Midsummer Classic's Homerun Derby

I. UNIT OVERVIEW & PURPOSE:
Students will be studying the path of a baseball after a major league slugger has hit it.

II. UNIT AUTHOR:
Patsy Dickerson, Christiansburg Middle School, Montgomery County

III. COURSE:
Mathematical Modeling: Capstone Course (the course title might change)

IV. CONTENT STRAND:
Algebra

V. OBJECTIVES:
To study a practical application of a quadratic formula and the meaning of several points on the graph.

VI. MATHEMATICS PERFORMANCE EXPECTATION(s):
MPE 12) Transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Select and use appropriate representations for analysis, interpretation, and prediction.
MPE 13) Investigate and describe the relationships among solutions of an equation, zeros of a function, x-intercepts of a graph, and factors of a polynomial expression.
MPE 16) Investigate and analyze functions (linear, quadratic, exponential, and logarithmic families) algebraically and graphically. Key concepts include
b) local and absolute maxima and minima;
c) domain and range, including limited and discontinuous domains and ranges;
d) zeros;
e) x- and y-intercepts
k) finding the values of a function for elements in its domain; and
l) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic.

VII. CONTENT:
Students will have an opportunity to research the game of baseball. There are many factors that influence how far a ball travels after it has been hit – the speed of the ball, the force the bat makes contact with the ball, the wind speed and direction, etc. This activity will give the students an opportunity to investigate the physics and

mathematics behind the hit.

VIII. REFERENCE/RESOURCE MATERIALS:

Students should have access to the internet and a graphing calculator

IX. PRIMARY ASSESSMENT STRATEGIES:

The main assessment will be the outcome of the attached activity and the discussion it generates.

X. EVALUATION CRITERIA:

Students should have a contextual understand of intercepts, maximum, and meaningless parts of the parabola.

XI. INSTRUCTIONAL TIME:

2- 45 minute class periods

Lesson 1 - Midsummer Classic's Homerun Derby

Strand
Algebra

Mathematical Objective(s)

- To graph using a given quadratic equation on a graphing calculator
- To understand the values in the table that accompanies the graph
- To understand the meaning of intercepts, maximum, x-and y- axes, useful domain and range all within context of the problem

Mathematics Performance Expectation(s)

12) Transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Select and use appropriate representations for analysis, interpretation, and prediction.

13) Investigate and describe the relationships among solutions of an equation, zeros of a function, x-intercepts of a graph, and factors of a polynomial expression.

16) Investigate and analyze functions (linear, quadratic, exponential, and logarithmic families) algebraically and graphically. Key concepts include

b) local and absolute maxima and minima;

c) domain and range, including limited and discontinuous domains and ranges;

d) zeros;

e) x- and y-intercepts

k) finding the values of a function for elements in its domain; and

l) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic.

Related SOL

AFDA.1 The student will investigate and analyze function (linear, quadratic, exponential, and logarithmic) families and their characteristics. Key concepts include c) domain and range; d) zeros; e) intercepts

AII.7 The student will investigate and analyze functions algebraically and graphically. Key concepts include a) domain and range, including limited and discontinuous domains and ranges; b) zeros; c) x- and y-intercepts; and h) composition of multiple functions. Graphing calculators will be used as a tool to assist in investigation of functions.

A.4 The student will solve multistep linear and quadratic equations in two variables, including c) solving quadratic equations algebraically and graphically;

A.7 The student will investigate and analyze function (linear and quadratic) families and their characteristics both algebraically and graphically, including a) determining whether a relation is a function; b) domain and range; c) zeros of a function; d) *x*- and *y*-intercepts; e) finding the values of a function for elements in its domain; and f) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic.

NCTM Standards List all applicable NCTM standards related to each lesson.

- represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules;
- understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions;
- interpret representations of functions of two variables

Additional Objectives for Student Learning (include if relevant; may not be math-related):

Materials/Resources

- Classroom set of graphing calculators
- Access for each person or pair of students to the Internet

Assumption of Prior Knowledge

- How to graph a quadratic equation
- To be able to identify intercepts and maximum or minimum
- Some basic knowledge of the game of baseball

Introduction: Setting Up the Mathematical Task

- In this lesson you will investigate the relationship of the speed that a baseball is hit and the time the baseball is in the air.
- There will be individual work as well as partner work.
- Two assumptions will be made throughout this lesson
    - The ball always leaves the bat at (to get maximum height and distance)
    - The beginning height h_o will always be zero as if the ball always leaves at ground level.

Student Exploration:

Begin the lesson by watching this YouTube video on hitting a baseball

http://www.youtube.com/watch?v=dg5AuYCsg98

Individual Work (attached in Appendix – Midsummer Classic's Homerun Derby – Lesson 1)
Small Group Work (attached – same as above)

Whole Class Sharing/Discussion (if relevant)

After the students have had an opportunity to think about and answer these questions, lead a whole class discussion on what the students think is the answers.

- What are some factors that you need to know to find the solution to this problem? (Think-Pair-Share)
- Sketch the path that a hit baseball will travel. Label home plate and the outfield.

Student/Teacher Actions:

- The students should be following the directions given in the Lesson – Midsummer Classic's Homerun Derby – Lesson 1
- The teacher should be moving throughout the room facilitating the execution of the lesson and leading class discussion

Monitoring Student Responses

Assessment

- **Questions**

  How is graphing helpful in analyzing the flight of a baseball after it has been hit?
  What is a reasonable domain for this problem? Why does the real number system not make sense in context of this problem?
  What do the values of the maximum mean? What does the x- represent and what does the y- represent in the ordered pair for the maximum?
  Explain the x and y intercepts of this graph in context of hitting a baseball.

- **Journal/writing prompts**

  Any of the questions above could be used as a journal or writing prompt

Extensions and Connections (for all students)

- Extension could be to do another investigation but include changing the angle that the ball leaves the bat.

Strategies for Differentiation

The differentiation strategies might include but are not limited to the following list created specifically for ESL students. Feel free to adopt these to your lesson:

- Assign roles to students in collaborative activities. Discover the strengths of EOL students and assign appropriate roles.

- Be aware that their might be some differences in communicating the procedural knowledge of mathematics
- Focus on mathematical content rather than on linguistic form (simplify word problems without changing the math meaning)
- Language and content should be presented simultaneously
- Seating (near teacher or next to a buddy, native language if possible)
- Write legibly and in print
- Step by step instructions (orally and in writing) Ask students to repeat aloud for the rest of the class.
- Use gestures and visuals to help clarify the message.
- Repeat, rephrase, and paraphrase.
- Simplify the language used rather than the mathematical concepts taught (use known vocabulary and simple sentence constructions).
- When students speak, focus on their message rather than their grammatical skills and accuracy. Respond using the proper grammatical form rather than overtly correcting their mistakes.
- Observe and record students' participation in small group activities.
- Give LEP students (especially beginners) alternate ways to participate in whole-class discussions and respond to questions (think/pair/share).
- Because LEP students may not understand the rules of baseball, this lesson could be adapted for a soccer kick with similar assumptions (45 degree angle, ball leaves from the ground, no wind)
- Assess whether LEP students have mastered mathematical concepts rather than their English grammar and fluency.
- Give EOL student more time for questions and answers.
- The accommodations are adopted from the following source. http:// www.doe.virginia.gov/VDOE/Instruction/ESL/LEPmathResource.pdf

Lesson 2 - Midsummer Classic's Homerun Derby

Strand
Algebra

Mathematical Objective(s)

- To graph using a given quadratic equation on a graphing calculator
- To understand the meaning of intercepts, maximum, x-and y- axes, useful domain and range all within context of the problem
- To calculate the horizontal distance that the ball will travel

Mathematics Performance Expectation(s)

13) Investigate and describe the relationships among solutions of an equation, zeros of a function, x-intercepts of a graph, and factors of a polynomial expression.

Related SOL

AFDA.1 The student will investigate and analyze function (linear, quadratic, exponential, and logarithmic) families and their characteristics. Key concepts include c) domain and range; d) zeros; e) intercepts

AII.7 The student will investigate and analyze functions algebraically and graphically. Key concepts include a) domain and range, including limited and discontinuous domains and ranges; b) zeros; c) x- and y-intercepts; and h) composition of multiple functions. Graphing calculators will be used as a tool to assist in investigation of functions.

A.7 The student will investigate and analyze function (linear and quadratic) families and their characteristics both algebraically and graphically, including a) determining whether a relation is a function; b) domain and range; c) zeros of a function; d) *x*- and *y*-intercepts; e) finding the values of a function for elements in its domain; and f) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic.

NCTM Standards List all applicable NCTM standards related to each lesson.

- represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules;
- interpret representations of functions of two variables

Additional Objectives for Student Learning (include if relevant; may not be math-related):

Physics of baseball

Materials/Resources

- Classroom set of graphing calculators
- Access for each person or pair of students to the Internet

Assumption of Prior Knowledge

- How to graph a quadratic equation
- To be able to identify intercepts and maximum
- Some basic knowledge of the game of baseball
- Use of d = r*t formula

Introduction: Setting Up the Mathematical Task

- In this lesson you will investigate the relationship of the speed that a baseball is hit, the time the baseball is in the air, and the horizontal distance the ball travels.
- There will be individual work as well as partner work.

Student Exploration:

Begin the lesson by watching these YouTube videos:

http://www.youtube.com/watch?v=uS7Iq_I0i6M - Babe Ruth's 60th homerun

http://www.youtube.com/watch?v=4EXMh-kyvcw&feature=related - Barry Bond's 715th homerun

Individual Work (attached – in Appendix – Midsummer Classic's Homerun Derby – Lesson 2)
Small Group Work (attached – same as above)

Whole Class Sharing/Discussion (if relevant)

After the students have had an opportunity to think about and answer these questions, lead a whole class discussion on what the students think is the answers.

- What are some factors that you need to know to find the solution to this problem? (Think-Pair-Share)
- Velocity has a vertical component and a horizontal component. The vertical component changes but the horizontal constant for the flight of the ball.
- How what happens to the vertical component of the velocity as the ball travels? What happens to the horizontal component of the velocity as the ball travels?

Student/Teacher Actions:

- The students should be following the directions given in the Lesson – Midsummer Classic's Homerun Derby Lesson 2
- The teacher should be moving throughout the room facilitating the execution of the lesson

Monitoring Student Responses

Assessment

- **Journal/writing prompts**

  Any of the questions above could be used as a journal or writing prompt

Extensions and Connections (for all students)

- Extension could be to do another investigation but include changing the angle that the ball leaves the bat.

Strategies for Differentiation

The differentiation strategies might include but are not limited to the following list created specifically for ESL students. Feel free to adopt these to your lesson:

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- Give LEP students (especially beginners) alternate ways to participate in whole-class discussions and respond to questions (think/pair/share).
- Assess whether LEP students have mastered mathematical concepts rather than their English grammar and fluency.
- The accommodations are adopted from the following source. http:// www.doe.virginia.gov/VDOE/Instruction/ESL/LEPmathResource.pdf

Appendix

Midsummer Classic's Homerun Derby

Lesson 1
Lesson 2

Midsummer Classic's Homerun Derby – Lesson 1

"The Major League Baseball All-Star Game, also known as the "Midsummer Classic", is an annual baseball game between players from the National League and the American League, currently selected by a combination of fans, players, coaches, and managers.[1] The All-Star Game usually occurs on the second Tuesday in July and marks the symbolic halfway point in the Major League Baseball (MLB) season (though not the mathematical halfway point; in most seasons, that actually takes place one week earlier). The game is usually played on a Tuesday, with no regular-season games scheduled on the day before or the day after. ... Players usually wear their own team uniforms."
(http://en.wikipedia.org/wiki/Major_League_Baseball_All-Star_Game)

Since 1985, one of the events that occurs the week of the All-Star Game is the Homerun Derby where the leagues longest hitters compete for hitting the most homeruns.

Problem: How high and how far do the longest hitters hit the baseball?

What are some factors that you need to know to find the solution to this problem? (Think-Pair-Share)

Sketch the path that a hit baseball will travel. Label home plate and the outfield.

Look at this website after class discussion of hitting a baseball: http://whyfiles.org/2010/hit-a-home-run

To find the vertical distance a ball travels, use the following formula:

$$d = -16t^2 + v_o t + h_o$$

Use this formula to complete the chart. You may use a graphing calculator table to fill in the values.

V_0 = 50 ft/sec (Velocity the ball leaves contact with the bat.)
h_0 = 0 ft

Time (t) in seconds	$d = -16t^2 + 50\,t + 0$	Height (d) in feet
0		
1.25		
2		
2.5		
3		
4		

Which value above does not "make sense" in the context of this problem? (Think-Pair-Share)

Partner Questions:

Graph this equation in your calculator when the velocity is 60 ft./sec. Adjust the window so the vertex and the zeros can be seen.

What does the vertex indicate in context of this problem in terms of height and time – what happens to the ball at this point?

What do the Zeros (x-intercepts) indicate in context of this problem in terms of height and time?

If the velocity is increased, what happens to the graph. Explain what happens to the distance the ball travels and the height that the ball will reach. (Hint: Adjust the velocity in the formula and graph this in your calculator and compare.)

Sketch the two graphs. Part of this graph "makes no sense" in context of this problem. Shade it and explain why.

Midsummer Classic's Homerun Derby – Lesson 2

Problem: How far will the ball travel horizontally after it has been hit? Will it be a homerun?

THINK, PAIR, SHARE QUESTIONS:

1. Are all major league homerun's created equally? In other words, if you hit the ball 375 feet will it be a homerun in all major league stadiums? Why or why not?

2. The angle that you hit the ball is very important. What do you think the best angle to hit the ball is: 30, 45, 60 or 75 ? Why?

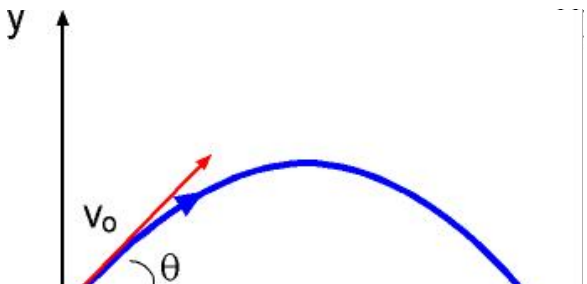
3. What happens to the speed of the baseball on its parabolic path after it has been hit?

4. Watch this video that will review the geometry of a 45-45-90 degree triangle.

http://www.youtube.com/watch?v=l6LUOVmix0c&feature=related

5. Label this drawing to illustrate the path of a baseball after it has been hit. Label the total time the ball travels is in flight, the maximum height, the horizontal distance the ball travels.

Assumptions:



- No wind
- No air resistance
- According to the graph, the height of the batter is 0 feet, how can you tell?

- The angle = 45

6. Make a sketch in the picture above of a right triangle that has v_o as the hypotenuse. Label the horizontal leg v_x and the vertical let v_y. If v_o is 50 feet per second, calculate the value of v_x and v_y. (HINT: remember in the tutorial video, the legs of a 45-45-90are equal and do not leave a square root in the denominator.)

$v_x =$ _____ $v_y =$ _____

7. Find the total distance traveled, use the formula: $d = v_x * t$; t = total time the ball is in the air (remember how to do this from Lesson 1)

8. Fill in the table below by calculating the total time of flight, maximum height of the ball, and distance the ball travels:

Initial Speed of ball (v_o) in ft/sec	**Time** the ball is in flight (Hint: find the zero of the equation $d = -16t^2 + v_o t$)	**Maximum height** of the ball in feet (Hint: from the parabola)	**Distance** in feet (horizontal) that the ball travels $d = v_x * t$	If a homerun is 425 yards, is this a homerun?
100 ft/sec				
Initial Speed of ball (v_o) in ft/sec	**Time** the ball is in flight (Hint: find the zero of the equation $d = -16t^2 + v_o t$	**Maximum height** of the ball in feet (Hint: from the parabola)	**Distance** in feet (horizontal) that the ball travels $d = v_x * t$	If a homerun is 425 yards, is this a homerun?
75 ft/sec				
150 ft/sec				
132 ft/sec				
88 ft/sec				

9. Are any of these problems in the chart un-reasonable for major league baseball? Why or why not.

10. How would changing the angle of the hit (more than 45 or less than 45) change the horizontal distance? Refer to this website to help answer this question. (**http://whyfiles.org/2010/hit-a-home-run)**

Key for Chart

8. Fill in the table below by calculating the total time of flight, maximum height of the ball, and distance the ball travels:

Initial Speed of ball (v_o) in ft/sec	**Time** the ball is in flight (Hint: find the zero of the equation $d = -16t^2 + v_o t$)	**Maximum height** of the ball in feet (Hint: from the parabola)	**Distance** in feet (horizontal) that the ball travels $d = v_x * t$	If a homerun is 425 feet, is this a homerun?
100 ft/sec	6.25 seconds	156 feet	70.7 * 6.25 = 441.9 ft	Yes
75 ft/sec	4.69 sec	87.9 feet	248.7 ft	No
150 ft/sec	10.4 sec	347 feet	1103.1 ft	Yes
132 ft/sec	8.46 sec	272 feet	789.1 ft	Yes
88 ft/sec	5.5 sec	121 feet	342.25 ft	No