Ping Pong Packing

1. **UNIT OVERVIEW & PURPOSE:**
   Students will be asked to package ping pong balls in the most efficient packaging possible.
2. **UNIT AUTHOR:**
   Patsy Dickerson, Christiansburg Middle School, Montgomery County
3. **COURSE:**
   Mathematical Modeling: Capstone Course (the course title might change)
4. **CONTENT STRAND:**
   Geometry
   Measurement
   Problem Solving, Decision Making, and Integration
5. **OBJECTIVES:**
   The student will be able to use his knowledge of volume and find the size of a container that will best be used to ship ping pong balls.
6. **MATHEMATICS PERFORMANCE EXPECTATION(s):**
   MPE.1 Solve practical problems involving rational numbers (including numbers in scientific notation), percents, ratios, and proportions. (SOL 8.3, 8.1b)
   MPE.6 Use formulas for surface area and volume of three- dimensional objects to solve real-world problems. (SOL G.13)
   MPE.7 Use similar geometric objects in two- or three-dimensions to
   a) compare ratios between side lengths, perimeters, areas, and volumes; b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;
   c) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and d) solve real-world problems about similar geometric objects. (SOL G.14)
7. **CONTENT:**
   Students will investigate various shapes of packing boxes – rectangular or triangular prism or cylindrical – to maximize the number of ping pong balls that will fit into a container to ship.
8. **REFERENCE/RESOURCE MATERIALS:**
9. **PRIMARY ASSESSMENT STRATEGIES:**
   The assessment will be the dimensions of the final box and the number of ping pong balls that will fit in the box for shipping.
10. **Evaluation Criteria:**
    The mathematical support contained in the final journal entry and class presentation

will serve as the evaluation of this unit.

XI. INSTRUCTIONAL TIME:

One or two 45 minute class periods depending on discussion

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Ping Pong Packing

Strand
Geometry
Measurement

Mathematical Objective(s)

- To use knowledge of calculating volume
- To determine the best size of a 3-dimensional container to ship ping pong balls

Mathematics Performance Expectation(s)
MPE.1 Solve practical problems involving rational numbers (including numbers in scientific notation), percents, ratios, and proportions. (SOL 8.3, 8.1b)
MPE.6 Use formulas for surface area and volume of three- dimensional objects to solve real-world problems. (SOL G.13)
MPE.7 Use similar geometric objects in two- or three-dimensions to
a) compare ratios between side lengths, perimeters, areas, and volumes; b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;
c) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and d) solve real-world problems about similar geometric objects. (SOL G.14)

Related SOL

8.3 The student will

a) solve practical problems involving rational numbers, percents, ratios, and proportions;

8.7 The student will

a) investigate and solve practical problems involving volume and surface area of prisms, cylinders, cones, and pyramids; and
b) describe how changing one measured attribute of a figure affects the volume and surface area.

G.13 The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

G.14 The student will use similar geometric objects in two- or three-dimensions to

a) compare ratios between side lengths, perimeters, areas, and volumes;
b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;
c) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and
d) solve real-world problems about similar geometric objects.

NCTM Standards

Grades 9–12 Expectations: In grades 9–12 all students should–analyze properties and determine attributes of two- and three-dimensional objects;

Grades 6–8 Expectations: In grades 6–8 all students should– use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume; use visual tools such as networks to represent and solve problems; use geometric models to represent and explain numerical and algebraic relationships; recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life.

Materials/Resources

- Ping pong balls – minimum 6 per group
- Paper towel cylinders – minimum 1 per group
- Rectangular Prism boxes – dimensions and volume will vary
- Internet Access and GeoGebra
- Graphing Calculator

Assumption of Prior Knowledge

- What a ping pong ball and paper towel cylinder look like
- Calculation of volume of a sphere (V = $\frac{4}{3}\pi r^3$)
- Calculate the volume of a rectangular prism, triangular prism, cylinder given the formulas
- Determine the change of the volume if one dimension (i.e. length) is changed (i.e. doubled, halved, etc.)
- A gross is a dozen dozen or 12 dozen which equals 144
- Previous knowledge of GeoGebra

Introduction: Setting Up the Mathematical Task

- Introduction: The teacher is the project manager for a company that manufactures and ships ping pong balls. The company has just received an order for 9 gross of ping pong balls. Your task, as members of the design team is to determine what size the box should be used to minimize the un-used space inside of the box when shipping the order. Your team will report back to the project manager and present your proposal.
- Instructional Techniques:
    - Create design teams of 3-4 people to design

  - Each design team will create a presentation for the project manager including graphics for their container size
  - The only restrictions for the container shape and/or size is that it will accommodate 9 gross of ping pong balls.

- Activities:
  - Each design team will calculate the volume of a ping pong ball and investigate the best shape for a box that will hold 9 gross ping pong balls.
  - Design teams should find the volume of one ping pong balls, 9 gross ping pong balls (all measurements should be in centimeters or meters)
  - Design teams should investigate the shape of rectangular prisms, triangular prisms and cylinders that would minimize the amount of space needed to ship the order
- How will you help them to understand the task?
  - Students will be given 6 ping pong balls per design team and several boxes and a paper towel tube to investigate various shapes
  - Hands on trials should be completed before the students transition to GeoGebra.
- Final Presentation:
  - Students/groups will need to make a presentation to the class based on their findings and decisions. It should include information like the volume of a ping pong ball, volume of 9 gross ping pong balls, "empty space" around each ping pong ball
  - The presentation may be in the form of poster or PowerPoint or smart board presentation.

Student Exploration :

- Goal: In this investigation students will investigate the shape of shipping boxes to determine the best shape to ship ping pong balls
- Problem: Create a container that will most efficiently hold 9 gross of ping pong balls
- Students should be given time to investigate packing ping pong balls into the containers that they have been given
- Follow directions written on the handout.
- The teacher should "walk through" the directions and then should monitor student's progress and be available for questions.
- During the discussions, the teacher should ask what happens to the volume as one of the dimensions changes.

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- Students should also be gathering data for their presentation of the package that they decide to create.
- The teacher should preview and investigate the wiki on GeoGebra to review how to find surface area and volume and how to use this wiki for the instsrution. http://www.geogebra.org/en/wiki/index.php/Volume_%26_Surface_Area
- This website can be used or students may do the calculations by hand using the graphing calculator and the formula sheet for surface area and volume of 3-dimensional shapes.

Assessment

- **Journal/writing prompts**
    - How might this investigation change if you were packing dominoes instead of ping pong balls?
    - The post office and UPS has imposed a limit to the size of the shipping container. It may not be more than 1 **cubic meter**. Does your container fit this parameter? What is the volume in cubic meters of your container?
    - How many square centimeters of cardboard will you need to construct your container?
- **Other**

  Each design team will create a class presentation including graphics with dimensions and mathematical explanation of the selected shape of container. They may include a model of their container.

Extensions and Connections (for all students)

- Lesson extensions/follow-up
    - Change the number of ping pong balls to 1 dozen or 100 dozen. How will this affect the container?
    - Ship standard brick instead of ping pong balls. Find the standard dimensions and weight for the most common brick on line. Remember to keep in mind a reasonable weight to make the package. You may need to research UPS or USPO to find weight restrictions.
    - Compare the cost of creating the packing container if 1 square meter of cardboard costs $3.00.

Strategies for Differentiation

The differentiation strategies might include but are not limited to the following list created specifically for ESL students. Feel free to adopt these to your lesson:

- Make instruction more concrete, visual, collaborative, and hands-on
- Assign roles to students in collaborative activities. Discover the strengths of EOL students and assign appropriate roles.
- Be aware that their might be some differences in communicating the procedural knowledge of mathematics
- Focus on mathematical content rather than on linguistic form (simplify word problems without changing the math meaning)
- Language and content should be presented simultaneously
- Seating (near teacher or next to a buddy, native language if possible)
- Write legibly and in print
- Step by step instructions (orally and in writing) Ask students to repeat aloud for the rest of the class.
- Give EOL student more time for questions and answers.
- Use gestures and visuals to help clarify the message.
- Repeat, rephrase, and paraphrase.
- Simplify the language used rather than the mathematical concepts taught (use known vocabulary and simple sentence constructions).
- When students speak, focus on their message rather than their grammatical skills and accuracy. Respond using the proper grammatical form rather than overtly correcting their mistakes.
- Observe and record students' participation in small group activities.
- Give LEP students (especially beginners) alternate ways to participate in whole-class discussions and respond to questions (think/pair/share, flashcards to raise over head, hand and/or body movements, individual chalkboards for solving computations).
- Assess whether LEP students have mastered mathematical concepts rather than their English grammar and fluency.

The accommodations listed are adopted from the following source.
http://www.doe.virginia.gov/VDOE/Instruction/ESL/LEPmathResource.pdf

Ping-Pong Packing

Problem: The teacher is the project manager for a company that manufactures and ships ping pong balls. The company has just received an order for 9 gross of ping pong balls. Your task, as members of the design team is to determine what size the box should be used to minimize the un-used space inside of the box when shipping the order. Your team will report back to the project manager and present your proposal.

Procedure:

1. Discuss with your design team the shape or shapes of packing container you would like to investigate.

2. What is the volume of one ping pong ball? Use centimeters for the unit of measurement
   a. **Formula**: $V = \frac{4}{3} p r^3$

   b. Calculate the number of ping pong balls in a gross and the total volume of the ping pong balls.

3. Investigate packing the ping pong balls in a cylinder. Use the following wiki from GeoGebra to help your investigation:
   http://www.geogebra.org/en/wiki/index.php/Volume_%26_Surface_Area
   Change the height or the radius to see how it changes the volume.

4. Do the same type of investigation with a rectangular prism and a rectangular prism. Change the height or the width or the length to see how it changes the volume.

5. What is the shape of your container that your design team has decided and given three reasons for selecting this shape and dimensions of the container?

6. Make a poster and/or a model of your container. You may use poster board or cardboard to make your model. Include the dimensions in your illustration and/or your container.

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Math - Problem Solving : Box Project for Ping-Pong Ball Container

- Design Team Name: ______________________________________

CATEGORY	4	3	2	1
Mathematical Reasoning	Uses complex and refined mathematical reasoning.	Uses effective mathematical reasoning	Some evidence of mathematical reasoning.	Little evidence of mathematical reasoning.
Mathematical Errors	90-100% of the steps and solutions have no mathematical errors.	Almost all (85-89%) of the steps and solutions have no mathematical errors.	Most (75-84%) of the steps and solutions have no mathematical errors.	More than 75% of the steps and solutions have mathematical errors.
Diagrams and Sketches	Diagrams and/or sketches are clear and greatly add to the reader\'s understanding of the procedure(s).	Diagrams and/or sketches are clear and easy to understand.	Diagrams and/or sketches are somewhat difficult to understand.	Diagrams and/or sketches are difficult to understand or are not used.
Neatness and Organization	The work is presented in a neat, clear, organized fashion that is easy to read.	The work is presented in a neat and organized fashion that is usually easy to read.	The work is presented in an organized fashion but may be hard to read at times.	The work appears sloppy and unorganized. It is hard to know what information goes together.
Mathematical Terminology and Notation	Correct terminology and notation are always used, making it easy to understand what was done.	Correct terminology and notation are usually used, making it fairly easy to understand what was done.	Correct terminology and notation are used, but it is sometimes not easy to understand what was done.	There is little use, or a lot of inappropriate use, of terminology and notation.
Explanation	Explanation is detailed and clear.	Explanation is clear.	Explanation is a little difficult to understand, but includes critical components.	Explanation is difficult to understand and is missing several components OR was not included.

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