2 & 3 Dimensional Shapes

I. UNIT OVERVIEW & PURPOSE:

Students will analyze 2 and 3 dimensional shapes in order to solve real world math problems. The first lesson will review formulas for area and allow them to develop volume formulas. The second lesson will compare ratios of perimeter and area of similar figures. The third lesson will compare ratios of volume of similar solids.

II. UNIT AUTHOR:

Michael Carrow, Radford High School, Radford City Public Schools

III. COURSE:

Mathematical Modeling: Capstone Course

IV. CONTENT STRAND:

Geometry, Measurement

V. OBJECTIVES:

Students will gather dimensions of figures to formulate ideas on area and volume of similar two- and three-dimensional shapes. Students will use these skills to solve real world math problems which deal with similarity ratios.

VI. MATHEMATICS PERFORMANCE EXPECTATION(s):

MPE.7 The student will use similar geometric objects in two- or three-dimensions to:

a) Compare ratios between side lengths, perimeters, areas, and volumes;
b) Determine how changes in one or more dimensions of an object affect area and/or volume of the object
c) Determine how changes in area and/or volume of an object affect one or more dimensions of the object; and
d) Solve real-world problems about similar geometric objects.

VII. CONTENT:

Students will examine various changes in dimensions and scale factors to investigate the relationships with areas and volumes.

VIII. REFERENCE/RESOURCE MATERIALS:

Classroom set of graphing calculators, rulers, protractors, classroom set of laptops or computer lab access, graph paper.

IX. PRIMARY ASSESSMENT STRATEGIES:

Similar problems will be given. No grades are earned for assessments for this unit. Journal entries to describe their learning, which will be graded based on the attached rubric.

X. EVALUATION CRITERIA:

Similar problems will be given for students to complete.

XI. INSTRUCTIONAL TIME:

3-5 classes (45-50 minutes)

Rubric for Journal Entries (Max of 10 pts.)

5 pts	3 pts	1 pt	0 pts
Able to put journal entries in their own words and provided accurate definitions of concepts learned.	Use of some of students own words and copied definitions, etc. from book or other source. Demonstrating that the student understood most of the concept, but still seems a little unclear on some things,	Copied journal entries from textbook or other source. Did not choose their own words.	No journal entries.
5 pts	**3 pts**	**1 pt**	**0 pts**
3 journal entries for the unit.	2 journal entries for the unit.	1 journal entry for the unit.	No journal entries.

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Lesson 1: Area and Volume Formulas

Strand
Geometry, Measurement

Mathematical Objective(s)
Students will recall area formulas for rectangles, triangles, circles, and trapezoids and develop the formulas for the volume of prisms, pyramids, spheres and cylinders. Students will also compare volumes of similar shapes and the ratios of various dimensions.

Mathematics Performance Expectation(s)
MPE.7 The student will use similar geometric objects in two- or three-dimensions to:

e) Compare ratios between side lengths, perimeters, areas, and volumes;
f) Determine how changes in one or more dimensions of an object affect area and/or volume of the object
g) Determine how changes in area and/or volume of an object affect one or more dimensions of the object; and
h) Solve real-world problems about similar geometric objects.

Related SOL

G.13 The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

G.14 The student will use similar geometric objects in two- or three-dimensions to
a) compare ratios between side lengths, perimeters, areas, and volumes;
b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;
c) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and
d) solve real-world problems about similar geometric objects.

NCTM Standards

- Analyze properties and determine attributes of two- and three-dimensional objects;
- Explore relationships (including congruence and similarity) among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them;

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- Understand and use formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders;
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others

Materials/Resources

- Classroom set of graphing calculators
- Rulers
- Classroom set of laptops or computer lab access.
- Classroom set of compasses.

Assumption of Prior Knowledge

- The student should have already completed the Geometry course offered at his/her school
- Students should understand the concepts of area and volume.
- Students should be able to accurately construct a rectangle, circle and triangles
- Students may have difficulty recalling formulas for area and volume.
- Students should understand similarity and all of its properties.
- Students should know the difference between and the types of 2- and 3-dimensional shapes.

Introduction: Setting Up the Mathematical Task

- In this lesson, students will investigate the area formulas and develop the volume formulas
- Recall area formulas (5 minutes)
- Construct various shapes and individual work (20 minutes)
- Classroom discussion (10 minutes)
- Developing Volume formulas (10 minutes)
- Discuss what square footage is and what it represents
- Students will work individually to construct various shapes
- Students will work in groups to develop the volume formulas

Student/Teacher Actions:

- Ask students for their meaning of area and the formulas for a rectangle, circle, and triangle (Should come up with something close to the amount of space inside a two dimensional figure. $A_R = lw$ $A_C = \pi r^2$ $A_T = \frac{1}{2}bh$)
- Students will accurately construct 5 similar figures of each: rectangle, circle, and triangle.
- Have students set up proportions using each of their 5 figures for each shape.

- Divide students into groups to develop the formulas for volume of a rectangular prism, sphere and pyramid. Give each group a three dimension object that has been sliced to allow them to make conjectures for the volume of the figures by building layer upon layer.

Monitoring Student Responses

- Students should determine that similar figures are proportional
- Students should communicate ideas to simplify the formula for volume
- Teacher simplifies student formulas to develop the generally used formula
- Teacher prompts students to formulate ideas of the volume of a figure based on squaring a dimension.

Assessment

- Give the students the dimensions of the Washington Monument and have students find the volume and draw a similar scaled drawing. Students will also pick another historical building to find volume and again draw another scaled drawing.
- Journal – Students will write in their journals their own definition for volume without using a formula

Strategies for Differentiation

- Allow students to use drawing software to construct their similar shapes.
- Also provide struggling students with pairs of figures to reinforce similarity or not.

Lesson 2: Areas and Perimeters of Similar Figures

Strand
Geometry, Measurement

Mathematical Objective(s)
Students will discover the relationships between the similarity ratio and the perimeters and areas of similar figures.

Mathematics Performance Expectation(s)
MPE.7 The student will use similar geometric objects in two- or three-dimensions to:

i) Compare ratios between side lengths, perimeters, areas, and volumes;
j) Determine how changes in one or more dimensions of an object affect area and/or volume of the object
k) Determine how changes in area and/or volume of an object affect one or more dimensions of the object; and
l) Solve real-world problems about similar geometric objects.

Related SOL

G.13 The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

G.14 The student will use similar geometric objects in two- or three-dimensions to
a) compare ratios between side lengths, perimeters, areas, and volumes;
b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;
c) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and
d) solve real-world problems about similar geometric objects.

NCTM Standards

- Analyze properties and determine attributes of two- and three-dimensional objects;
- Explore relationships (including congruence and similarity) among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them;

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- Understand and use formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders;
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others

Materials/Resources

- Classroom set of graphing calculators
- Classroom set of laptops or computer lab access (optional).
- Graph paper
- Classroom set of protractors

Assumption of Prior Knowledge

- The student should have already completed the Geometry course offered at his/her school
- Students should understand the concepts of area and volume.
- Students should be able to accurately construct rectangles, circles and triangles.
- Students should understand similarity and all of its properties.
- Students should be able to use right triangle geometry. It may be necessary to spend a day reviewing this topic. Online site for practice if necessary http://nlvm.usu.edu/en/nav/frames_asid_335_g_4_t_3.html?from=category_g_4_t_3.html

Introduction: Setting Up the Mathematical Task

Modeling Problem:

A farmer has a chance to add property to his existing farm. He can only afford to buy $9000\ yd^2$. To make matters even worse, he only has a scaled drawing of the prospective real estate. The scaled drawing is a triangular field with one angle of 30° and another angle of 50° with the included side at 300 yds. Will he be able to buy this piece of property?

- Partner work, drawing similar rectangles (10 minutes)
- Discussion of partner work (5 minutes)
- Looking at the ratios algebraically (10 minutes)
- Modeling Problem and assessment (20 minutes)

Student/Teacher Actions:

- Have students work with a partner to draw a 4 by 5 unit rectangle on a sheet of graph paper
- Students will then draw 3 more rectangles using a scale factor between 2 and 10.
- Have students create the following table and complete

Rectangle	Scale Factor	Width	Length	Perimeter	Area
Original	-	**4**	**5**		
2nd					

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3rd					
4th					

- Discuss how the ratios compare with the scale factor
- Look at the perimeters and area ratios algebraically $\frac{Perimeter\ of\ Originial}{Perimeter\ of\ Rect.x} = \frac{2l+2w}{2(kl)+2(kw)}$ $\frac{Area\ of\ Original\ Rect.}{Area\ of\ Rect.\ x} = \frac{lw}{(kl)(kw)}$ (x is any rectangle and k is the scale factor)
- Students should see that the perimeter of similar rectangles have a ratio of $1: k$ and the area of similar rectangles have a ratio of $1: k^2$.
- Discuss modeling problem

Monitoring Student Responses

- Students should come to the conclusion that a similar triangle needs to be drawn first.
- Students may get confused with the correct units.
- Students should realize that right triangles will have to be drawn and recall the Pythagorean Theorem.

Assessment

- Change the dimensions and wording of the modeling problem. It is now a piece of property that needs fencing and challenge students to calculate the amount of fencing (perimeter) that is needed for the property.
- Journal – Show algebraically how the ratios of the circumferences of similar circles and the ratios of perimeters of similar triangles relate.

Strategies for Differentiation

- Allow students to use drawing software or geoboards to construct their similar shapes.
- Allow students to use spreadsheets to develop their table for the partner work.

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Lesson 3: Volumes of Similar Solids

Strand
Geometry, Measurement

Mathematical Objective(s)
Students will find the relationships between the similarity ratio and the ratio of volumes of similar figures.

Mathematics Performance Expectation(s)
MPE.7 The student will use similar geometric objects in two- or three-dimensions to:

m) Compare ratios between side lengths, perimeters, areas, and volumes;
n) Determine how changes in one or more dimensions of an object affect area and/or volume of the object
o) Determine how changes in area and/or volume of an object affect one or more dimensions of the object; and
p) Solve real-world problems about similar geometric objects.

Related SOL

G.13 The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

G.14 The student will use similar geometric objects in two- or three-dimensions to
a) compare ratios between side lengths, perimeters, areas, and volumes;
b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;
c) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and
d) solve real-world problems about similar geometric objects.

NCTM Standards
- Analyze properties and determine attributes of two- and three-dimensional objects;
- Explore relationships (including congruence and similarity) among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them;
- Understand and use formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders;

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- Communicate mathematical thinking coherently and clearly to peers, teachers, and others

Materials/Resources

- Classroom set of graphing calculators
- Classroom set of laptops or computer lab access (Microsoft Excel).

Assumption of Prior Knowledge

- The student should have already completed the Geometry course offered at his/her school
- Students should understand the concepts of area and volume.
- Students should be able to accurately construct rectangles, circles and triangles.
- Students should understand similarity and all of its properties.
- Students should be able to use right triangle geometry.

Introduction: Setting Up the Mathematical Task

Modeling Problem:

You currently store sugar in a cylindrical tube that measures 5 in. in diameter and is 7 in. tall and holds 32 oz. Yesterday you stopped by Wal-Mart and noticed that sugar was on sale, so you purchased an 8 lb. bag. Now being a symmetrical person, you would like to find another cylindrical container to replace your current sugar container. You only have 2 ft. between your counter and cabinets to fit the new container. Can you buy a similar cylindrical can to fit in between the counter and cabinets?

- Microsoft Excel (20 minutes)
- Modeling Problem and assessment (25 minutes)

Student/Teacher Actions:

- Have students create a spreadsheet such as the one below. However allow students to pick the numbers for the length, width, & height of rectangular prism A. The rest of the numbers will be calculated with the formulas they create.

	A	B	C	D	E	F	G
1		L	W	H	V	Similarity ratio (B:A)	Ratio of Volumes (B:A)
2	Rect. Prism A	3	6	8		2	
3							
4							
5	Rect. Prism B						

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- Discuss student knowledge of excel and ways to enter formulas
- In Cell e2, students should enter b2*c2*d2
- In Cell b5, students should enter b2*f2
- Explore different ways to enter and copy formulas within Excel to continue filling out row 5.
- In g5, students should enter e5/e2.

- Once spreadsheet is completed, students should change the numbers to investigate how the similarity ratio is related to the ratio of volumes. Students should realize that the ratio of the volumes is a cube of the similarity ratio.

Monitoring Student Responses

- Students should be able to develop formulas in the spreadsheet for volume for various shapes
- Students should change the columns in their spreadsheet for the assessment.

Assessment

- Students will build a spreadsheet to investigate other shapes such as square pyramids and right cylinders.

Strategies for Differentiation

- Allow students to work in pairs (someone with weak knowledge of spreadsheets and another student with more experience).
- Allow students to use actual models of the figures they are investigating.