners.
- There is an opportunity for peer tutoring of the dynamic geometry software for those students who are more proficient with it.
- English language learners (ELLs)—the actual working with the software should be acceptable; however, some of the vocabulary in the commands may take further explanation

Lesson 6: Home for a Wizard

Strand

Geometry, Measurement

Mathematical Objective(s)

- Apply knowledge of similar figures and similar 3-dimensional objects to a real-world scenario.
- Constructions of irregular polygons using dynamic geometry software
- Transformations of polygons using dynamic geometry software
- Students will use scale factors to determine cost of similar objects.
- Students will read blueprints and construct blueprints using their choice of software.

Virginia College and Career Ready Mathematics Performance Expectation(s)

MPE 7: Use similar geometric objects in two- or three- dimensions to

a) compare ratios between side lengths, perimeters, areas, and volumes;

b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;

c) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and

d) solve real-world problems about similar geometric objects.

Related SOL

G.3d, G.4c,d,g, G.9, G.13, G.14d

NCTM Standards

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
- Understand meanings of operations and how they relate to one another.
- Understand measurable attributes of objects and the units, systems, and processes of measurement.

- Make decisions about units and scales that are appropriate for problem situations involving measurement.
- Apply appropriate techniques, tools, and formulas to determine measurements.
- Understand and use formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders.
- Build new mathematical knowledge through problem solving. Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.
- Select and use various types of reasoning and methods of proof.
- Organize and consolidate their mathematical thinking through communication.
- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognize and apply mathematics in contexts outside of mathematics.
- Create and use representations to organize, record, and communicate mathematical ideas.
- Select, apply, and translate among mathematical representations to solve problems.
- Use representations to model and interpret physical, social, and mathematical phenomena.

Materials/Resources

- Class set of laptops or computer lab access; LCD projector for demonstration
- Geometer's Sketchpad, Geogebra, or a CAD program
- Actual blueprints showing floor design of home.

Assumption of Prior Knowledge

- Area of various polygons (regular, non-regular, irregular).
- Understanding of congruent shapes.
- Transformations of polygons.
- Similar figures and scale factors.
- Use of a ruler to measure and record measurements with precision.
- Familiarity with dynamic geometry software previously.
- Level 2/3 on the Van Hiele scale concerning polygons.
- Concepts of floor design.

Introduction: Setting Up the Mathematical Task

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- Goal of the lesson:

  "In this lesson, you will investigate the impact of dimension changes on the size and cost of a home."

- Warm up:
- Timeline for lesson: One 90-minute block

What	Who	Amount of Time
Warm up	Students work with a partner to complete the warm up; share strategies when prompted by teacher. If necessary teacher should facilitate discussion reviewing the impact of scale changes on area and volume of an object.	15 min
Student Exploration 1	A Home for a Wizard	60 minutes
Writing Prompt	How do dimension changes impact the size and cost of a home?	15 minutes

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Warm up:

Scenario: A manufacturer of various food items has decided to decrease the size of its packaging by 25 percent. Find the following information for the food box with which you have been provided. (Each pair of students receives a food box that is in the shape of a rectangular prism.) Apply what you have learned in previous lessons to determine the indicated information for the new food box. Be able to explain your work.

	Original Food Box	New Food Box
Length		
Width		
Height		
Total Surface Area		
Volume		

Have students share the techniques they used to determine the dimensions of the new food box.

Student Exploration I

Real world scenario: The architect of the blue prints from the last lesson has been contacted by a player of the Washington Wizards about building a home with the same floor plan, but modified to fit the above-average size of basketball players. To maintain aesthetics, the basketball player would like for all dimensions represented in the new floor plan to reflect the same change in scale factor. The developer asks you to make a new blueprint for him reflecting the new dimensions.

Given that the average height of a male in the United States is 5’ 10” and the average height of an NBA basketball player is 6’5”, determine an appropriate scale factor to use to change the house dimensions. Modify an existing floor plan of your choice to reflect this change.

A. Make a table listing:

- Dimensions of each room on the existing floor plan. Remember to include height!
- Dimensions of each room on the new floor plan. Remember to include height!

B. Using the software of your choice, draw the new floor plan.

C. The developer is unsure of how much more to charge the basketball player for this house. If the original house sold for an amount equivalent to $100 per square foot, make a mathematical

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argument for how much per square foot the builder should charge for the larger scale home. (Remember that the height of the house has changed, too!)

Monitoring Student Responses

- Students will communicate their thinking and new knowledge verbally and in writing during the writing prompt.
- Students will encourage each other to explain their work, especially when two students disagree as to an outcome to a question.
- The teacher will circulate during the group activities and question students about the processes they are using and how they know that these processes work.
- The teacher will assist students who have difficulties by phrasing the directions in different words and asking probing questions of students to determine where the difficulties lie. As appropriate, the teacher will provide background knowledge or model a similar problem to help students move to the next level of thought.
- Students who are ready to move forward can be prompted to think about how the cabinetry and plumbing fixtures may need to be altered to accommodate the changes in scale.

Assessment

- Journal/writing prompts rubric (to follow)
- Rubric for Student Exploration 1: A Home for a Wizard (to follow)

Extensions and Connections (for all students)

- Lesson extensions/follow-up: How would decreasing the house dimensions by a scale factor of 10% impact the cost of the house?
- Connections to other mathematics content: This connects to students previous learning of similar figures (Math 7) and solving proportions (Math 7, Pre-Algebra, and Algebra I)
- Connections to content in other subject areas: landscaping, architectural drawing

Strategies for Differentiation

- List ideas for addressing needs of a diverse population of students such as:
  - Working with carefully chosen partners helps meet the needs of students with processing, memory, motor issues;
  - English language learners (ELLs); ELL students may need additional time to translate the directions and outcomes.
  - High-ability students: How does an increase by a scale factor of 20% increase costs as compared to an increase of 10%.
- Provide graph paper for blueprints and tables instead of having students use software to construct them.

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Rubric for "A Home for a Wizard"

	Excellent – 4 points	Good – 3 points	Average –2 points	Poor – 1 points
Table or chart of measurements.	Table or chart includes accurate measurements of dimensions of All rooms, including bathrooms, utility rooms, etc.	Table or chart includes accurate measurements of dimensions of At least 80% of the rooms	Table or chart includes accurate measurements of dimensions of two of the following: Between 50 and 80% of the rooms.	Table or chart includes accurate measurements of dimensions of one of the following: Fewer than 50% of the rooms
Scaled values	Table or chart includes accurate measurements of dimensions of All rooms, including bathrooms, utility rooms, etc.	Table or chart includes accurate measurements of dimensions of At least 80% of the rooms	Table or chart includes accurate measurements of dimensions of two of the following: Between 50 and 80% of the rooms.	Table or chart includes accurate measurements of dimensions of one of the following: Fewer than 50% of the rooms
Blueprint	Blueprint accurately shows scaled models of All rooms, including bathrooms, utility rooms, etc.	Blueprint accurately shows scaled models of At least 80% of the rooms	Blueprint accurately shows scaled models of two of the following: Between 50 and 80% of the rooms.	Blueprint accurately shows scaled models of one of the following: Fewer than 50% of the rooms
Cost of larger scaled home	Student presents a mathematical argument that correctly incorporates the cost of the additional height of the room.	Student presents a mathematical argument that incorporates the cost of the additional height of the room, but makes arithmetic errors.	Student recognizes that the height of the room would impact the cost, but does not present an argument that take this cost into consideration.	Student calculates new cost using only the changes in length and width of the rooms, but does not incorporate the height.

Rubric for Journal/Writing Prompt

Excellent – 4 points	Good – 3 points	Average – 2 points	Poor – 1 points	0 points
Student demonstrates mastery of the concept of similar figures. Examples are specific, relevant, and explained.	Student demonstrates some knowledge of the concept of similar figures. Examples are present, but may not be as specific as possible or explained as clearly as possible.	Student's knowledge of similar figures is unclear. Examples are present, but they may be vague or undeveloped.	Student's knowledge of the similar figures is limited. Examples are not present or do not relate to the concept of similar figures.	Student chose not to participate.

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