Transformational Graphing in the Real World

I. UNIT OVERVIEW & PURPOSE:

This unit specifically addresses the concept of transformational graphing. Absolute value, polynomial, and square root functions will be examined.

II. UNIT AUTHOR:

Donna Deplazes
Craig County Public Schools

III. COURSE:

Mathematical Modeling: Capstone Course

IV. CONTENT STRAND:

Algebra

V. OBJECTIVES:

- To recognize and translate the graph of an absolute value function.
- To recognize and translate the graph of polynomial (specifically quadratic) functions.
- To establish a pattern for easy recognition of higher order polynomials.
- To recognize and translate the graph of square root function.
- To expose students to possible real world situations involving transformational graphing.

VI. MATHEMATICS PERFORMANCE EXPECTATION(s):

MPE.2: Collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

MPE.12: Transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Select and use appropriate representations for analysis, interpretation, and prediction.

MPE.14: Recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors or these functions.

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in collaboration with
Mr. Michael Bolling, Virginia Department of Education

VII. CONTENT:

This unit specifically addresses the topic of transformation graphing and general recognition of absolute value, polynomial (specifically quadratic), and exponential functions. All lessons are discussed in the context of a real world application.

VIII. REFERENCE/RESOURCE MATERIALS:

Graphing calculators will be required. Student Exploration Worksheets and Exit Slip Assessments will be needed for all three lessons. Computers to access GeoGebra would also be beneficial when presenting if you find or make your own animation applets to help students visualize the concepts.

IX. PRIMARY ASSESSMENT STRATEGIES:

Please see Exit Slip Assessments attached to each lesson. Upon completion of more functions a unit assessment on the transformation of functions will be given.

X. EVALUATION CRITERIA:

Students will complete a five question exit slip at the end of the lesson. Documents will be attached to each individual lesson. Students will earn a classwork grade out of 10 points – 5 of these points are from the answers to the exit slip, 2 points are for each group's participation in the class discussion, and 3 points are for each student's participation in their group.

XI. INSTRUCTIONAL TIME:

Three 90-minute class periods.

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Air Traffic Control

Strand

Algebra

Mathematical Objective(s)

Functions, Absolute Value Functions, Transformational Graphing

In this lesson students will discuss transformational graphing by examining an air traffic control situation. Students will explore multiple representations of absolute value functions. They will solve given problems by transforming the graph of an absolute value function. Tables, graphs, and equations will be used by students to aide in finding the necessary solutions.

Mathematics Performance Expectation(s)

MPE.2: Collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

MPE.12: Transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Select and use appropriate representations for analysis, interpretation, and prediction.

MPE.14: Recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.

Related SOL

AII.6 The student will recognize the general shape of function (absolute value) families and will convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.

AFDA.4 The student will transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Students will select and use appropriate representations for analysis, interpretation, and prediction.

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Please note that transformation graphing will be applied in all lessons within the unit. The type of function addressed will change daily. It is intended for students to use basic transformation graphing techniques to help them on a day-to-day basis.

NCTM Standards

- Understand relations and functions and select, convert flexibly among, and use various representations for them.
- Interpret representations of functions of two variables.
- Use symbolic algebra to represent and explain mathematical relationships.

Materials/Resources

- Classroom set of graphing calculators.
- Students need to know the vertical separation minimum for aircraft. According to the FAA the minimum safe vertical separation distance is 1000 feet. (http://www.faa.gov/about/office_org/headquarters_offices/ato/service_units/enroute/rvsm/)
- Student Exploration WS #1
- Student Exploration WS #2

Assumption of Prior Knowledge

- Students should have completed Algebra II.
- Students should have experience using a graphing calculator, specifically finding a specific viewing window.
- Students might find it difficult to realize that horizontal transformations are represented "reversely" from graphic to symbolic representation.
- The relevant real life context in this problem is air traffic control.

Introduction: Setting Up the Mathematical Task

In this lesson, students will investigate how vertical and horizontal translations affect the symbolic representation of an absolute value function.

Introduction – 10 minutes
Student Exploration #1 – 25 minutes

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Discussion of SE #1 – 10 minutes
Student Exploration #2 – 25 minutes
Discussion of SE #2 – 10 minutes
CW Assignment: Exit Slip Assessment – 10 minutes

- To introduce the task students will be asked to define absolute value. Students will be asked to consider if any expression containing an absolute value can ever be a negative value. Students will also be reminded of the shape of the absolute value graph by examining the graph of $f(x) = |x|$.
- Students will work the exploration activities and then the class will discuss their conclusions. Each group will be asked to answer one of the four questions on each exploration worksheet and comment on the answers provided by other groups. All students will have the correct answers before the end of class.
- Students are asked to explore transformational graphing in the two exploration worksheets attached.
- The figure given in the exploration activities is designed to assist students who struggle to "picture" the actual situation.

Student Exploration 1:

Group Work (groups of 2 or 3)

Student/Teacher Actions:

- Students should use the information from the introduction to complete a worksheet (Air Traffic Control – Student Exploration #1) that addresses vertical translations of a given function.
- Teacher will be guiding students as needed if questions/problems arise, but will not answer the questions for the students.
- Students should realize that that the second airplane is always 1500 feet above the given airplane. This would then add 1500 to the function rule!
- If time allows and you have access to a lab or a classroom set of computers… The dynamic Explore Learning activity gizmo allows students to change $a, h,$ and k and see how the graph changes as well. This website is not free – a subscription is required. You may view free for five minutes.
  http://fahrenbacher.com/webpage/alg-1222/unit6/absolute-value-geogebra/absolute_value_geogebra.html

Monitoring Student Responses

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- Students are expected to discuss the exploration activities together in their groups and then discuss the questions as a class at the end of the activity.
- The teacher will assist students who have difficulties and extend the material for students that are ready to move forward.

Student Exploration 2:

Group Work (groups of 2 or 3)

Student/Teacher Actions:

- Students should use the information from the introduction to complete a worksheet (Air Traffic Control – Student Exploration #2) that addresses horizontal translations of a given function.
- Teacher will be guiding students as needed if questions/problems arise, but will not answer the questions for the students.
- Students should realize that that the new path is "centered" at 200 miles instead of 375. Therefore the vertex of the absolute value function would be found at a x value of 200. Remember that to show this correctly a +200 should be represented by -200 in the symbolic form.
- If time allows and you have access to a lab or a classroom set of computers… The dynamic Explore Learning activity gizmo allows students to change $a, h,$ and k and see how the graph changes as well. This website is not free – a subscription is required. You may view free for five minutes.
  http://fahrenbacher.com/webpage/alg-1222/unit6/absolute-value-geogebra/absolute_value_geogebra.html

  - Students are expected to discuss the exploration activities together in their groups and then discuss the questions as a class at the end of the activity.
  - At the end of the second exploration students are asked to make generalizations about how horizontal and vertical shifts (translations) affect the function rule.
  - The teacher will assist students who have difficulties and extend the material (add a step of difficulty) for students that are ready to move forward.

Assessment

Students will complete a five question exit slip at the end of the lesson. Please see attached document. Students will earn a CW grade out of 10 points – 5 of these points are from the answers to the exit slip, 2 points are for each group's participation in the class discussion, and 3 points are for each student's participation in their group.

Extensions and Connections (for all students)

The concept of vertical and horizontal stretching and compressing was brought up in the last exploration question and will be discussed in a classroom setting. The topic of transformational graphing will continue in subsequent days' lessons.

Strategies for Differentiation

The graphic organizer/worksheet was designed with the needs of a diverse classroom of students in mind. There is a visual representation (graph) of the situation. Tables were created to assist students as well. Use of the graphing calculator is also encouraged and even prompted.

- For ELL learners, teachers should work with the ELL teacher to provide bridges between mathematics vocabulary and the student's primary language.
- Learning disabled students may benefit if the teacher provides multiple choice answers to the student explorations.
- Visual learners will benefit from the graphical representations and the ability to dynamic exploration allowed within Explore Learning as well as the graphing calculator.
- Auditory learners will benefit from the classroom and group discussions.
- Kinesthetic learners will benefit from movement from individual work to group work and the involvement in classroom presentation.
- High ability students may start to begin to compare groups for similarities or differences and offer opinions to lead into tomorrow's lessons.

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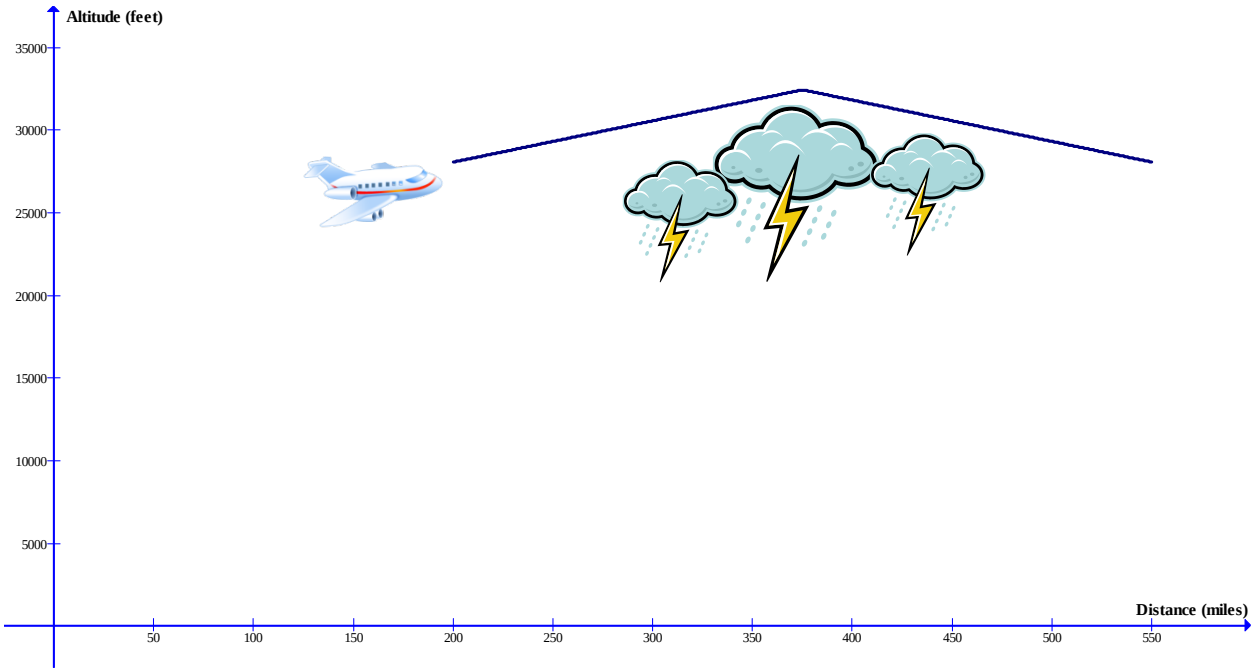
NAME: _____ DATE: _____

Air Traffic Control – Student Exploration #1



The above graph shows the altitude of an airplane over the distance it flies for a specific part of its flight. At a certain point in the scheduled flight path the pilot is told to change his course to fly above a powerful storm in the flight's original path. Please use this scenario to answer the questions below.

1. If given the flight path of the airplane as $f(x) = -25|x - 375| + 32450$ where x represents the distance flown in miles since departure and $f(x)$ represents the altitude of the airplane at a given distance, please find the altitude of the airplane at the following distance increments.

Distance (miles)	Altitude (feet) – Airplane 1
200	_____
250	_____
300	_____
350	_____
400	_____
450	_____
500	_____
550	_____

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2. Using the table constructed in question 1, please fill in the altitude of the second plane that is 1500 feet above the one described above.

Distance (miles)	Altitude (feet) - Airplane 2
200	_____
250	_____
300	_____
350	_____
400	_____
450	_____
500	_____
550	_____

3. Using the table constructed in question 2 how could you modify the function that represents airplane 1, $f(x) = -25|x - 375| + 32450$, to represent airplane 2. Please use your graphing calculator to assist you.

$$f(x) = \_\_\_\_\_$$

4. What is the vertex of the absolute value function representing the flight path of the airplane during this segment of its flight path? Justify your answer.

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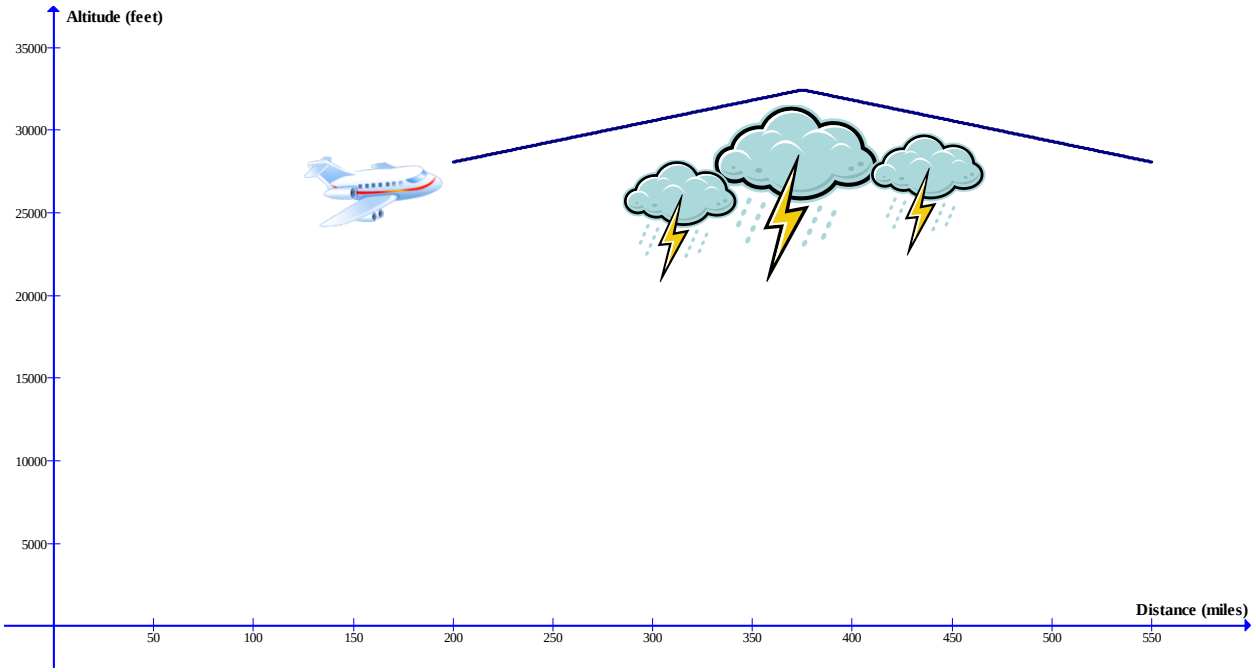
Air Traffic Control – Student Exploration #2



The above graph shows the altitude of an airplane over the distance it flies for a specific part of its flight. At a certain point in the scheduled flight path the pilot is told to change his course to fly above a powerful storm in the flight's original path. Please use this scenario to answer the questions below.

1. The flight path of the airplane is given as $f(x) = -25|x - 375| + 32450$ where x represents the distance flown in miles since departure and $f(x)$ represents the altitude of the airplane at a given distance. What if the storm had been centered at a distance of 200 miles from departure instead of at mile 375… How could you modify the function $f(x) = -25|x - 375| + 32450$ to represent the earlier change in altitude? Please use your graphing calculator to assist you.

$$f(x) = \_\_\_\_\_$$

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2. Using the function determined in question 1, please fill in the altitude of the airplane if the storm had occurred earlier in its flight path.

Distance (miles)	Altitude (feet)
50	_____
100	_____
150	_____
200	_____
250	_____
300	_____
350	_____

3. Using the two exploration activities from the day write a generalization to discuss how a vertical shift on the graph is represented in the function rule.

4. Using the two exploration activities from the day write a generalization to discuss how a horizontal shift on the graph is represented in the function rule.

5. Using your graphing calculator experiment with the given function, $f(x) = -25|x - 375| + 32450$, to determine how to create a steeper (or less steep) climb and decent. Write down your findings.

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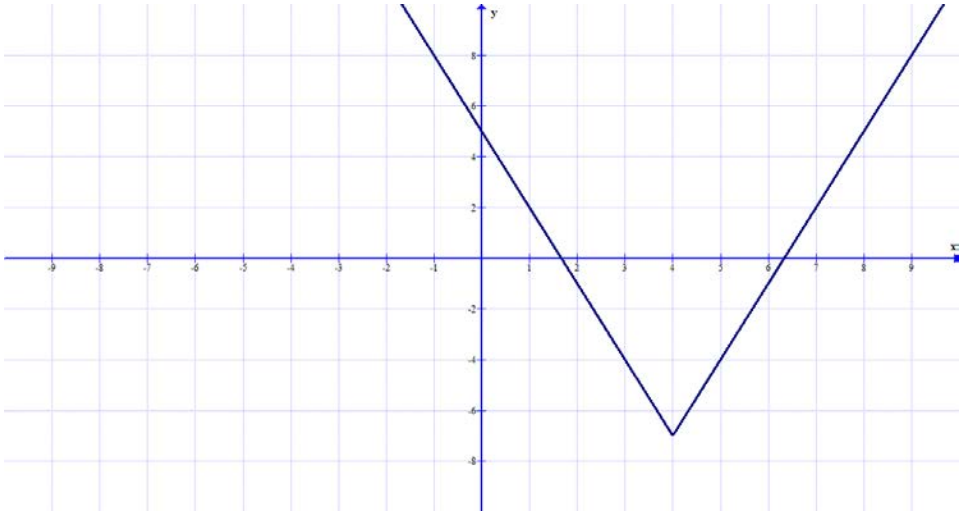
Air Traffic Control – Exit Slip

1. If given the absolute value function $y = |x|$, write two different function rules that would move the vertex to $(-3, 8)$.

2. Write the equation of the absolute value function shown in the graph below.



3. Given the following table write the function rule of the absolute value functions it represents.

X	y
2	14
4	4
5	-1
6	4
8	14

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4. If given the absolute value function, $f(x) = |x - 4| + 7$, what would be the function rule if it was shifted 2 units to the left and 6 units up.

5. Explain what each number in the following function represents graphically.

$$f(x) = 5|x + 9| - 3$$

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Building Bridges

Strand

Algebra

Mathematical Objective(s)

Functions, Quadratic Functions, Transformational Graphing

In this lesson students will discuss transformational graphing by examining the construction of suspension bridges. Students will explore multiple representations of quadratic functions. They will solve given problems by transforming the graph of a quadratic function. Tables, graphs, and equations will be used by students to aide in finding the necessary solutions.

Mathematics Performance Expectation(s)

MPE.2: Collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

MPE.12: Transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Select and use appropriate representations for analysis, interpretation, and prediction.

MPE.14: Recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.

Related SOL

AII.6 The student will recognize the general shape of function (quadratic) families and will convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.

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AFDA.3 The student will collect data and generate an equation for the curve (quadratic) of best fit to model real-world problems or applications. Students will use the best fit equation to interpolate function values, make decisions, and justify conclusions with algebraic and/or graphical models.

AFDA.4 The student will transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Students will select and use appropriate representations for analysis, interpretation, and prediction.

Please note that transformation graphing will be applied in all lessons within the unit. The type of function addressed will change daily. It is intended for students to use basic transformation graphing techniques to help them on a day-to-day basis.

NCTM Standards

- Understand relations and functions and select, convert flexibly among, and use various representations for them.
- Interpret representations of functions of two variables.
- Use symbolic algebra to represent and explain mathematical relationships.

Materials/Resources

- Classroom set of graphing calculators.
- Students need to be familiar with the structure of suspension bridges. Below are a couple of the famous American suspension bridges.
  - Golden Gate Bridge
  - Brooklyn Bridge
- SmartBoard or LCD Projector
- Building Bridges – Student Exploration WS #1
- Building Bridges – Student Exploration WS #2
- Building Bridges – Exit Slip Assessment

Assumption of Prior Knowledge

- Students should have completed Algebra II.
- Students should have experience using a graphing calculator, specifically finding a specific viewing window.
- Students might find it difficult to realize that horizontal transformations are represented "reversely" from graphic to symbolic representation.

- The relevant real life context in this problem is related to the construction of a suspension bridge.

Introduction: Setting Up the Mathematical Task

In this lesson, students will investigate how vertical and horizontal translations affect the symbolic representation of a quadratic function.

Introduction – 20 minutes
Student Exploration #1 – 20 minutes
Discussion of SE #1 – 10 minutes
Student Exploration #2 –20 minutes
Discussion of SE #2 – 10 minutes
CW Assignment: Exit Slip Assessment – 10 minutes

- Have students discuss the similarities and differences between the graphs of absolute value functions and quadratic functions. Students will be reminded about the symmetric characteristics of the graph of a quadratic function. It is important to note that only three points are needed to model a quadratic function. The TI-84+ will give a symbolic representation found through regression that is in standard form. However, if you use the symmetry of the parabola you can find the symbolic representation in vertex form.
    - First use the parent function $y = x^2$ to introduce students to quadratic functions.
    - Then use $y = 2(x-4)^2 + 7$ for an in class example. Have students examine the symmetry (centered around the vertex) in the table that represents the function.
    - Have students find the quadratic regression using their calculators and these given points: $(4, 7), (2,\ 15), (6,\ 15)$.
    - Then have the students verify that $y = 2(x-4)^2 + 7$ is equal to $y = 2x^2 - 16x + 39$.
- Students will work the exploration activities and then the class will discuss their conclusions. Each group will be asked to answer discussion questions on each exploration worksheet and comment on the answers provided by other groups. All students will have the correct answers before the end of class.
- Students are asked to explore transformational graphing of quadratic functions in the two exploration worksheets attached.
- The figure given in the exploration activities is designed to assist students who struggle to "picture" the actual situation.

Student Exploration 1:

Group Work (groups of 2 or 3)

Student/Teacher Actions:

- Students should use the information from the introduction to complete a worksheet (Building Bridges – Student Exploration #1) that asks students to explain vertical and horizontal translations using the vertex form of a quadratic function.
- Teacher will be guiding students as needed if questions/problems arise, but will not answer the questions for the students.
- Students should realize that the vertex of the quadratic in the given situation is $(200,\ 15)$.
- If time allows and you have access to a lab or a classroom set of computers… The dynamic Explore Learning activity gizmo allows students to change $a, h,$ and k and see how the graph changes as well. This website is not free – a subscription is required. You may view free for five minutes.http://www.explorelearning.com/index.cfm?method=cResource.dspDetail&ResourceID=150

Monitoring Student Responses

- Students are expected to discuss the exploration activities together in their groups and then discuss the questions as a class at the end of the activity.
- The teacher will assist students who have difficulties and extend the material (add a step of difficulty) for students that are ready to move forward.

Student Exploration 2:

Group Work (groups of 2 or 3)

Student/Teacher Actions:

- Students should use the information from the introduction and the discussion of the first student exploration to complete a worksheet (Building Bridges – Student Exploration #2) that addresses horizontal and vertical translations as well as stretches and compressions of a given quadratic function.
- Teacher will be guiding students as needed if questions/problems arise, but will not answer the questions for the students.
    - Students should realize that they only need to change the location of the two towers (leaving the distance between the two towers and the height of the towers the same).
    - Students should realize that they need to change the height of the towers and the height of the cable above the bridge surface to form a vertical translation.

  - Changing the distance between the towers or just the height of the towers would result in a stretch or compression.
- If time allows and you have access to a lab or a classroom set of computers… The dynamic Explore Learning activity gizmo allows students to change $a, h,$ and k and see how the graph changes as well. This website is not free – a subscription is required. You may view free for five minutes.
  http://www.explorelearning.com/index.cfm?method=cResource.dspDetail&ResourceID=150

Monitoring Student Responses

  - Students are expected to discuss the exploration activities together in their groups and then discuss the questions as a class at the end of the activity.
  - At the end of the second exploration students are asked to make generalizations about how horizontal and vertical shifts (translations) affect the function rule.
  - The teacher will assist students who have difficulties and extend the material (add a step of difficulty) for students that are ready to move forward.

Assessment

Students will complete a five question exit slip at the end of the lesson. Please see attached document. Students will earn a CW grade out of 10 points – 5 of these points are from the answers to the exit slip, 2 points are for each group's participation in the class discussion, and 3 points are for each student's participation in their group.

Extensions and Connections (for all students)

The concept of vertical and horizontal stretching and compressing was brought up in the last exploration question and will be discussed in a classroom setting. The topic of transformational graphing will continue in subsequent days' lessons.

Strategies for Differentiation

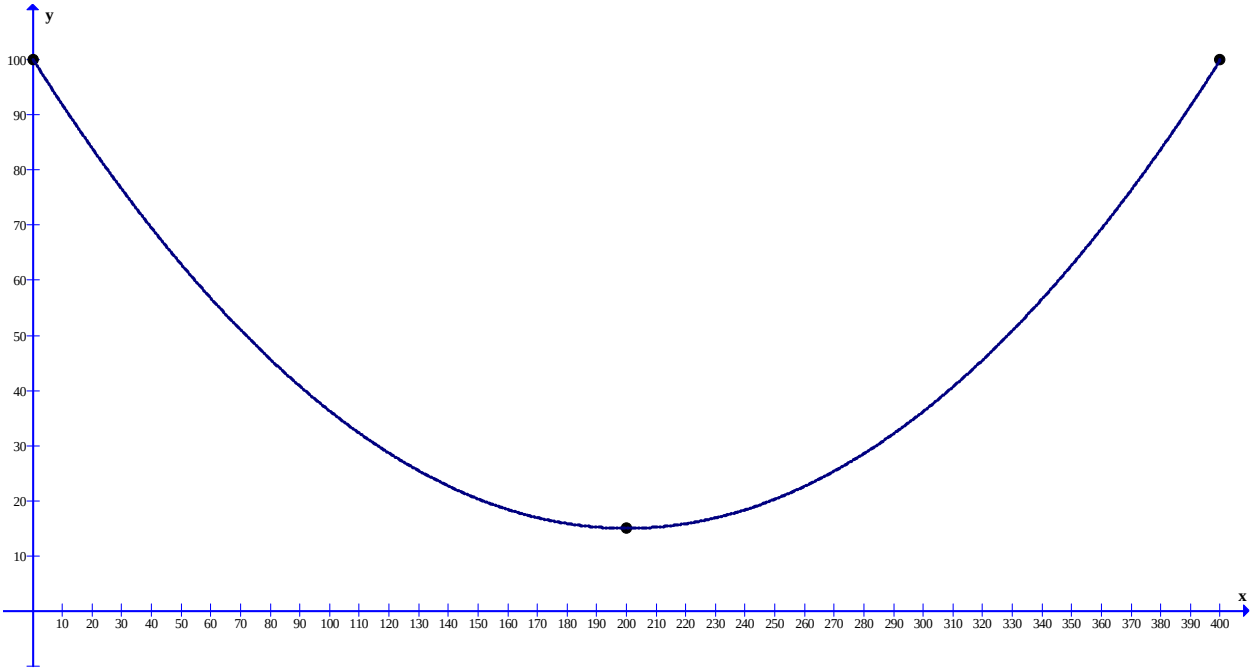
- For ELL learners, teachers should work with the ELL teacher to provide bridges between mathematics vocabulary and the student's primary language.
- Learning disabled students may benefit if the teacher provides multiple choice answers to the student explorations.
- Visual learners will benefit from the graphical representations and the ability to dynamic exploration allowed within Explore Learning as well as the graphing calculator.

- Auditory learners will benefit from the classroom and group discussions.
- Kinesthetic learners will benefit from movement from individual work to group work and the involvement in classroom presentation.
- High ability students may start to begin to compare groups for similarities or differences and offer opinions to lead into tomorrow's lessons.

NAME: _____ DATE: _____

Building Bridges – Student Exploration #1

The figure shows a suspension bridge. We are going to find the quadratic function to symbolize the path of the support cable between the two towers that are 100 feet tall. There is a distance of 400 feet between the two towers. The cable reaches its lowest point at the middle of the span at a height of 15 feet above the bridge's surface. Below is a scale drawing on a coordinate plane of what the situation looks like that should help you.



1. Using the graphing calculator and the three points indicated on the graph above find a quadratic function to represent the situation.

$f(x) =$ _____

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2. Now using the symmetry of the graph you should be able to find the vertex of the parabola given and then use another given point to find the symbolic representation in vertex form, $f(x) = a(x-h)^2 + k$ where the vertex is (h, k).

   Fill in the values of the vertex (h, k).

   $$f(x) = a(x - \_\_\_\_\_)^2 + \_\_\_\_\_$$

   Then pick another point, $\big(x, f(x)\big)$, on the graph to use.

   $$\_\_\_\_\_ = a(\_\_\_\_\_ - \_\_\_\_\_)^2 + \_\_\_\_$$

   If you fill in for $h,\ k,\ x,\ and\ f(x)$ there is only one variable left, a.
   Solve for a, then write the vertex form of the quadratic function.

$$f(x) = \_\_\_\_\_$$

3. Do you think that the numbers used for $h,\ k, and\ a$ are reasonable given the graph (visual representation) of the situation? Why or why not?

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NAME: _____ DATE: _____

Building Bridges – Student Exploration #2

The figure shows a suspension bridge. We are going to find the quadratic function to symbolize the path of the support cable between the two towers that are 100 feet tall. There is a distance of 400 feet between the two towers. The cable reaches its lowest point at the middle of the span at a height of 15 feet above the bridge's surface. Below is a scale drawing on a coordinate plane of what the situation looks like that should help you.

Based on what was learned from Building Bridges – Student Exploration #1 use the same situation to answer the following questions.

1. What number or numbers in the situation would you change to move the quadratic function horizontally only (no stretching or compressing the actual shape)? Explain why or how your change(s) would move the function horizontally.

2. What number or numbers in the situation would you change to move the quadratic function vertically only (no stretching or compressing the actual shape)? Explain why or how your change(s) would move the function horizontally.

3. What number or numbers in the situation would you change to stretch the function horizontally? Explain why or how your change(s) would stretch the function horizontally.

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4. What number or numbers in the situation would you change to compress the function horizontally? Explain why or how your change(s) would compress the function horizontally.

5. What number or numbers in the situation would you change to stretch the function vertically? Explain why or how your change(s) would stretch the function vertically.

6. What number or numbers in the situation would you change to compress the function vertically? Explain why or how your change(s) would compress the function vertically.

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NAME: _____ DATE: _____

Building Bridges – Exit Slip

1. Draw a graph on a coordinate plane to symbolize the path of the support cable of a suspension bridge. The height of the two towers is 80 feet. There is a distance of 350 feet between the two towers. The cable reaches its lowest point at the middle of the span at a height of 22 feet above the bridge's surface. Be sure to label and divide your axis using an appropriate scale. Give the coordinates of three keys points of interest on the graph.

2. Using the three points identified in the graph use your graphing calculator to find the quadratic regression to represent the situation.

3. Now using the symmetry of the graph, develop the quadratic function that represents the situation in vertex form.

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4. Verify that the regression equation in question 2 is equivalent to the function you found in question 3.

5. Discuss how this quadratic function has been transformed (horizontally or vertically translated, compressed or stretched) compared to the quadratic function from exploration activity #1 – $y = \frac{17}{8000}(x - 200)^2 + 15$.

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Bacteria – Growth or Decay?

Strand

Algebra

Mathematical Objective(s)

Functions, Exponential Functions, Transformational Graphing

In this lesson students will discuss transformational graphing by examining the growth and decay of bacteria. Students will explore multiple representations of exponential functions. They will solve given problems by transforming the graph of an exponential function. Tables, graphs, and equations will be used by students to aide in finding the necessary solutions.

Mathematics Performance Expectation(s)

MPE.2: Collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

MPE.12: Transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Select and use appropriate representations for analysis, interpretation, and prediction.

MPE.14: Recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.

Related SOL

AII.6 The student will recognize the general shape of function (exponential) families and will convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.

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AFDA.4 The student will transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Students will select and use appropriate representations for analysis, interpretation, and prediction.

Please note that transformation graphing will be applied in all lessons within the unit. The type of function addressed will change daily. It is intended for students to use basic transformation graphing techniques to help them on a day-to-day basis.

NCTM Standards

- Understand relations and functions and select, convert flexibly among, and use various representations for them.
- Interpret representations of functions of two variables.
- Use symbolic algebra to represent and explain mathematical relationships.

Materials/Resources

- Classroom set of graphing calculators.
- Bacteria: Growth or Decay? – Student Exploration WS #1
- Bacteria: Growth or Decay? – Student Exploration WS #2
- Bacteria: Growth or Decay? – Exit Slip Assessment

Assumption of Prior Knowledge

- Students should have completed Algebra II.
- Students need to be familiar with the fast growth and/or decay of bacteria.
- Students should have experience using a graphing calculator, specifically finding a specific viewing window.
- Students might find it difficult to realize that horizontal transformations are represented "reversely" from graphic to symbolic representation.
- The relevant real life context in this problem is related to the behavior of bacteria over time.

Introduction: Setting Up the Mathematical Task

In this lesson, students will investigate how vertical and horizontal translations affect the symbolic representation of a quadratic function.

Introduction – 15 minutes
Student Exploration #1 – 25 minutes

Discussion of SE #1 – 10 minutes
Student Exploration #2 –15 minutes
Discussion of SE #2 – 10 minutes
CW Assignment: Exit Slip Assessment – 10 minutes

- To introduce the task we will discuss the general form of an exponential function, $y = a(b)^x$, and how to tell when it is an example of growth or decay. It is important to note that a minimum of two points are needed to graph an exponential function. The TI-83, 84, or 84+ will give a symbolic representation found through regression that is in standard form.
    - First use the function $y = 2^x$ to discuss exponential growth functions.
    - Then use the function $y =. 5^x$ to discuss exponential decay functions.
    - The class will discuss a (the initial value or y-intercept) and b (the growth or decay factor.
- Students will work the exploration activities and then the class will discuss their conclusions. Each group will be asked to answer a discussion questions on each exploration worksheet and comment on the answers provided by other groups. All students will have the correct answers before the end of class.
- Students are asked to explore transformational graphing of quadratic functions in the two exploration worksheets attached.

Student Exploration 1:

Group Work (groups of 2 or 3)

Student/Teacher Actions:

- Students should use the information from the introduction to complete a worksheet (Bacteria: Growth or Decay? – Student Exploration #1) that asks students to determine if examples (given in symbolic form, word problems, tables, and graphs) of exponential function as examples of growth or decay. Students will be asked to place all of the questions into the other three representations of the function.
- Teacher will be guiding students as needed if questions/problems arise, but will not answer the questions for the students.
- Multiple representations are used to help students who normally struggle see the relationship between the different representations. This should help students be able to convert given information that is confusing into a form that they understand.
- If time allows and you have access to a lab or a classroom set of computers… The dynamic Explore Learning activity gizmo allows students to change $a, b, h,$ and k and see how the graph changes as well. This website requires an initial software download, but is free. http://www.geogebra.org/en/upload/files/Line/Exponential.html

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Monitoring Student Responses

- Students are expected to discuss the exploration activities together in their groups and then discuss the questions as a class at the end of the activity.
- The teacher will assist students who have difficulties and extend the material (add a step of difficulty) for students that are ready to move forward.

Student Exploration 2:

Group Work (groups of 2 or 3)

Student/Teacher Actions:

- Students should use the information from the introduction and the discussion of the first student exploration to complete a worksheet (Bacteria: Growth or Decay? – Student Exploration #2) that addresses horizontal and vertical stretches and compressions as well as translations of a given exponential function.
- Teacher will be guiding students as needed if questions/problems arise, but will not answer the questions for the students.
    - Students should realize that a horizontal translation doesn't always make sense conceptually (in the real world). Moving the graph to the left will technically start when time is negative.
    - Students should realize that most transformations of exponential functions involve vertical stretches and compressions.
    - Students will also find it difficult to learn that if $a < 1$ and b> 1 the exponential function is classified as growth.
- If time allows and you have access to a lab or a classroom set of computers… The dynamic Explore Learning activity gizmo allows students to change $a, b, h,$ and k and see how the graph changes as well. This website requires an initial software download, but is free. http://www.geogebra.org/en/upload/files/Line/Exponential.html

Monitoring Student Responses

- Students are expected to discuss the exploration activities together in their groups and then discuss the questions as a class at the end of the activity.
- At the end of the second exploration students are asked to make generalizations about how horizontal and vertical shifts (translations) affect the function rule.
- The teacher will assist students who have difficulties and extend the material (add a step of difficulty) for students that are ready to move forward.

Assessment

Students will complete a five question exit slip at the end of the lesson. Please see attached document. Students will earn a CW grade out of 10 points – 5 of these points are from the answers to the exit slip, 2 points are for each group's participation in the class discussion, and 3 points are for each student's participation in their group.

Extensions and Connections (for all students)

The concept of vertical and horizontal stretching and compressing was brought up in the last exploration question and will be discussed in a classroom setting. The topic of transformational graphing will continue in subsequent days' lessons.

Strategies for Differentiation

- For ELL learners, teachers should work with the ELL teacher to provide bridges between mathematics vocabulary and the student's primary language.
- Learning disabled students may benefit if the teacher provides multiple choice answers to the student explorations.
- Visual learners will benefit from the graphical representations and the ability to dynamic exploration allowed within Geogebra as well as the graphing calculator.
- Auditory learners will benefit from the classroom and group discussions.
- Kinesthetic learners will benefit from movement from individual work to group work and the involvement in classroom presentation.
- High ability students may start to begin to compare groups for similarities or differences or summarize their findings over the past three lessons about transformational graphing.

NAME: _____ DATE: _____

Bacteria: Growth or Decay? – Student Exploration #1
Please give three other representations of the following exponential functions.

1. Given the following word problem involving an exponential function determine whether it is a sample of growth or decay. Then given three other representations (graph, table, and function rule) of the exponential function.

   The population of a bacteria sample at the start of an experiment is 64. After one hour the population is 16. Three hours into the experiment the bacteria population is 1.

   Growth or Decay? Justify your decision.

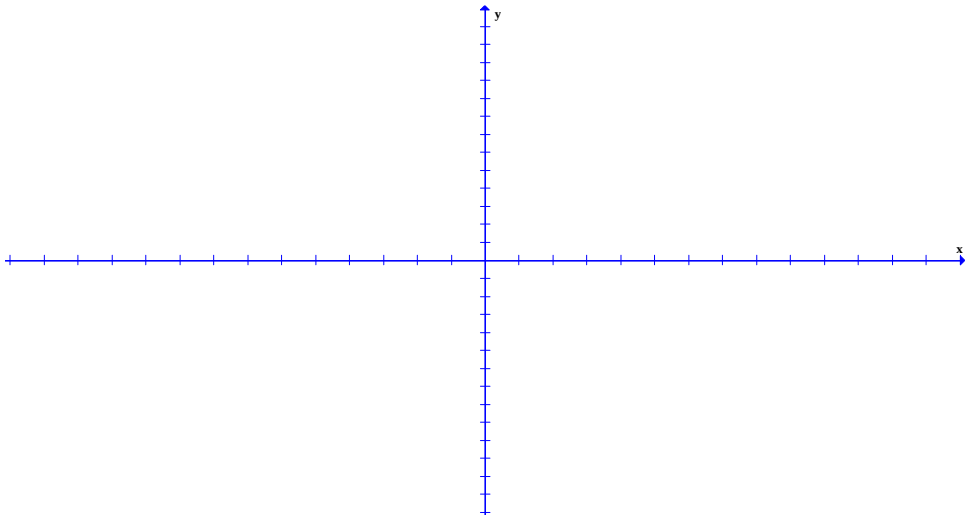
   _____

   _____

   Function notation: $f(x) =$ _____
   Hint – You might want to use the graphing calculator to run an exponential regression on the data.
   Table:

X	Y

   Graph: Be sure the scale on your axes allows all three points found above to be graphed.



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2. Given the following three points of an exponential function determine whether it is a sample of growth or decay. Then give three other representations (graph, words, and function rule) of the exponential function.

X	y
0	10
2	15
5	22.5

Growth or Decay? Justify your decision.

_____

_____

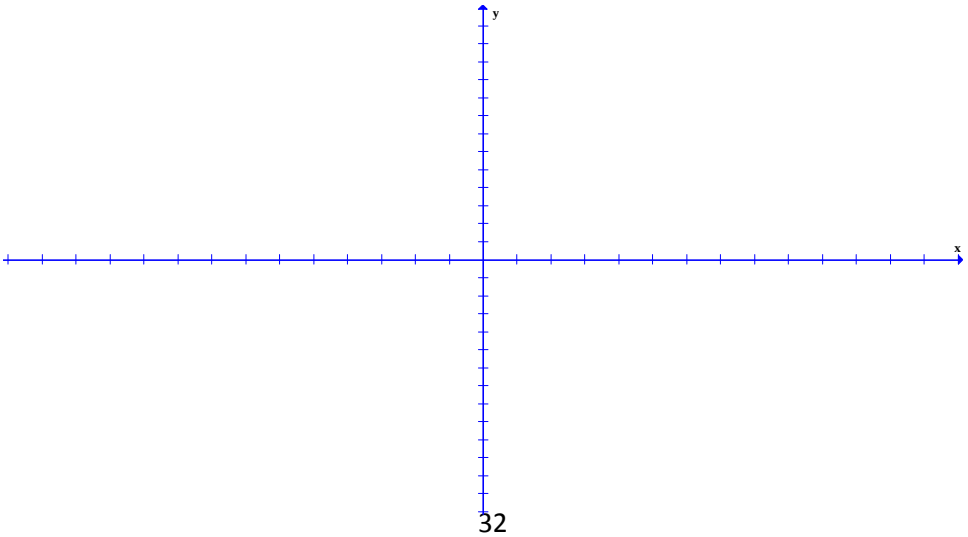
Function notation: $f(x) =$ _____
Hint – You might want to use the graphing calculator to run an exponential regression on the data.

Words: Write a two sentence word problem that uses two of the points given in the table.

_____

_____

Graph: Be sure the scale on your axes allows all three points found above to be graphed.



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3. Given the function notation determine whether this example is growth or decay. Then give three other representations (graph, words, and table) of the exponential function.

   Function notation: $f(x) = 5(3)^x$

   Growth or Decay? Justify your decision.

   _____

   _____

   Table:

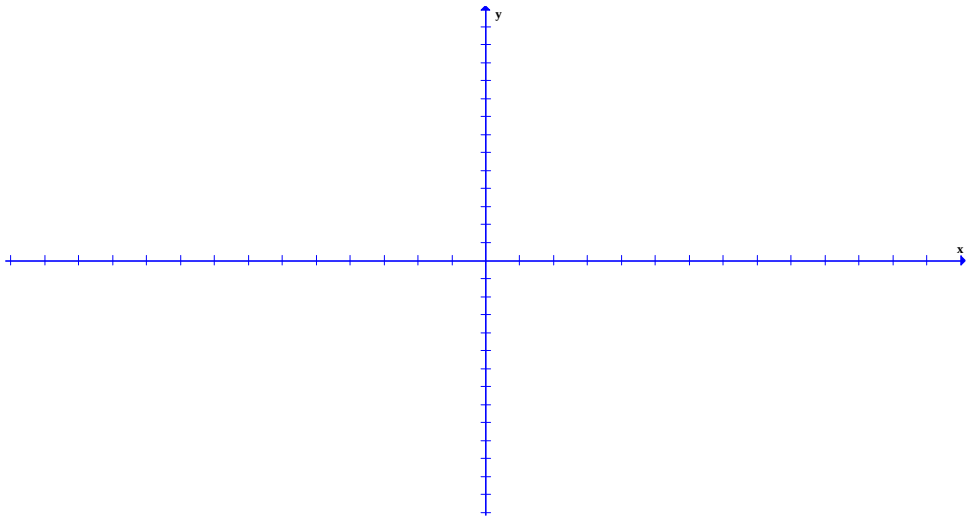
x	y

   Words: Write a two sentence word problem that uses two of the points given in the table.

   _____

   _____

   Graph: Be sure the scale on your axes allows all three points found above to be graphed.



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NAME: _____ DATE: _____

Bacteria: Growth or Decay? – Student Exploration #2

1. Given the parent function, $y = 3^x$, predict what would happen if the following transformations were done.

   a. $y = 3^{x+4}$ _____

   b. $y = 3^{x-2}$ _____

   c. $y = 3^x + 7$ _____

   d. $y = 3^x - 5$ _____

   e. $y = 2(3)^x$ _____

   f. $y = .5(3)^x$ _____

   g. $y = 4(3)^{x-1} + 6$ _____

2. Now using your calculator check to see if your predictions were correct. If not correct them with what you see happening from the graphs of the parent function and the transformation.

   a. _____

   b. _____

   c. _____

   d. _____

   e. _____

   f. _____

   g. _____

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3. Which of the transformations in question 1 would ***NOT*** be appropriate to use to represent a bacteria population? Please justify your choices.
   (Remember that x values represent time and y values represent the number of bacteria present.)

4. Would you describe $y = -1(2)^{x+3} - 4$ as exponential growth or decay? Justify your answer.

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Bacteria: Growth or Decay? – Exit Slip

1. Describe the transformations of the following exponential function.
   $y = (4)^{x+6} + 1$ _____

2. Describe the transformations of the following exponential function.
   $y = 5(.875)^{x} + 7$ _____

3. Describe the transformations of the following exponential function.
   $y = .475\left(\frac{3}{4}\right)^{x+5} - 3$ _____

4. Would the exponential function given in question 3 be appropriate to model a real life situation involving bacteria population? Why or why not?

5. What determines in an exponential function is classified as growth or decay? (Hint – Does it matter what a is?)

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