

Transformational Graphing in the Real World

I. UNIT OVERVIEW & PURPOSE:

This unit specifically addresses the concept of transformational graphing. Absolute value, polynomial, and square root functions will be examined.

II. UNIT AUTHOR:

Donna Deplazes
Craig County Public Schools

III. COURSE:

Mathematical Modeling: Capstone Course

IV. CONTENT STRAND:

Algebra

V. OBJECTIVES:

- To recognize and translate the graph of an absolute value function.
- To recognize and translate the graph of polynomial (specifically quadratic) functions.
- To establish a pattern for easy recognition of higher order polynomials.
- To recognize and translate the graph of square root function.
- To expose students to possible real world situations involving transformational graphing.

VI. MATHEMATICS PERFORMANCE EXPECTATION(s):

MPE.2: Collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

MPE.12: Transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Select and use appropriate representations for analysis, interpretation, and prediction.

MPE.14: Recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.

VII. CONTENT:

This unit specifically addresses the topic of transformation graphing and general recognition of absolute value, polynomial (specifically quadratic), and exponential functions. All lessons are discussed in the context of a real world application.

VIII. REFERENCE/RESOURCE MATERIALS:

Graphing calculators will be required. Student Exploration Worksheets and Exit Slip Assessments will be needed for all three lessons. Computers to access GeoGebra would also be beneficial when presenting if you find or make your own animation applets to help students visualize the concepts.

IX. PRIMARY ASSESSMENT STRATEGIES:

Please see Exit Slip Assessments attached to each lesson. Upon completion of more functions a unit assessment on the transformation of functions will be given.

X. EVALUATION CRITERIA:

Students will complete a five question exit slip at the end of the lesson. Documents will be attached to each individual lesson. Students will earn a classwork grade out of 10 points – 5 of these points are from the answers to the exit slip, 2 points are for each group's participation in the class discussion, and 3 points are for each student's participation in their group.

XI. INSTRUCTIONAL TIME:

Three 90-minute class periods.

Air Traffic Control

Strand

Algebra

Mathematical Objective(s)

Functions, Absolute Value Functions, Transformational Graphing

In this lesson students will discuss transformational graphing by examining an air traffic control situation. Students will explore multiple representations of absolute value functions. They will solve given problems by transforming the graph of an absolute value function. Tables, graphs, and equations will be used by students to aide in finding the necessary solutions.

Mathematics Performance Expectation(s)

MPE.2: Collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

MPE.12: Transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Select and use appropriate representations for analysis, interpretation, and prediction.

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Related SOL

All.6 The student will recognize the general shape of function (absolute value) families and will convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.

AFDA.4 The student will transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Students will select and use appropriate representations for analysis, interpretation, and prediction.

Please note that transformation graphing will be applied in all lessons within the unit. The type of function addressed will change daily. It is intended for students to use basic transformation graphing techniques to help them on a day-to-day basis.

NCTM Standards

- Understand relations and functions and select, convert flexibly among, and use various representations for them.
- Interpret representations of functions of two variables.
- Use symbolic algebra to represent and explain mathematical relationships.

Materials/Resources

- Classroom set of graphing calculators.
- Students need to know the vertical separation minimum for aircraft. According to the FAA the minimum safe vertical separation distance is 1000 feet.
(http://www.faa.gov/about/office_org/headquarters_offices/ato/service_units/enroute/rvsm/)
- Student Exploration WS #1
- Student Exploration WS #2

Assumption of Prior Knowledge

- Students should have completed Algebra II.
- Students should have experience using a graphing calculator, specifically finding a specific viewing window.
- Students might find it difficult to realize that horizontal transformations are represented “reversely” from graphic to symbolic representation.
- The relevant real life context in this problem is air traffic control.

Introduction: Setting Up the Mathematical Task

In this lesson, students will investigate how vertical and horizontal translations affect the symbolic representation of an absolute value function.

Introduction – 10 minutes

Student Exploration #1 – 25 minutes

Discussion of SE #1 – 10 minutes

Student Exploration #2 – 25 minutes

Discussion of SE #2 – 10 minutes

CW Assignment: Exit Slip Assessment – 10 minutes

- To introduce the task students will be asked to define absolute value. Students will be asked to consider if any expression containing an absolute value can ever be a negative value. Students will also be reminded of the shape of the absolute value graph by examining the graph of $f(x) = |x|$.
- Students will work the exploration activities and then the class will discuss their conclusions. Each group will be asked to answer one of the four questions on each exploration worksheet and comment on the answers provided by other groups. All students will have the correct answers before the end of class.
- Students are asked to explore transformational graphing in the two exploration worksheets attached.
- The figure given in the exploration activities is designed to assist students who struggle to “picture” the actual situation.

Student Exploration 1:

Group Work (groups of 2 or 3)

Student/Teacher Actions:

- Students should use the information from the introduction to complete a worksheet (Air Traffic Control – Student Exploration #1) that addresses vertical translations of a given function.
- Teacher will be guiding students as needed if questions/problems arise, but will not answer the questions for the students.
- Students should realize that the second airplane is always 1500 feet above the given airplane. This would then add 1500 to the function rule!
- If time allows and you have access to a lab or a classroom set of computers... The dynamic Explore Learning activity gizmo allows students to change a , h , and k and see how the graph changes as well. This website is not free – a subscription is required. You may view free for five minutes.

http://fahnenbacher.com/webpage/alg-1222/unit6/absolute-value-geogebra/absolute_value_geogebra.html

Monitoring Student Responses

- Students are expected to discuss the exploration activities together in their groups and then discuss the questions as a class at the end of the activity.

- The teacher will assist students who have difficulties and extend the material for students that are ready to move forward.

Student Exploration 2:

Group Work (groups of 2 or 3)

Student/Teacher Actions:

- Students should use the information from the introduction to complete a worksheet (Air Traffic Control – Student Exploration #2) that addresses horizontal translations of a given function.
- Teacher will be guiding students as needed if questions/problems arise, but will not answer the questions for the students.
- Students should realize that the new path is “centered” at 200 miles instead of 375. Therefore the vertex of the absolute value function would be found at a x value of 200. Remember that to show this correctly a +200 should be represented by -200 in the symbolic form.
- If time allows and you have access to a lab or a classroom set of computers... The dynamic Explore Learning activity gizmo allows students to change a , h , and k and see how the graph changes as well. This website is not free – a subscription is required. You may view free for five minutes.
http://fahnenbacher.com/webpage/alg-1222/unit6/absolute-value-geogebra/absolute_value_geogebra.html
 - Students are expected to discuss the exploration activities together in their groups and then discuss the questions as a class at the end of the activity.
 - At the end of the second exploration students are asked to make generalizations about how horizontal and vertical shifts (translations) affect the function rule.
 - The teacher will assist students who have difficulties and extend the material (add a step of difficulty) for students that are ready to move forward.

Assessment

Students will complete a five question exit slip at the end of the lesson. Please see attached document. Students will earn a CW grade out of 10 points – 5 of these points are from the answers to the exit slip, 2 points are for each group’s participation in the class discussion, and 3 points are for each student’s participation in their group.

Extensions and Connections (for all students)

The concept of vertical and horizontal stretching and compressing was brought up in the last exploration question and will be discussed in a classroom setting. The topic of transformational graphing will continue in subsequent days’ lessons.

Strategies for Differentiation

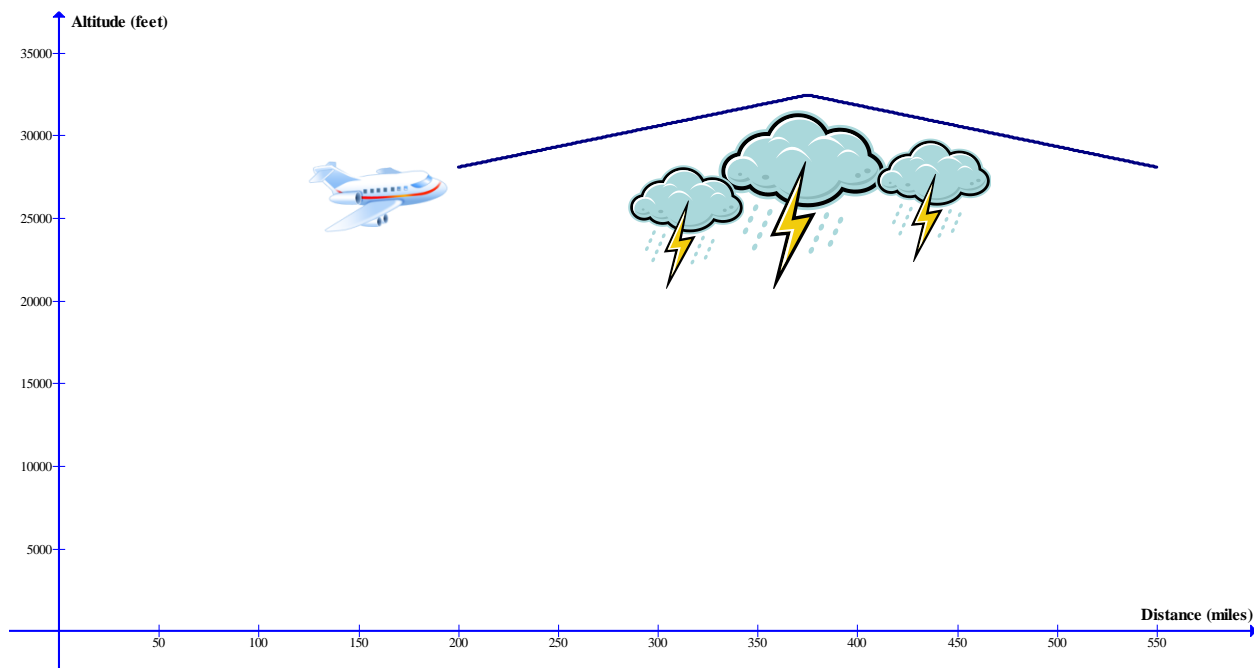
The graphic organizer/worksheet was designed with the needs of a diverse classroom of students in mind. There is a visual representation (graph) of the situation. Tables were created to assist students as well. Use of the graphing calculator is also encouraged and even prompted.

- For ELL learners, teachers should work with the ELL teacher to provide bridges between mathematics vocabulary and the student's primary language.
- Learning disabled students may benefit if the teacher provides multiple choice answers to the student explorations.
- Visual learners will benefit from the graphical representations and the ability to dynamic exploration allowed within Explore Learning as well as the graphing calculator.
- Auditory learners will benefit from the classroom and group discussions.
- Kinesthetic learners will benefit from movement from individual work to group work and the involvement in classroom presentation.
- High ability students may start to begin to compare groups for similarities or differences and offer opinions to lead into tomorrow's lessons.

NAME: _____

DATE: _____

Air Traffic Control – Student Exploration #1



The above graph shows the altitude of an airplane over the distance it flies for a specific part of its flight. At a certain point in the scheduled flight path the pilot is told to change his course to fly above a powerful storm in the flight's original path. Please use this scenario to answer the questions below.

1. If given the flight path of the airplane as $f(x) = -25|x - 375| + 32450$ where x represents the distance flown in miles since departure and $f(x)$ represents the altitude of the airplane at a given distance, please find the altitude of the airplane at the following distance increments.

Distance (miles)	Altitude (feet) – Airplane 1
200	_____
250	_____
300	_____
350	_____
400	_____
450	_____
500	_____
550	_____

2. Using the table constructed in question 1, please fill in the altitude of the second plane that is 1500 feet above the one described above.

Distance (miles)	Altitude (feet) - Airplane 2
200	_____
250	_____
300	_____
350	_____
400	_____
450	_____
500	_____
550	_____

3. Using the table constructed in question 2 how could you modify the function that represents airplane 1, $f(x) = -25|x - 375| + 32450$, to represent airplane 2. Please use your graphing calculator to assist you.

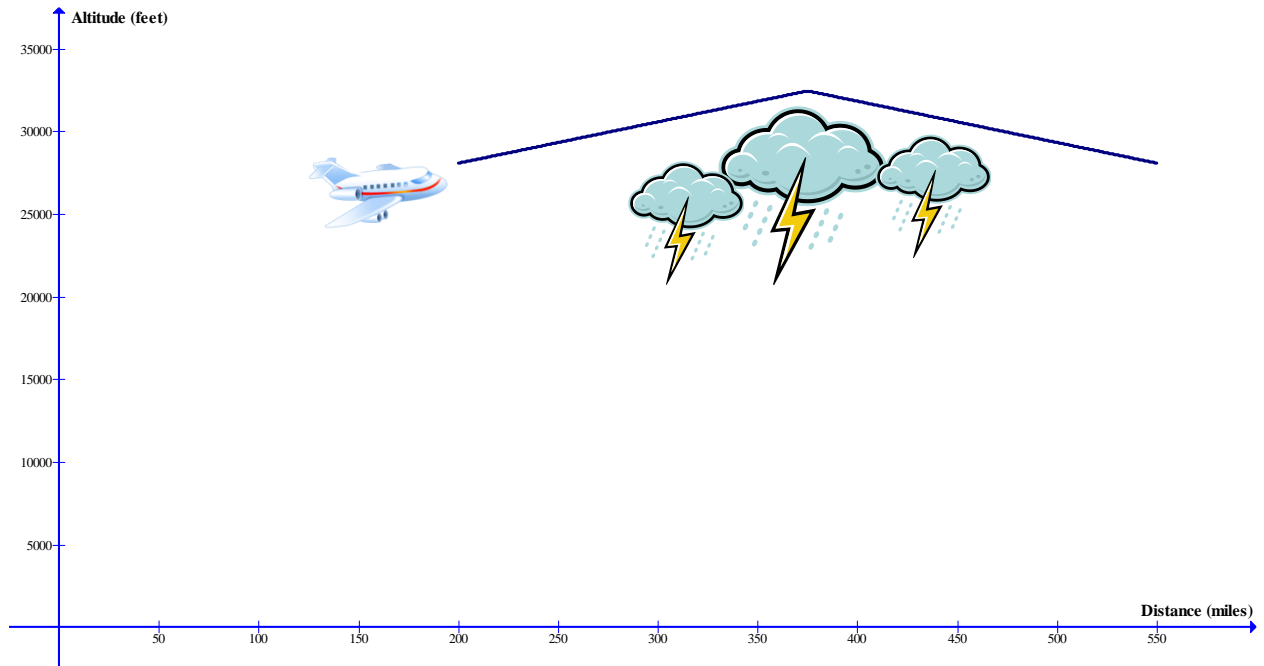
$$f(x) = \underline{\hspace{2cm}}$$

4. What is the vertex of the absolute value function representing the flight path of the airplane during this segment of its flight path? Justify your answer.

NAME: _____

DATE: _____

Air Traffic Control – Student Exploration #2



The above graph shows the altitude of an airplane over the distance it flies for a specific part of its flight. At a certain point in the scheduled flight path the pilot is told to change his course to fly above a powerful storm in the flight's original path. Please use this scenario to answer the questions below.

1. The flight path of the airplane is given as $f(x) = -25|x - 375| + 32450$ where x represents the distance flown in miles since departure and $f(x)$ represents the altitude of the airplane at a given distance. What if the storm had been centered at a distance of 200 miles from departure instead of at mile 375... How could you modify the function $f(x) = -25|x - 375| + 32450$ to represent the earlier change in altitude? Please use your graphing calculator to assist you.

$$f(x) = \underline{\hspace{10em}}$$

2. Using the function determined in question 1, please fill in the altitude of the airplane if the storm had occurred earlier in its flight path.

Distance (miles)	Altitude (feet)
50	_____
100	_____
150	_____
200	_____
250	_____
300	_____
350	_____

3. Using the two exploration activities from the day write a generalization to discuss how a vertical shift on the graph is represented in the function rule.
4. Using the two exploration activities from the day write a generalization to discuss how a horizontal shift on the graph is represented in the function rule.
5. Using your graphing calculator experiment with the given function, $f(x) = -25|x - 375| + 32450$, to determine how to create a steeper (or less steep) climb and decent. Write down your findings.

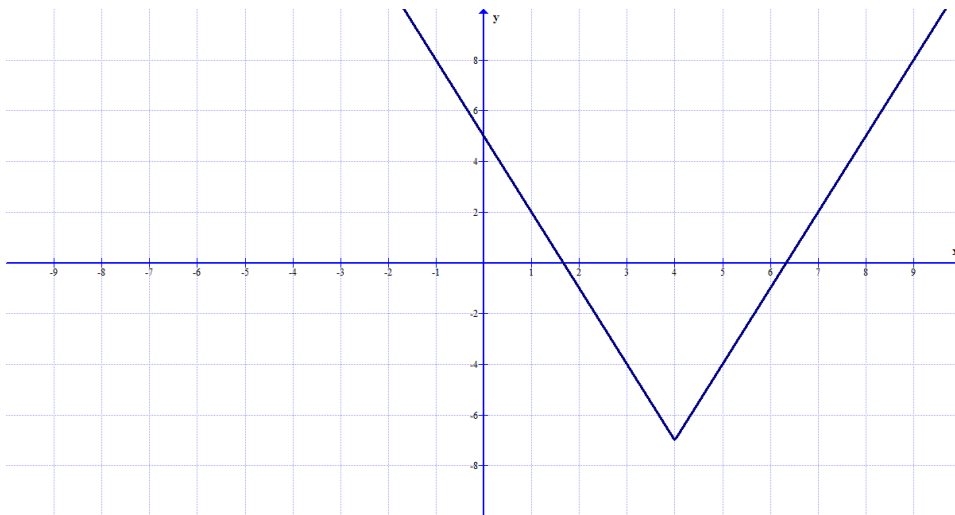
NAME: _____

DATE: _____

Air Traffic Control – Exit Slip

1. If given the absolute value function $y = |x|$, write two different function rules that would move the vertex to $(-3, 8)$.

2. Write the equation of the absolute value function shown in the graph below.



3. Given the following table write the function rule of the absolute value functions it represents.

X	y
2	14
4	4
5	-1
6	4
8	14

4. If given the absolute value function, $f(x) = |x - 4| + 7$, what would be the function rule if it was shifted 2 units to the left and 6 units up.

5. Explain what each number in the following function represents graphically.

$$f(x) = 5|x + 9| - 3$$