Fun with Solids

I. UNIT OVERVIEW & PURPOSE:

The purpose of this lesson is to solve real-world problems related to surface area and volume of three-dimensional objects. Students will begin each lesson by "discovering" the formulas for surface area and volume of specific three-dimensional objects: rectangular prisms, cylinders, cones, spheres, and pyramids. After each formula has been found, students will then use those formulas to investigate real-world problems. Lesson 1 – 'Fun with Cakes', Lesson 2 – 'Fun with Volcanoes', Lesson 3 – Senior Prank, and Lesson 4 – The Egyptian Tomb. As a unit project, students will then plan, build, and present a toy they have made using various three-dimensional solids in Lesson 5.

II. UNIT AUTHORS:

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III. COURSE:

Mathematical Modeling: Capstone Course (the course title might change)

IV. CONTENT STRAND:

Geometry

V. OBJECTIVES:

Students will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

VI. MATHEMATICS PERFORMANCE EXPECTATION(s):

MPE.6) Students will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

VII. CONTENT:

In this unit, students will discover the formulas for the surface area and volume of rectangular prisms, cylinders, cones, spheres, and pyramids. The students will then apply those formulas to investigate miscellaneous "fun" real-world problems. The unit will end with a culminating activity of students planning, building, and presenting a toy using various solids that have been explored.

VIII. REFERENCE/RESOURCE MATERIALS:

VA SOL Geometry Standards
NCTM Standards
Teacher generated worksheets

Various other VA SOL Standards

IX. PRIMARY ASSESSMENT STRATEGIES:

The formula worksheet provided will be filled in each time the surface area and volume formulas are discovered for each object. It will be graded at the end of the unit for completeness and accuracy.

Each lesson will end with student presentations and will be graded using a rubric.

Each lesson will also conclude with a journal reflection, to be turned in and graded for completion at the end of the unit.

X. EVALUATION CRITERIA:

Rubrics will be provided in each lesson to aid in scoring student presentations.

XI. INSTRUCTIONAL TIME:

5-7 days for a 90 minute block or 10-14 days for a 45 minute class. Each lesson, 1 through 4, should take one day on a 90 minute block, but lesson 5 may require more than one day.

Lesson 3 – Senior Prank

Strand
Geometry

Mathematical Objective(s)
Students will discover the formulas for surface area and volume of three-dimensional objects through hands on investigations. Students will then use those formulas to investigate real-world problems.

Mathematics Performance Expectation(s)
MPE.6) Students will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

Related SOL
G.13 - Students will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

NCTM Standards
In grades 9-12 all students should:

- Analyze properties and determine attributes of two- and three-dimensional objects;
- Explore relationships among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them;
- Use geometric models to gain insight into, and answer questions in, other areas of mathematics;
- Use geometric ideas to solve problems in, and gain insight into, other disciplines and other areas of interest such as art and architecture.

Career and Technical Education Standards
Architectural Drawing and Design 016 – Students will demonstrate job-specific mathematics skills.
Architectural Drawing and Design 038 – Students will calculate square footage.

Materials/Resources

- Formula Worksheet
- Water Tower Worksheet
- Geometric Solids – cylinders and spheres
- Small dried beans
- Disposable trays
- Paper, pencil, rulers, and calculators

Assumption of Prior Knowledge

- Students will have taken Algebra 1, Geometry, and Algebra 2 or Algebra Functions & Data Analysis.
- Students will know the area formulas for rectangles and circles and cylinders.
- Students will know the surface area of a three-dimensional object is the sum of the areas of all of its faces.
- Students will know the volume of a three-dimensional object is the number of unit cubes that will fill the object.
- Students are expected to know how to calculate the surface area and volume of cylinders and spheres once they have the formulas.
- Students should be operating on at least Level 2 of the Van Hiele Scale in relationship to geometric solids.

Introduction: Setting Up the Mathematical Task

- Introduction: "You are going to pull off the greatest Senior Prank ever! The city has a spherical water tower that you are going to steal, paint the school colors, and place on the football field. You'll be legends (if you don't go to jail)."
- The entire lesson will take approximately 90-180 minutes.
- Students will be pair off into teams of two.
- Students will discover the surface area and volume formulas for spheres using math manipulatives.
- Students will have to use prior knowledge of area of circles to derive the surface area formula. They need to use the cylinder to derive the volume of a sphere.
- Students will use the formulas to complete their "senior prank".
- Students will illustrate and report their findings at the end of class and write a reflective journal entry for homework.

Student Exploration 1:

Small Group Work

Students will work in groups of two.

Student/Teacher Actions:

- Each pair of students will have a tray of small dried beans, geometric solids including cylinders and spheres.
- Students will use the manipulatives and prior knowledge to derive the surface area and volume formulas of spheres.

    - To come up with the formula for surface area of a sphere, students will relate it to the surface area of a cube, except the surfaces are circles. Teachers may have to give this hint.
    - To figure out the formula for the volume of a sphere, students will fill a cylinder with dry beans and then fill a sphere of the same height with those same beans. Students should notice there aren't quite enough beans to fill the sphere, in fact, it needs one third more beans. They will use the volume formula for a cylinder, change the height to radius, and multiply by four thirds to get the volume of a sphere.
- Students will write the formulas down on the worksheet provided in Lesson 1. Each student will have his/her own worksheet.
- Teacher will offer little assistance during the exploration so the students can struggle through and assemble the new information. However, students may discuss what they are discovering with each other. Teacher may give more hints, as this is more difficult to derive.
- Have a class discussion to ensure correct formulas for each student.

Monitoring Student Responses

- Students should discuss their discoveries with each other during the exploration.
- Random students will be called on to present what they have derived to the class, and allow the class to determine if they had similar answers. The same student should explain how they came up with their formula, and see if other students derived the formula a different way.
- The teacher will confirm the results found and the results will be used in the second activity.

Student Exploration 2:

Student/Teacher Actions:

- "Now that we have discovered the formulas for volume and surface area of cones, let's use these formulas to help us paint the water tower and move it to the football field."
- Students will have to do a little research online to find out the dimensions of the water tower in their town, or a town nearby, including the dimensions of the spherical tank by itself.
- Students will first calculate the volume of the water tank.
- Then they will calculate how many 55-gallong barrels they will need to drain the tank to make it easier to move.

- Students will need to calculate the surface area of the cylindrical water tower so they can get a sheet big enough to cover it when they steal it.
- Students will have to calculate the surface area of the tank to determine how much paint they will need to cover the entire tank in their school colors.
- Students will illustrate their calculations on a poster or piece of paper and will present their findings to the class.

Monitoring Student Responses

- Teacher will walk around the room and monitor group progress and answer questions as needed.

Assessment

- Formula Worksheet from Lesson 1
- Water Tower Worksheet
- Students will illustrate their painted water towers on a poster or piece of paper.
- Students will present their findings to the class and be graded with the following rubric:

	4	3	2	1	0
Correct Calculations	All correct	Most correct	Half correct	Few correct	None correct
Show Appropriate Work	All work shown	Most work shown	Half of the work shown	Little work shown	No work shown
Drawing of Water Tower	Excellent	Very Good	Good	Fair	Poor
Participation/ Group Cooperation	Excellent	Very Good	Good	Fair	Poor

Neatness/ Presentation	Excellent	Very Good	Good	Fair	Poor

Extensions and Connections (for all students)

- For homework, students will write a one page reflection about what they learned and how they could apply this to another real world application or talk about what other things they could figure out for this problem, like the cost of paint or barrels to store the water for moving or cost to move it (gas prices), etc.

Strategies for Differentiation

- This lesson is visual, collaborative, and hands on; which by nature will allow for multiple learning styles. Students who struggle with this activity may be provided more prompts and tips for calculations.
- Students who need extra help may be paired with students who are good at peer coaching rather than openly supplying answers.
- In a collaborative classroom, the additional teacher will provide extra assistance to students as needed.
- Students who do not work well with others may have to work by themselves and discuss conclusions directly with the teacher.

Stealing the Water Tower!

(Best Senior Prank Ever)

You and your friends have decided to pull off the best senior prank ever: steal the water tower and put it on the football field. But before you do the deed, you need to do a few computations to make sure everything will run smoothly.

1. Your first problem is the water tower is full of water (thanks Captain Obvious). You don't want water spraying everywhere so you'll need to drain it into 55-gallon drums. How many drums do you need?

2. Now that it's drained, you need to cover it for transportation (you can't just drive down the street with a water tower in the back of your truck). You decide to disguise the tower by making a cylindrical frame around it and cover it with a sheet. If the diameter of the frame is 6 inches larger than the tower and just as high, how big does your sheet need to be? (You don't need to cover the bottom)

3. Even though you are already the coolest kids in school, you're going to go one step further and paint the tank in school colors. How many 1 gallon paint cans do you need to buy?

_____ City Water Tower

Dimensions:

Tower Height: _____

Tank Volume: _____

Tank Surface Area: _____

Conversion to consider:

1 gallon = 0.13368 Cubic Feet

Lesson 4 – Egyptian Tomb

Strand
Geometry

Mathematical Objective(s)
Students will discover the formulas for surface area and volume of three-dimensional objects through hands on investigations. Students will then use those formulas to investigate real-world problems.

Mathematics Performance Expectation(s)
MPE.6) Students will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

Related SOL
G.13 - Students will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

G.14 - Students will use similar geometric objects in two- or three-dimensions to

a) compare ratios between side lengths, perimeters, areas, and volumes;
b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;
c) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and
d) solve real-world problems about similar geometric objects.

WHI.1 – Students will improve skills in historical research and geographical analysis by

b) using maps, globes, artifacts, and pictures to analyze the physical and cultural landscapes of the world;
c) identifying major geographic features important to the study of world history.

NCTM Standards
In grades 9-12 all students should:

- Analyze properties and determine attributes of two- and three-dimensional objects;
- Explore relationships among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them;
- Use geometric models to gain insight into, and answer questions in, other areas of mathematics;
- Use geometric ideas to solve problems in, and gain insight into, other disciplines and other areas of interest such as art and architecture.

Materials/Resources

- Formula Worksheet

- Egyptian Tomb Worksheet
- Geometric Solids – cubes and pyramids
- Small dried beans
- Disposable trays
- Paper, pencil, rulers, and calculators
- Computers with internet access

Assumption of Prior Knowledge

- Students will have taken Algebra 1, Geometry, and Algebra 2 or Algebra Functions & Data Analysis.
- Students will know the area formulas for rectangles and triangles.
- Students will know the surface area of a three-dimensional object is the sum of the areas of all of its faces.
- Students will know the volume of a three-dimensional object is the number of unit cubes that will fill the object.
- Students are expected to know how to calculate the surface area and volume of cubes and pyramids once they have the formulas.
- Students should be operating on at least Level 2 of the Van Hiele Scale in relationship to geometric solids.

Introduction: Setting Up the Mathematical Task

- Introduction: "You are on the prom committee and this year's theme is "The Egyptian Tomb". Your job is to construct the Great Pyramid of Giza to scale to fit inside the high school gym.
- The entire lesson will take approximately 90-180 minutes.
- Students will be paired off into teams of two.
- Students will discover the surface area and volume formulas for pyramids using math manipulatives.
- Students will have to use prior knowledge of area of triangles and rectangles to derive the surface area formula. They need to use the cube to derive the volume of a pyramid.
- Students will use the formulas to complete their Egyptian Tomb.
- Students will illustrate and report their findings at the end of class and write a reflective journal entry for homework.

Student Exploration 1:

Small Group Work
Students will work in groups of two.

Student/Teacher Actions:

- Each pair of students will have a tray of small dried beans, geometric solids including rectangular prisms and square pyramids.
    - Students will use the manipulatives and prior knowledge to derive the surface area and volume formulas of pyramids. To figure out the formula for the surface area of a square pyramid, student will find the area of each side, which is a triangle, and the area of the base, and add them up and turn it into a nicer multiplication formula.
    - To come up with the formula for the volume of a square pyramid, students will fill a cube with dry beans then pour those beans into a pyramid of same base and height. Students should notice only one third of the beans fit into the pyramid thus showing them that the volume of a square pyramid is one third of the volume of a cube.
- Students will write the formulas down on the formula worksheet provided in Lesson 1. Each student will have his/her own worksheet.
- Teacher will offer little assistance during the exploration so the students can struggle through and assemble the new information. However, students may discuss what they are discovering with each other.
- Have a class discussion to ensure correct formulas for each student.

Monitoring Student Responses

- Students should discuss their discoveries with each other during the exploration.
- Random students will be called on to present what they have derived to the class, and allow the class to determine if they had similar answers. The same student should explain how they came up with their formula, and see if other students derived the formula a different way.
- The teacher will confirm the results found and the results will be used in the second activity.

Student Exploration 2:

Student/Teacher Actions:

- “Now that we have discovered the formulas for volume and surface area of pyramids, let’s use these formulas to help us build a replica of The Great Pyramid of Giza.”
- Students will research the dimensions of the Great Pyramid of Giza.
- Students will use ratios to scale the Great Pyramid of Giza to a smaller, more manageable size to construct. This ties into the geometry SOL G14.
- Students will calculate how much cloth they need to cover the pyramid (Surface Area)
- Students will calculate how much volume the pyramid has to determine if they can fill it with balloons or candy.
- Students will also compare the volume/surface area of similar square pyramids.

Monitoring Student Responses

- The teacher will walk around the room and monitor group progress and answer questions as needed.

Assessment

- Formula Worksheet from Lesson 1.
- Students will research The Great Pyramid of Giza and answer questions on the Pyramid worksheet – attached.
- Students will draw a small scale replica of the pyramid on a poster or a piece of paper.
- Students will present their findings to the class and be graded with the following rubric:

	4	3	2	1	0
Correct Calculations	All correct	Most correct	Half correct	Few correct	None correct
Show Appropriate Work	All work shown	Most work shown	Half of the work shown	Little work shown	No work shown
Drawing of Water Tower	Excellent	Very Good	Good	Fair	Poor
Participation/ Group Cooperation	Excellent	Very Good	Good	Fair	Poor
Neatness/ Presentation	Excellent	Very Good	Good	Fair	Poor

Extensions and Connections (for all students)

- For homework, students will write a one page reflection about what they learned and how they could apply this to another real world application.
- If time allows and/or the teacher is willing, students could build a scaled replica of the pyramid they chose.

Strategies for Differentiation

- This lesson is visual, collaborative, and hands on; which by nature will allow for multiple learning styles. Students who struggle with this activity may be provided more prompts and tips for calculations.
- Students who need extra help may be paired with students who are good at peer coaching rather than openly supplying answers.
- In a collaborative classroom, the additional teacher will provide extra assistance to students as needed.
- Students who do not work well with others may have to work by themselves and discuss conclusions directly with the teacher.

The Egyptian Tomb!

(Senior Prom)

You and your classmates are on Planning Committee for this year's senior prom. Because of your awesomeness at math, you've been assigned to construct the Great Pyramid of Giza. Because Prom is in the Gym, you're obviously going to need to scale back. Fortunately, you have the tools to do so! Follow the steps below and good luck!

1. RESEARCH:

Lateral Height: _____ Base: ____ x ____

(Don't forget units of measurement)

Source of Information:

_____

Interesting Facts:

_____

_____

2. SCALE:

Your pyramid must be much smaller (try a ratio of 1:32). What are your new dimensions?

Lateral Height: _____ Base: ____ x ____

3. Material:

Now you need to figure out your material list for, most importantly, covering the pyramid. What is the surface area of your pyramid?

How does the surface area of your pyramid compare to the surface area of the actual Pyramid of Giza?

4. Extra money in the budget:

   You want to fill your pyramid with candy but reality slaps you in the face and you need to down-grade your size again so you don't go broke. What is a good ratio to compare the actual pyramid to a piñata-sized pyramid?

   Actual: Height_____ Base _____ x _____

   Ratio: 1: ___

   Piñata: Height_____ Base _____ x _____

5. What is the surface area and volume of your Piñata pyramid?

   Surface Area:_____ Volume: _____

Lesson 5 – Toy Time

Strand
Geometry

Mathematical Objective(s)
Students will discover the formulas for surface area and volume of three-dimensional objects through hands on investigations. Students will then use those formulas to investigate real-world problems.

Mathematics Performance Expectation(s)
MPE.6) Students will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

Related SOL
G.13 - Students will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

NCTM Standards
In grades 9-12 all students should:

- Analyze properties and determine attributes of two- and three-dimensional objects;
- Explore relationships among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them;
- Use geometric models to gain insight into, and answer questions in, other areas of mathematics;
- Use geometric ideas to solve problems in, and gain insight into, other disciplines and other areas of interest such as art and architecture.

Materials/Resources

- Geometric Solids
- Paper, pencil, rulers, and calculators

Assumption of Prior Knowledge

- Students will have taken Algebra 1, Geometry, and Algebra 2 or Algebra Functions & Data Analysis.
- Students will know the surface area and volume formulas for prisms, cylinders, cones, spheres, and pyramids.
- Students should be now operating on at least Level 3 of the Van Hiele Scale in relationship to geometric solids.

Introduction: Setting Up the Mathematical Task

- Introduction: "If you look around, everything you see can be broken down into basic solids such as cylinders, prisms, cones, spheres and pyramids. Today you will get to invent, design and build a toy!"
- The entire lesson will take approximately 120 minutes.
- Students will work in groups of two or three.
- Students will draw a picture of their toy using at least 5 solids.
- Students will give the toy a catchy name and describe how it works.
- Students will build the toy in class.
- Students will calculate the volume and surface area of their toy.
- Students will present their toy to the class (3-5 minutes long).

Student Exploration 1:

Small Group Work

Students will work in groups of two or three.

Student/Teacher Actions:

- Students in groups of two or three will brainstorm ideas for a toy.
- Students will draw up "blueprints" of their design on paper and determine what solids they will need to build their toy.
- Students will bring the solids needed to make the toy to class the next day. They will use household items such as paper towel rolls, tissue boxes, etc. to build their toy.
- Students will build their toy in class.
- Students will calculate the volume and surface area of their toy.
- Students will present their toy to the class.

Monitoring Student Responses

- The teacher will assist students by answering questions and providing suggestions along the way.
- The teacher will have some extra household items on hand in case students forget their materials at home: toilet paper / paper towel rolls, tissue boxes, ping pong balls, medicine bottles, etc.
- The students will share some of their ideas with each other.

Assessment

- Students will be graded with the following rubric:

	4	3	2	1	0
Blue Print of the toy	Excellent	Very Good	Good	Fair	Poor
Number of figures used	5 figures used	4 figures used	3 figures used	2 figures used	1 figure used
Identification of each figure	All figures identified correctly	Most figures identified correctly	Half of the figures identified correctly	Only one or two figures identified correctly	None of the figures are identified correctly
Name of toy and description	Creative and complete description	Creative and partial description	Not creative and partial description	Not creative and poor description	No creativity and no description

Volume and Surface Area calculations	All correct	Most correct	Half correct	Few correct	None correct
Participation and group cooperation	Excellent	Very Good	Good	Fair	Poor
Color and neatness	Excellent	Very Good	Good	Fair	Poor
Class presentation	Excellent	Very Good	Good	Fair	Poor
One page paper	Excellent	Very Good	Good	Fair	Poor

Extensions and Connections (for all students)

- For homework, students will write one page describing the toy they built, what solids they used, how they put it together, and what it does.
- This will help students in their other classes and/or later in their careers if/when they need to create a presentation for another teacher or a boss.

Strategies for Differentiation

- This lesson is visual, collaborative, and hands on; which by nature will allow for multiple learning styles.
- Students who need extra help may be paired with students who are good at peer coaching rather than openly supplying answers.
- In a collaborative classroom, the additional teacher will provide extra assistance to students as needed.
- Students who do not work well with others may have to work by themselves and discuss conclusions directly with the teacher.
- The teacher may or may not want to write out a list of what the students are supposed to do and/or incorporate into their toy (as stated in the introduction to this lesson).