Fun with Solids

I. UNIT OVERVIEW & PURPOSE:

The purpose of this lesson is to solve real-world problems related to surface area and volume of three-dimensional objects. Students will begin each lesson by "discovering" the formulas for surface area and volume of specific three-dimensional objects: rectangular prisms, cylinders, cones, spheres, and pyramids. After each formula has been found, students will then use those formulas to investigate real-world problems. Lesson 1 – 'Fun with Cakes', Lesson 2 – 'Fun with Volcanoes', Lesson 3 – Senior Prank, and Lesson 4 – The Egyptian Tomb. As a unit project, students will then plan, build, and present a toy they have made using various three-dimensional solids in Lesson 5.

II. UNIT AUTHORS:

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III. COURSE:

Mathematical Modeling: Capstone Course (the course title might change)

IV. CONTENT STRAND:

Geometry

V. OBJECTIVES:

Students will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

VI. MATHEMATICS PERFORMANCE EXPECTATION(s):

MPE.6) Students will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

VII. CONTENT:

In this unit, students will discover the formulas for the surface area and volume of rectangular prisms, cylinders, cones, spheres, and pyramids. The students will then apply those formulas to investigate miscellaneous "fun" real-world problems. The unit will end with a culminating activity of students planning, building, and presenting a toy using various solids that have been explored.

VIII. REFERENCE/RESOURCE MATERIALS:

VA SOL Geometry Standards
NCTM Standards
Teacher generated worksheets

Various other VA SOL Standards

IX. PRIMARY ASSESSMENT STRATEGIES:

The formula worksheet provided will be filled in each time the surface area and volume formulas are discovered for each object. It will be graded at the end of the unit for completeness and accuracy.
Each lesson will end with student presentations and will be graded using a rubric.
Each lesson will also conclude with a journal reflection, to be turned in and graded for completion at the end of the unit.

X. EVALUATION CRITERIA:

Rubrics will be provided in each lesson to aid in scoring student presentations.

XI. INSTRUCTIONAL TIME:

5-7 days for a 90 minute block or 10-14 days for a 45 minute class. Each lesson, 1 through 4, should take one day on a 90 minute block, but lesson 5 may require more than one day.

Lesson 2 – Fun with Volcanoes

Strand
Geometry

Mathematical Objective(s)
Students will discover the formulas for surface area and volume of three-dimensional objects through hands on investigations. The students will then use those formulas to investigate real-world problems.

Mathematics Performance Expectation(s)
MPE.6) Students will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

Related SOL
G.13 - Students will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.
WG.1a – Students will use maps, globes, satellite images, photographs and diagrams to obtain geographical information about the world's countries, cities, and environment.

NCTM Standards
In grades 9-12 all students should:

- Analyze properties and determine attributes of two- and three-dimensional objects;

- Explore relationships among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them;
- Use geometric models to gain insight into, and answer questions in, other areas of mathematics;
- Use geometric ideas to solve problems in, and gain insight into, other disciplines and other areas of interest such as art and architecture.

Materials/Resources

- Formula Worksheet
- Geometric Solids – cones and cylinders (all shapes have openings for easy filling)
- Small dried beans
- Disposable trays
- Paper, pencil, rulers, and calculators
- Computers with internet access

Assumption of Prior Knowledge

- Students will have taken Algebra 1, Geometry, and Algebra 2 or Algebra Functions & Data Analysis.
- Students will know the area formulas for rectangles and circles and cylinders.
- Students will know the surface area of a three-dimensional object is the sum of the areas of all of its faces.
- Students will know the volume of a three-dimensional object is the number of unit cubes that will fill the object.
- Students are expected to know how to calculate the surface area and volume of rectangular prisms and cylinders once they have the formulas.
- Students should be operating on at least Level 2 of the Van Hiele Scale in relationship to geometric solids.

Introduction: Setting Up the Mathematical Task

- Introduction: "You started a job at a museum as a projects manager. Your first task is to make a replica of a famous volcano. You will have to research a volcano of your choice and find the measurements. Part of your job is to find the volume and surface area, so materials can be purchased for your model."
- The entire lesson will take approximately 90-180 minutes.
- Students will be pair off into teams of two.
- Students will discover the surface area and volume formulas for cones using math manipulatives.

- Students will have to use prior knowledge of area of circles and sector area to derive the surface area formula. They need to use the cylinder to derive the volume of a cone.
- Students will use the formulas to complete the volcano activity.
- Students will illustrate and report their findings at the end of class and write a reflective journal entry for homework.

Student Exploration 1:

Small Group Work

Students will work in groups of two.

Student/Teacher Actions:

- Each pair of students will have a tray of small dried beans, geometric solids including cylinders and cones.
- Students will use the manipulatives and prior knowledge to derive the surface area and volume formulas of cones.
    - To figure out the formula for the surface area of the cone, students will have to find the lateral height of the cone using the height, radius, and Pythagorean Theorem. The teacher will have to provide extra guidance for this exploration and help the students assemble the information into the correct formula.
    - To come up with the formula for the volume of the cone, students will fill a cylinder with the dry beans. They will then try to fill a cone with the same height and radius of their cylinder with the beans from the cylinder. Students should notice only one third of the beans are needed to fill the cone, thus helping them come to the conclusion that the volume of a cone is one third of the volume of a cylinder.
- Students will write the formulas down on the formula worksheet provided in Lesson 1. Each student will have his/her own worksheet.
- Teacher will offer little assistance during the exploration so the students can struggle through and assemble the new information. However, students may discuss what they are discovering with each other. Teacher may give hints for the surface area, as this is more difficult to derive.
- Have a class discussion to ensure correct formulas for each student.
- The teacher may want to write instructions down on a handout for students, but directions for each exploration could also be given verbally to walk the students through the steps.

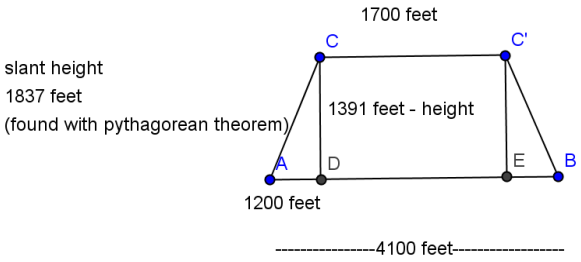
Monitoring Student Responses

- Students should discuss their discoveries with each other during the exploration.
- Random students will be called on to present what they have derived to the class, and allow the class to determine if they had similar answers. The same student should explain how he/she came up with their formula, and see if other students derived the formula a different way.
- The teacher will confirm the results found and the results will be used in the second activity.

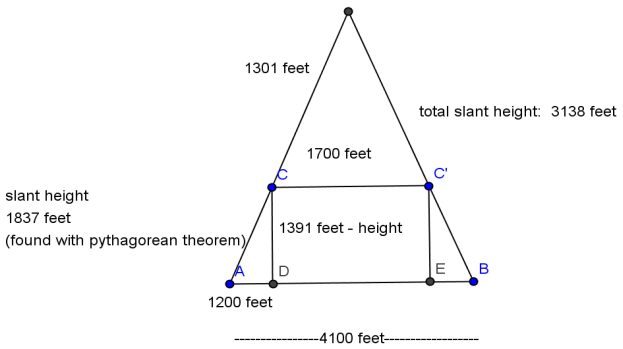
Student Exploration 2:

Student/Teacher Actions:

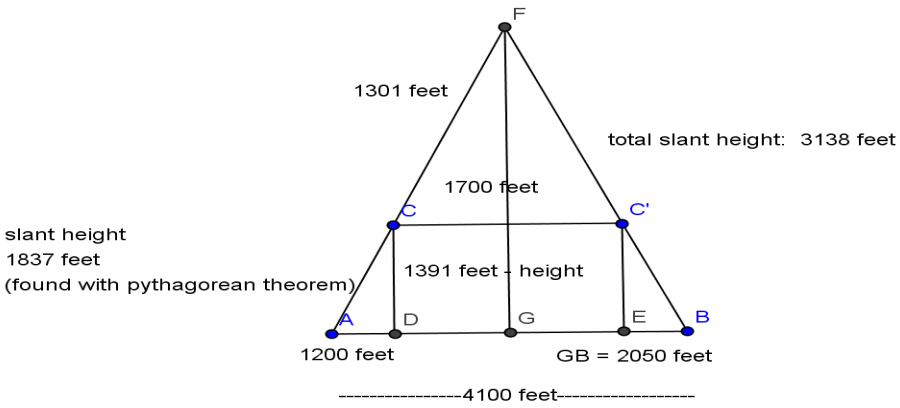
- "Now that we have discovered the formulas for volume and surface area of cones, let's use these formulas to find the surface area and volume of volcanoes."
- Students will work in groups of two and use computers and Google Maps to research and measure "famous volcanoes". Examples include:
    - Mount Vesuvius, Mount St. Helens, Crater Lake, El Chicon, Mount Tambora, Mauna Loa, Thera, Nevado del Ruiz, Mount Pinatubo, Mount Sinabung, Cotopaxi
- They will pick a volcano of their choice and write 3 fun facts about the volcano. Each group will have to pick a different volcano.
- They will research the diameter of the base and of the crater of the volcano. They will also need the height. (for example: the diameter is approx. 4100 feet across the base; diameter of the crater is approx. 1700 feet; height is approx. 1391 feet)
- They will need to calculate the slant height of the volcano from the base to the crater using a trapezoid and then cutting that into two right triangles. They will use the Pythagorean Theorem.
- Example:



- They will need to use similar triangle to calculate the slant height as if the crater did not exist.



- Next, they will have to calculate the height of the volcano as if the crater was not there (using the Pythagorean Theorem once more)



- They will calculate the surface area and volume of the volcano if it did not have a crater.
- They will have to calculate the volume and surface area of the invisible cone at the top of the volcano.
- They will subtract the volume of the volcano with no crater and the invisible cone at the top, and that will give them the volume of the volcano.
- To find the lateral surface area, students will have to find the surface area of the volcano with no crater – the surface area of the cone of the top minus the area of the circular base.
- Students will illustrate their calculations on a poster or piece of paper, along with their three fun facts, and their results. Then they will present their findings to the class.

Monitoring Student Responses

- Teacher will walk around the room and monitor group progress and answer questions as needed.

Assessment

- Formula Worksheet – attached.
- Students will illustrate their volcano, calculations, facts, and results on a poster or piece of paper. Depending on time left in class, this may have to be done for homework.
- Students will present their findings to the class and be graded with the following rubric:

	4	3	2	1	0
Correct Calculations	All correct	Most correct	Half correct	Few correct	None correct
Show Appropriate Work	All work shown	Most work shown	Half of the work shown	Little work shown	No work shown
Drawing of Volcano	Excellent	Very Good	Good	Fair	Poor
Participation/ Group Cooperation	Excellent	Very Good	Good	Fair	Poor
Neatness/ Presentation	Excellent	Very Good	Good	Fair	Poor

Extensions and Connections (for all students)

- For homework, students will write a one page reflection about what they learned and how they could apply this to another real world application.

Strategies for Differentiation

- This lesson is visual, collaborative, and hands on; which by nature will allow for multiple learning styles. Students who struggle with this activity may be provided more hints for calculations.
- Students who need extra help may be paired with students who are good at peer coaching rather than supplying answers.

- Teacher may want to make a handout with detailed directions of the investigations so students can remember what to do.
- In a collaborative classroom, the additional teacher will provide extra assistance to students as needed.
- Students who do not work well with others may have to work by themselves and discuss conclusions directly with the teacher.