Fun with Solids

I. UNIT OVERVIEW & PURPOSE:

The purpose of this lesson is to solve real-world problems related to surface area and volume of three-dimensional objects. Students will begin each lesson by "discovering" the formulas for surface area and volume of specific three-dimensional objects: rectangular prisms, cylinders, cones, spheres, and pyramids. After each formula has been found, students will then use those formulas to investigate real-world problems. Lesson 1 – 'Fun with Cakes', Lesson 2 – 'Fun with Volcanoes', Lesson 3 – Senior Prank, and Lesson 4 – The Egyptian Tomb. As a unit project, students will then plan, build, and present a toy they have made using various three-dimensional solids in Lesson 5.

II. UNIT AUTHORS:

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III. COURSE:

Mathematical Modeling: Capstone Course (the course title might change)

IV. CONTENT STRAND:

Geometry

V. OBJECTIVES:

Students will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

VI. MATHEMATICS PERFORMANCE EXPECTATION(s):

MPE.6) Students will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

VII. CONTENT:

In this unit, students will discover the formulas for the surface area and volume of rectangular prisms, cylinders, cones, spheres, and pyramids. The students will then apply those formulas to investigate miscellaneous "fun" real-world problems. The unit will end with a culminating activity of students planning, building, and presenting a toy using various solids that have been explored.

VIII. REFERENCE/RESOURCE MATERIALS:

VA SOL Geometry Standards
NCTM Standards
Teacher generated worksheets

Various other VA SOL Standards

IX. PRIMARY ASSESSMENT STRATEGIES:

The formula worksheet provided will be filled in each time the surface area and volume formulas are discovered for each object. It will be graded at the end of the unit for completeness and accuracy.
Each lesson will end with student presentations and will be graded using a rubric.
Each lesson will also conclude with a journal reflection, to be turned in and graded for completion at the end of the unit.

X. EVALUATION CRITERIA:

Rubrics will be provided in each lesson to aid in scoring student presentations.

XI. INSTRUCTIONAL TIME:

5-7 days for a 90 minute block or 10-14 days for a 45 minute class. Each lesson, 1 through 4, should take one day on a 90 minute block, but lesson 5 may require more than one day.

Lesson 1 – Fun with Cakes

Strand
Geometry

Mathematical Objective(s)
Students will discover the formulas for surface area and volume of three-dimensional objects through hands on investigations. Students will then use those formulas to investigate real-world problems.

Mathematics Performance Expectation(s)
MPE.6) Students will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

Related SOL
G.13 - Students will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

NCTM Standards
In grades 9-12 all students should:

- Analyze properties and determine attributes of two- and three-dimensional objects;
- Explore relationships among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them;

- Use geometric models to gain insight into, and answer questions in, other areas of mathematics;
- Use geometric ideas to solve problems in, and gain insight into, other disciplines and other areas of interest such as art and architecture.

Career and Technical Education Standards
Culinary Arts I 016 – Students will demonstrate job-specific mathematical skills.

Materials/Resources
- Formula Worksheet
- Geometric unit figures of cubes and rectangular prisms (all shapes have openings for easy filling)
- 1 inch cubes
- Canned goods (cylinders)
- Various shapes and sizes of cake pans
- Paper, pencil, rulers, and calculators

Assumption of Prior Knowledge
- Students will have taken Algebra 1, Geometry, and Algebra 2 or Algebra Functions & Data Analysis.
- Students will know the area formulas for rectangles and circles.
- Students will know the surface area of a three-dimensional object is the sum of the areas of all of its faces.
- Students will know the volume of a three-dimensional object is the number of unit cubes that will fill the object.
- Students are expected to know how to calculate the surface area and volume of rectangular prisms and cylinders once they have the formulas.
- Students should be operating on at least Level 2 of the Van Hiele Scale in relationship to geometric solids.

Introduction: Setting Up the Mathematical Task

- Introduction: "You love cake so much, that you decided to open a cake shop. You know you will be making three cakes a day for five days and you need to go shopping. You must figure out how many boxes of cake mix and cans of icing to purchase."
- The entire lesson will take approximately 90 minutes.
- Students will be paired off into teams of two.
- Students will discover the surface area and volume formulas of rectangular prisms and cylinders using math manipulatives.

- Students will have to use prior knowledge of area of rectangles and circles to derive the volume and surface area formulas.
- Students will use the formulas to complete the cake activity.
- Students will illustrate and report their findings at the end of class and write a reflective journal entry for homework.

Student Exploration 1:

Small Group Work
Students will be paired off into teams of two.

Student/Teacher Actions:

- Each pair of students will have 1 inch unit cubes, geometric solids including cubes and rectangular prisms, and some canned goods.
- Students will use the manipulatives and prior knowledge to derive the surface area and volume formulas of rectangular prisms, cubes, and cylinders.
    - To come up with the formulas for the surface area of cubes and rectangular solids, students will calculate the area of the individual sides. Students will then try to simplify this addition into more efficient formulas using multiplication.
    - To come up with the formulas for the volume of cubes and rectangular solids, students will fill the geometric solid with unit cubes and count how many unit cubes fit into the figure. They will then try to come up with a formula using multiplication that would yield that number every time. (length x width x height)
    - To come up with the formula for the surface area of the cylinder, students will use the canned goods. The top/bottom of the can is a circle and they will take off the label to notice that the side is just a rectangle. They will have to know the area of a circle for the top and the length of the rectangle is the circumference of the top.
    - To come up with the formula for the volume of a cylinder, students will make a connection to the volume of cubes from earlier. Then length times width of the top of the can is the area of a circle, times the height of the can yields its volume. The teacher will help make this connection if necessary.
- Students will write the formulas down on the formula worksheet provided. Each student will have his/her own worksheet.

- The instructor will offer little assistance during the exploration so the students can struggle through and assemble the new information. However, students may discuss what they are discovering with each other.
- Have a class discussion to ensure correct formulas for each student.
- This exploration should take no more than 30 minutes but should be adjusted according to the needs of the class.

Monitoring Student Responses

- Students should discuss their discoveries with each other during the exploration.
- Random students will be called on to present what they have derived to the class, and allow the class to determine if they had similar answers. The same student should explain how they came up with their formula, and see if other students derived the formula a different way.
- The teacher will confirm the results found and the results will be used in the second activity.

Student Exploration 2:

Student/Teacher Actions:

- "Now that we have discovered the formulas for volume and surface area of rectangular prisms and cylinders, let's answer the question of how much cake mix and icing we need to purchase."
- Have several cake pans of various sizes that are either rectangular prisms or cylinders on a table in the front of the room. (If you do not own that many cake pans, you could either borrow from department members; or students could volunteer to bring in a cake pan.)
- Each group of two will pick one or two cake pans from the collection.
- The students will measure the dimensions of the chosen cake pans in inches.
- Using the formulas they just derived, students will calculate the surface area and volume of the cake that would be produced by the pan (assuming the cake would fill the pan perfectly).
- They will calculate the surface area of the portion of the cake that will be iced.
- They will calculate the volume of a three layer cake that would be produced by their cake pan(s).
- They will calculate the surface area of the part of the three layer cake that will be iced. (careful, you must put icing between each layer)

- They will calculate the volume of all the cakes that will be produced in one week assuming they are making three cakes a days for 5 days, and estimate how many boxes of cake mix to buy.
- They will also calculate the surface area of the iced part of all of the cakes that will be produced in one week, and estimate how many cans of icing to buy.
- Students will be given the volume produced by 1 box of cake mix and the surface area covered by 1 can of icing:
    - 1 box of cake mix will perfectly fill a 9x13x2 cake pan
    - 1 can of icing will perfectly cover a the cake from the 9x13x2 pan
- Students will illustrate one of their cakes on a poster or piece of paper and present it to the class, make a shopping list based on their findings, and write a reflective journal entry for homework.

Monitoring Student Responses

- Teacher will walk around the room and monitor group progress and answer questions as needed.

Assessment

- Formula Worksheet – attached.
- Students will illustrate their cake and findings on a poster or piece of paper.
- Students will present their findings to the class and be graded with the following rubric:

	4	3	2	1	0
Correct Calculations	All correct	Most correct	Half correct	Few correct	None correct
Show Appropriate Work	All work shown	Most work shown	Half of the work shown	Little work shown	No work shown
Drawing of Cake	Excellent	Very Good	Good	Fair	Poor
Participation/ Group	Excellent	Very Good	Good	Fair	Poor

Cooperation					
Neatness/ Presentation	Excellent	Very Good	Good	Fair	Poor

Extensions and Connections (for all students)

- For homework, students will write a one page reflection about what they learned and how they could apply this to another real world application. The teacher may or may not provide a prompt for this reflection.

Strategies for Differentiation

- This lesson is visual, collaborative, and hands on; which by nature will allow for multiple learning styles.
- Students who need extra help may be paired with students who are good at peer coaching rather than supplying answers.
- In a collaborative classroom, the additional teacher will provide extra assistance to students as needed.
- Students who do not work well with others may have to work by themselves and discuss conclusions directly with the teacher.

Surface Area and Volume Formula Worksheet

I. For each solid, draw a picture of the solid then fill in the formula for the surface area and the volume of each. You will use this worksheet each time we investigate a new three-dimensional object.

1) Prism Picture

Surface Area _____

Volume _____

2) Cylinder

Surface Area _____

Volume _____

3) Cone

Surface Area _____

Volume _____

4) Sphere

Surface Area _____

Volume _____

5) Pyramid

Surface Area _____

Volume _____