Percentages

I.	**UNIT OVERVIEW & PURPOSE:**
	The unit has the purpose of students learning about percentages and its applications to personal finance. Students will be calculating net salaries, possible car payments, and developing and analyzing a personal budget.
II.	**UNIT AUTHOR:**
	Jessica Brevard, Floyd Elementary School, Floyd County Public Schools
III.	**COURSE:**
	Mathematical Modeling: Capstone Course
IV.	**CONTENT STRAND:**
	Number and Operations
V.	**OBJECTIVES:**
	Students will learn applications of percentages and will be applying this to personal financial situations. Students should see the real-world applications of percentages and how budgets will play a critical role in their future.
VI.	**MATHEMATICS PERFORMANCE EXPECTATION(s):**
	MPE 1.
	The student will solve practical problems involving rational numbers (including numbers in scientific notation), percents, ratios, and proportions.
	MPE 26.
	The student will solve, algebraically and graphically,
	a) absolute value equations and inequalities;
	b) quadratic equations over the set of complex numbers;
	c) equations containing rational algebraic expressions; and
	d) equations containing radical expressions.
	Graphing calculators will be used for solving and for confirming the algebraic solutions.
VII.	**CONTENT:**
	Students will be applying mathematical concepts to other content areas outside mathematics including economics and budgeting. Students will be provided with practical problems and will be asked to choose a method to solving them. Many of the problems can be solved by setting up an algebraic equation and solving for the unknown.
VIII.	**REFERENCE/RESOURCE MATERIALS:**
	Calculators, Internet access via classroom laptops or school computer lab.
IX.	**PRIMARY ASSESSMENT STRATEGIES:**
	Each lesson (3 lessons) will have an assessment collected in the form of worksheets. These assessments will be completed by the students as individuals or in pairs (it is up to the students and to the teacher).
	The last day, there will be a final assessment and it will be completed individually.
X.	**EVALUATION CRITERIA:**
	For grading the whole unit, it is suggested that each of the three lesson assessments count as 20% each and the final assessment count as 40%. Possible correct solutions are included in this document. There may be variations in the answers. Teachers should determine how to distribute the points (some points given for accuracy of the answer, neatness in presentation, clarity in explanations, etc.). On the final assessment, students

will be asked to complete two Excel spreadsheets. It is suggested that these each count for 50% of the final assessment grade.

XI.	**INSTRUCTIONAL TIME:**
	Four-45 minutes classes (including the final assessment).

Lesson 1--Percentages

Strand
Number and Operations

Mathematical Objective(s)
Percentages. In this lesson students will develop ways to apply percent concepts to calculating salaries and income tax.

Mathematics Performance Expectation(s)
>MPE 1.
>The student will solve practical problems involving rational numbers (including numbers in scientific notation), percents, ratios, and proportions.
>MPE 26.
>The student will solve, algebraically and graphically,
>a) absolute value equations and inequalities;
>b) quadratic equations over the set of complex numbers;
>c) equations containing rational algebraic expressions; and
>d) equations containing radical expressions.
>Graphing calculators will be used for solving and for confirming the algebraic solutions.

Related SOL

A.4   The student will solve multistep linear and quadratic equations in two variables, including
>a) solving literal equations (formulas) for a given variable;
>b) justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets;
>c) solving quadratic equations algebraically and graphically;
>d) solving multistep linear equations algebraically and graphically;
>e) solving systems of two linear equations in two variables algebraically and graphically; and
>f) solving real-world problems involving equations and systems of equations.
>Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.

NCTM Standards:

- Apply and adapt a variety of appropriate strategies to solve problems
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others

Additional Objectives for Student Learning (include if relevant; may not be math-related):
Students will gain a general understanding of income taxes.

Materials/Resources
- Classroom set of graphing calculators.
- Access to a classroom set of laptops (or the class will need to take place in a computer lab).
- Internet access.

Assumption of Prior Knowledge
- Students should already have the basic concept of percents (out of 100) and how to find percent of a number. Students should also know how to calculate a percent.
- Students should have already completed Algebra 1.
- Students should also have prior knowledge of equations. This prior knowledge includes setting up (modeling) an equation given a practical problem and then solving it.
- Students should understand the basic idea behind gross versus net pay and should have prior knowledge of what income tax is (not necessarily how to calculate it).
- The relevant real life context in this problem involves salaries, income tax, gross and net pay.

Introduction: Setting Up the Mathematical Task

- "In this lesson, you will investigate the applications of percents when calculating salaries."

Begin by presenting the 2011 tax brackets (http://www.taxbrackets2011.com/)

Take a few moments to discuss, in a whole group setting, what tax brackets are, what income tax is and how to calculate it, gross and net pay, etc. Make sure to explain that the tax brackets in the United States are graduated. Also, students will need to be told that this is a simple model; things like state and local taxes and deductions are not taken into account. This background information is important to share to ensure that students can understand and use the appropriate vocabulary to minimize confusion about what the questions are asking. Do a few examples with the students using any salary. (Example: "If you earn $28,000/yr and you get a 10% raise, how much extra take-home pay would receive after taxes? Students will most likely be quick to respond with $2,800. Explain that if a person receives a raise of $2,800 a year, it does not mean that they will pocket an extra $2,800 a year).

Student Exploration 1 and Assessment:

Give students a copy of the following questions. It is suggested that students work together (no more than 2 students). Teachers should circulate around the room and provide hints and ask leading questions. This is to be collected at the end of the class and is the assessment for the class period. Require students to show their work and write down their calculations. Simply giving an answer should not be acceptable. Encourage them to explain their reasoning. Also, if the teacher prefers, one could start the class out as a whole group and have a discussion of methods, what the unknowns are, etc.

1) Your salary is $35,000. What is your net pay per year (filing as a single person with no deductions)? Per month?

One method of finding the solution:

35,000 – 0.1*8,500 – 0.15*(34,500 – 8,500) – 0.25*(35,000 – 34,500) = $30,125

$30,125/12 = $2,510.42 per month

Explanation of the equation: This equation demonstrates the graduated income tax system in the US. We pay 10% for every dollar up to $8,500. We pay 15% for every dollar between $8,500 and $34,500, and so forth.

2) Due to inflation and an increase in your monthly bills, you find that you need to bring home $167 more a month. How much of an increase in your annual gross pay will you need to make ends meet?

One method of finding the solution:

167 * 12 = 2,004

0.75 * x = 2,004

x = $2,672

Another method of finding the solution (with an equation):

(35,000 + x) – 0.1*8,500 – 0.15*(34,500 – 8,500) – 0.25*(35,000 + x – 34,500)=$32,129

Explanation of the equation: The (35,000 + x) represents what the salary will be with the raise. The two middle terms stay the same (as they were for the equation in #1) because a raise will not affect them. The 0.25*(35,000 + x – 34,500) or (0.25*(x + 500)) represents that 25% will be paid on the amount above $34,500. The $32,129 is the sum of the original net pay and $2,004 (the amount of extra income you now need).

3) Consider the problem given in #2. Complete the problem again (you still need an increase of $167 per month of your net pay). However, this time your salary is $33,000. You will see that this will put your needed annual gross pay into the next tax bracket.

One method of finding the solution:
1,500 *0.85 = 1,275 (how much net pay will be generated in the 15% tax bracket)
2004 – 1,275 = 729 (how much net pay is needed to be generated in the 25% tax bracket).
0.75x = 729 (75% of what number will generated $729 in net pay?)

x = 972

1500+972 = $2472 (the sum two gross amounts)

Another method of finding the solution (with an equation):

First, find the original net pay of a salary of $33,000:

$33,000 – 0.1*8,500 – 0.15*(33,000 – 8,500) = $28,475$

Now to find how much of a raise needed:

$(33,000 + x) – 0.1*8,500 – 0.15*(34,500 – 8,500) – 0.25*(33,000 + x – 34,500) = $30,479$

     Explanation of the equation: The $(33,000 + x)$ represents what the salary will be with the raise. The two middle terms stay the same (as they were for the equation in #1) because a raise will not affect them. The $0.25*(33,000 + x – 34,500)$ or $(0.25*(x – 1,500))$ represents that 25% will be paid on the amount above $34,500. The $30,479 is the sum of the original net pay and $2,004 (the amount of extra income you now need).

Extensions and Connections (for all students)

- This lesson could correspond with an economics class. Collaborating with economics teachers would be helpful to provide continuity. In economics, teachers cover income tax, but rarely have time to show students how to calculate it. If this lesson could take place around the same time an economics teacher is covering income tax, it would provide greater understanding for students. The students would already have the background knowledge for the math class (which would cut down on the introduction portion of the lesson) and also would enable students to truly understand "income tax" and it simply not be another vocabulary term to memorize in their economics class.

Strategies for Differentiation

- Lower ability students: If students are struggling to come up with any solutions, consider giving them the equations to find the solutions. After studying the equation and processing how it was developed, ask students to journal (or verbally communicate) from where the equation came and to document their own understanding of the equation.
- ELL students: Make a simply-worded vocabulary list and provide it to the ELL students. Students could use it to quickly reference the meaning of the new terms (net pay, gross pay, etc).
- Higher ability students:

These questions are not related to income taxes, however are related to salaries. These are practical problems and they provide an opportunity for the students to express their findings in journal form.

As a journal entry, please investigate and find the solutions to the following problems. Please show your work and clearly state why the solutions are correct.

Your boss offers everyone in the company to choose between receiving $110 monthly increase in pay or to receive a 4% raise. Your salary is $32,000. One of your co-workers salary is $37,000. Which option should you choose? Which option should your co-worker choose? Why?

It would be beneficial for you to choose the first option and your co-worker to take the 4% raise.

If your boss offered you this choice two years in a row, which choices would be the most beneficial to you (you can choose the same choice both years or choose different choices each year). Write a journal response explaining how you would choose to take your increase in pay and provide reasons why.

Students should see that the last choice would provide them with the most in gross pay.
$34,611, 4% both times
$34,640, $110/mo., both times
$34,600, 4% first, $110/mo. second
$34,652, $110/mo. first, 4% second