

Real Life Parallels

I. UNIT OVERVIEW & PURPOSE:

Each lesson uses the definition and properties of parallel lines cut by a transversal to explore real world problems and situations and create projects using these concepts. The first has students finding actual examples of real-life parallel lines, the second uses them to make an artistic creation using parallel lines and then uses them to solve real-world problems, and finally in the third parallel lines are used to create maps of make-believe cities and then their properties are used to create a tour of their city.

II. UNIT AUTHOR:

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III. COURSE:

Mathematical Modeling: Capstone Course

IV. CONTENT STRAND:

Geometry

V. OBJECTIVES:

Students will be able to identify parallel lines versus skew lines, will be able to discuss the properties of parallel lines cut by a transversal, and will be able to find examples of parallel lines in the real world and discuss how and why they are parallel and will be able to create their own parallel lines using multiple strategies.

VI. MATHEMATICS PERFORMANCE EXPECTATION(s):

MPE.32: The student will use the relationships between angles formed by two lines cut by a transversal to a) determine whether two lines are parallel; b) verify the parallelism, using algebraic and coordinate methods as well as deductive proofs; and c) solve real-world problems involving angles formed when parallel lines are cut by a transversal.

VII. CONTENT:

Each of these three lessons will use the concepts of parallel lines using a different situation. The first will have students find where parallel lines are used in everyday life and then have them assess why it is important that the lines are parallel and how to ensure that they are parallel. The second lesson will have students use strategies they discussed in the previous lesson to create their own artwork using parallel lines, and also use parallel lines to solve problems involving indirect measurement. The third lesson will show city maps with parallel streets and 'transversal' streets. We will navigate a sightseeing tour through a city and then students will create their own city maps with parallel and transversal streets created on graph paper using coordinate methods and then create a tour through their city using properties of parallel lines.

VIII. REFERENCE/RESOURCE MATERIALS:

Students will utilize art supplies, protractors, rulers, and graph papers to create projects. They will also use digital cameras, a laptop with an LCD projector, calculators, and hypsometers to solve problems and work on projects.

One problem is taken from the following website:

<http://www.nexuslearning.net/books/ML-Geometry/Chapter3/ML%20Geometry%203-3%20Parallel%20Lines%20and%20Transversals.pdf>

IX. PRIMARY ASSESSMENT STRATEGIES:

The assessment will be done three ways. The first will be teacher assessment from what students are saying, doing, and producing. The second will be through exit tickets/journal entries, and the third will be with the projects/products they turn in from each lesson.

X. EVALUATION CRITERIA:

The assessment will be done by the teacher evaluating student responses and student work and projects.

XI. INSTRUCTIONAL TIME:

Three 90 minute block periods (could possibly go into a fourth 90 minute block if more time is desired for various activities)

Lesson 1: Parallel in the Real World

Strand

Geometry

Mathematical Objective(s)

The goal of this lesson is for students to see where parallel lines are used in real life. First they will brainstorm and find examples, and then they will write an explanation of the best and easiest way to ensure that these lines truly are parallel. They will see how the mathematical concepts apply to real world situations.

Mathematics Performance Expectation(s)

MPE.32: The student will use the relationships between angles formed by two lines cut by a transversal to a) determine whether two lines are parallel; b) verify the parallelism, using algebraic and coordinate methods as well as deductive proofs; and c) solve real-world problems involving angles formed when parallel lines are cut by a transversal.

Related SOL G2a, c: The student will use the relationship between angles formed by two parallel lines cut by a transversal to a) determine whether two lines are parallel and c) solve real-world problems involving angles formed when parallel lines are cut by a transversal.

NCTM Standards

- Apply and adapt a variety of appropriate strategies to solve problems
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others

Materials/Resources

- Digital cameras
- Laptop computer with LCD projector

Assumption of Prior Knowledge

- The student should have taken and passed Geometry successfully.
- A student at a Van Hiele level 1 should be able to complete this lesson successfully, but most of the students should be at a level 3.
- Students should know the definition of parallel lines, be able to recognize parallel lines, and be able to create parallel lines based on properties of parallel lines cut by a transversal. They should also know the various types of angles created by parallel lines cut by a transversal and their properties.
- Students may struggle to come up with examples of parallel lines in the real world that they cannot physically find around the school building. They may also have trouble deciding why the lines in their examples should be parallel and as well as figuring out ways to ensure that

they are parallel. Some examples they may be able to find include power lines, the mortar between bricks in a wall, and the lines on a road.

- Students should have basic knowledge of how to take and save photos using a digital camera.
- This lesson focuses students on how parallel lines are used in the real world.

Introduction: Setting Up the Mathematical Task

“In this lesson, you will see how parallel lines and their properties are used in the everyday real world.”

- Have students write down everything they can think of about parallel lines for five minutes, and then have students share with a partner for five minutes what they have come up with. After this, each pair will share one thing they said with the whole class that hasn’t already been said.
- Draw a pair of parallel lines with a transversal on a poster board. Color one angle and ask students which other angle should be colored to create each type of angle pair (alternate interior, alternate exterior, consecutive interior, and corresponding) and ask what is known about each pair of angles when the lines creating them are parallel. Write the angle pair and its’ properties on the poster board. Do this for each type of angle and post these on the walls of the room. (10 minutes)

Student Exploration 1:

Student/Teacher Actions:

- Give each student pair from earlier a digital camera and ask the pairs to go around (classroom, school building, school grounds) taking pictures of any parallel lines they see. In addition, have them come up with at least one more example of parallel lines in real life that are not pictured. (20 minutes)
- Looking at their pictures, they must pick 2 plus their example not pictured and write down why it is important the lines are parallel and brainstorm how it can be ensured that the lines are parallel. If there is time, students could try to find pictures of the example they came up with on their own online. (10 minutes)
- Students then need to pick one of their three examples to share with the class. The pictures can be shown through the computer and LCD projector. Students need to share what their example is, where they found it, and what they wrote about why it is important for the lines to be parallel and how it can be ensured that they are. Allow time for student comments. Students could search for a picture on the internet to show if they are using the example they came up with on their own. (30 minutes)

Monitoring Student Responses

- Students should be encouraged to use mathematical terminology as much as possible and may need to be reminded of this throughout the lesson.
- Teacher will help clarify students' thoughts during student interactions with each other and while they present their findings to the class. The teacher can also help summarize concepts after all have shared.
- Students who have difficulties should be paired with a student who has a better grasp of the material for the main activity.
- Summary
 - For the last ten minutes, have students summarize at least five things they know about parallel lines in a journal or exit ticket.
 - Students will turn in their journal/exit ticket and the three examples of parallel lines and the writing they did for each one.

Assessment

- Describe and attach the assessments for each lesson objective.
 - **Journal/writing prompts**
 - List/summarize at least five things you know about parallel lines from today's lesson. (No rubric, read over student responses and mark right or wrong and grade accordingly. May just want to grade as a 'participation' grade and check over student responses and address them in the next lesson.)
 - **Three Parallel Line Examples**
 - Grade each out of three points for a total of nine points for the assignment. One point for the picture/example being a correct example of parallel lines, one point for an appropriate explanation of why the lines should be parallel, and the third point for an appropriate explanation of how to be sure they are parallel.

Extensions and Connections (for all students) Students can research a profession that may involve parallel lines or interview someone who works in a profession involving parallel lines. Students should find out why parallel lines are important for this profession and how it is insured that the lines are parallel, if possible. Have them share their findings with the class and/or write a report and create a power point presentation about what they learned.

Strategies for Differentiation

- Assign students specific tasks (such as ESL students are the photographers and the other student is the writer).
- Focus on mathematical content and not necessarily grammar.
- Summarize and paraphrase ideas after being presented.

Lesson 2: The Art & Problems of Parallel Lines

Strand

Geometry

Mathematical Objective(s)

The goal of this lesson is for students to see how to use parallel lines and their properties to create artwork. They can use their own strategies to be sure their lines are parallel. They will then see how using parallel lines and their properties can help them to solve problems.

Mathematics Performance Expectation(s)

MPE.32: The student will use the relationships between angles formed by two lines cut by a transversal to a) determine whether two lines are parallel; b) verify the parallelism, using algebraic and coordinate methods as well as deductive proofs; and c) solve real-world problems involving angles formed when parallel lines are cut by a transversal.

Related SOL G2b, c: The student will use the relationship between angles formed by two parallel lines cut by a transversal to b) verify the parallelism, using algebraic and coordinate methods as well as deductive proofs and c) solve real-world problems involving angles formed when parallel lines are cut by a transversal.

NCTM Standards:

- Apply and adapt a variety of appropriate strategies to solve problems
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others

Materials/Resources

- Protractors
- Art supplies (pipe cleaners, popsicle sticks, glue, paint, paper, glitter, etc.)
- Tape measures
- Calculators

Assumption of Prior Knowledge

- The student should have taken and passed Geometry successfully.
- A student at a Van Hiele level 1 should be able to complete this lesson successfully, but most of the students should be at a level 3.
- Students should know the definition of parallel lines versus skew lines, be able to recognize parallel lines, and be able to create parallel lines based on properties of parallel lines cut by a transversal.
- Students may have trouble connecting the idea of parallel lines creating similar triangles and using them to create indirect measurements.

- Students should know the various types of angles created by parallel lines cut by a transversal and their properties.
- This lesson focuses students on how parallel lines are used to create and build as well as to solve problems in the real world.

Introduction: Setting Up the Mathematical Task

“In this lesson, you will use parallel lines to build your own creation and then use their properties to take measurements of objects that would be hard to measure.”

- Give students a ‘hurdle’ activity. Give students two minutes to complete each hurdle on their own, then call on a student to share their response and discuss (except the final hurdle, which is discussed in the second bullet point below). Be sure posters from last class are covered so students have to remember on their own.
 - Write the definition of parallel from last class.
 - Write down the types of angles parallel lines create when intersected by a transversal.
 - Write the property of parallel lines and each type of angle.
 - Write the difference between parallel and skew lines. (11 minutes)
- Discuss the answers the students came up with for the last hurdle. Using a Venn Diagram with two intersecting circles created on a poster board, one labeled ‘Parallel’ and the other labeled ‘Skew’ have students put in properties of each type of line in the diagram where they go. Display this along with the posters from last class with angles created by parallel lines cut by a transversal and their properties. (4 minutes)

Student Exploration 1:

Student/Teacher Actions:

- Give students art supplies & protractors. Tell them they must use any art supplies in any means they want to make an artistic creation of some kind- 2D or 3D. Their creations must include parallel lines cut by a transversal, with bonus points for skew lines being included. They have 25 minutes to be creative.
- After they are done, they must write a short paragraph telling about their creation and the method they used to make sure their lines were truly parallel. (5 minutes)

Monitoring Student Responses

- Students should be encouraged to use mathematical terminology as much as possible and may need to be reminded of this.
- Teacher may need to help students struggling with making sure the lines in their creations are parallel by reminding them of the properties of parallel lines discussed earlier.

- Students who have difficulties could ask the teacher or another student to proof his/her paragraph before turning it in.

Student Exploration 2:

Student/Teacher Actions:

- Ask students, “Are rays from the sun like parallel lines? Why?” Allow student answers. Then ask, If you stand outside in the sun, what would happen? What would you create? Allow student responses. Draw a person, the ground and shadow, and the sun and draw the triangle created. (Students may need to be reminded that shadows appear on the ground- not in mid-air! Students may also need to be reminded that the hypotenuse of the triangle would be the sun’s ray, which would not necessarily be visible.) Then draw another object (a tree, flagpole, etc.) on the same ground with the sun and create the triangle, and then ask “What do you know about these triangles?” Allow for student responses. Hopefully students will realize that the two triangles are similar due to the Angle-Angle postulate, since they would both contain a right angle (from the person/tree standing straight up from the ground) and corresponding congruent angles created from the parallel rays of the sun and the ground. Then ask students, “What do you know about the sides of similar triangles?” Allow for student response, hopefully they will recall that sides of similar triangles are proportional. Then ask, “Why might this be useful? What could we find with this knowledge?” Allow student responses. Hopefully students will realize that they can use this information to find measurements of objects indirectly. If there is time and/or if necessary, do an example problem using the person and flagpole triangles drawn on the board and inserting measurements for their shadows and the person’s height and use them to find the height of the flagpole. (10 minutes)
- Put students in pairs. Tell them to find the measurements of three things outside using this method. Give each a tape measure and calculator and have them take paper and pencil. Have them find the measurement of one in the pair’s height and the length of their shadow. Then have them find the length of the shadow of the object they want to measure. Using a proportion, have them find the height of the object. They need to draw a picture for each example and show the proportion and the work they do to turn in. (25 minutes)

Monitoring Student Responses

- Students should be encouraged to use mathematical terminology as much as possible and may need to be reminded of this.
- Teacher may need to help students struggling with the concept of parallel sunrays creating similar triangles. While other students start the project, they may need to hold some students back to go over the ideas again and then send them out to do 1 or 2 instead of 3 examples.

- Students may want teacher to look over their problems and answers before turning them in to be sure they make sense and have been set up correctly.
- Summary
 - For the last ten minutes, have students summarize the difference between skew and parallel lines and also describe why knowing about parallel lines and their properties can be useful in solving problems - they may want to include the kinds of things that could be measured- in a journal or exit ticket. They should also write a few sentences discussing how skew lines may be used in the real world.
 - Students will turn in their journal/exit ticket, their creation and the paragraph they wrote, and the three items they measured with the work they did for each.

Assessment

- Describe and attach the assessments for each lesson objective.
 - **Journal/writing prompts**
 - Summarize the difference between skew and parallel lines and also describe why knowing about parallel lines and their properties can be useful in solving problems- they may want to include the kinds of things that could be measured in a journal or exit ticket. They should also write a few sentences discussing how skew lines may be used in the real world. (No rubric, read over student responses and mark right or wrong and grade accordingly. May just want to grade as a 'participation' grade and check over student responses and address them in the next lesson.)
 - **Parallel Line Creations**
 - No rubric, read over student paragraphs and mark right or wrong and grade accordingly. May just want to grade as 'participation' and perhaps give bonus points for creativity of creations.
 - **Indirect Measurement Problems**
 - Grade each out of three points for a total of nine points for the assignment. One point for the picture being drawn, one point for the problem being set up correctly based on the measurements each group took, and one for a correct answer.

Extensions and Connections (for all students) Have students work out the 'rainbow problem'. See <http://www.nexuslearning.net/books/ML-Geometry/Chapter3/ML%20Geometry%203-3%20Parallel%20Lines%20and%20Transversals.pdf> page 6 number 30. Display a picture showing that when sunlight hits raindrops/water, it bounces off at an angle to our eyes to create each different color. This looks like two parallel lines being cut by a transversal (see picture in problem from website). Have students figure out what kind of angles this creates, and if one angle's measure is known find the measure of the other angle.

Strategies for Differentiation

- Students who are having difficulty can work in pairs on the first task- an ESL student could create the art project and the other could help with the mathematical part and writing the paragraph.
- After the explanation about similar triangles, have any students who still are struggling stay behind for some extra help, then send them out only having to complete 1 or 2 examples instead of 3.
- Focus on mathematical content and not necessarily grammar.

Lesson 3: Mapping with Parallel Lines

Strand

Geometry

Mathematical Objective(s)

The goal of this lesson is for students to practice using the properties of parallel lines correctly by applying them to a real-world situation. Students will use maps of cities with parallel streets and 'transversal' streets to take a tour of the city using the properties of parallel lines and then will create their own city with streets and landmarks by using slopes and equations of lines.

Mathematics Performance Expectation(s)

MPE.32: The student will use the relationships between angles formed by two lines cut by a transversal to a) determine whether two lines are parallel; b) verify the parallelism, using algebraic and coordinate methods as well as deductive proofs; and c) solve real-world problems involving angles formed when parallel lines are cut by a transversal.

Related SOL G2: The student will use the relationship between angles formed by two parallel lines cut by a transversal to a) determine whether two lines are parallel; b) verify the parallelism, using algebraic and coordinate methods as well as deductive proofs and c) solve real-world problems involving angles formed when parallel lines are cut by a transversal.

NCTM Standards:

- Apply and adapt a variety of appropriate strategies to solve problems
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others

Materials/Resources

- Maps of cities like Washington D.C. or Chicago or simple maps of portions of cities (such as the Mall in Washington D.C.)
- Graph paper or poster board with 'ghost grid'
- Rulers
- Graphing Calculators
- Markers/colored pencils

Assumption of Prior Knowledge

- The student should have taken and passed Geometry successfully.
- A student at a Van Hiele level 2 should be able to complete this lesson successfully, but most of the students should hopefully be at a level 3.
- Students should know the definition of parallel lines, be able to recognize parallel lines, and be able to create parallel lines based on properties of parallel lines cut by a transversal.

- Students may have trouble handling multiple sets of parallel lines and transversals.
- Students should know the various types of angles created by parallel lines cut by a transversal and their properties. They should also know about slopes of parallel lines.
- This lesson focuses students on how parallel lines are used to create city layouts and their properties can be used to move through the city.

Introduction: Setting Up the Mathematical Task

“In this lesson, you will use parallel lines to take a tour of a city and then to create your own city based on properties of parallel lines.”

- Give students 5 minutes to write everything they know about parallel lines- definition, properties, etc. Review all with students for 5 minutes.
- Review equations of lines with students in $y = mx + b$ form and how each part represents changes on the graph.
- Show students a city map with parallel streets. Discuss what the students notice about the map. Why are parallel lines used? (5 minutes)
- Go through a ‘tour’ of the city created by the teacher. For example, “Start at the Visitor’s Center. Now go to the site on the angle that is alternate exterior with the angle where the Visitor’s Center is, using parallel streets Main and Second. Where are you?” Give each direction, give students a few minutes to figure out the answer, then have them share their answers. Go through one whole tour together. (15 minutes)
- Put up a different map. Put the students in pairs and give them a tour worksheet. Have them write down their location on the worksheet for each step of the tour, including the final destination. If you wanted, you could give several different tours and see which pair can complete each of them the fastest. (10 – 20 minutes)

Student Exploration 1:

Student/Teacher Actions:

- Give pairs of students graph paper or poster board with a ‘ghost grid’, rulers, colored pencils or markers, and graphing calculators. They are to create their own make-believe city (which they should name). It can be as realistic and creative as they want, but the streets of the city must be made up of parallel lines that are created using equations of lines and labeled with the equation used. They should create an x and y axis and then create equations to graph, some that are parallel and others that are not. There must be at least 10 streets (which students are free to name if they choose) with the equations for each written along each line somewhere, and 15 locations/landmarks (which could include schools, libraries, museums, town halls, or anything else that may be appropriate). Students are then to create a tour of their city like the tours they completed of cities with

the class and with a partner. Their tour must have at least 10 steps and stops. If desired, have them create it like a brochure with the map inside and the tour outside. (30 minutes)

Monitoring Student Responses

- Students should be encouraged to use mathematical terminology as much as possible and may need to be reminded of this.
- Teacher may need to help students struggling with equations of lines and do a more extensive review with a few students who need extra help with reviewing the concept.
- Summary
 - For the last ten minutes, have students go through the tour of a another pair's city to be sure it all works out correctly in their journal/exit ticket. They will then check with the creators to be sure their tour turned out correctly.
 - Students will turn in their journal/exit ticket, the tour worksheet, and their city map and tour they created.

Assessment

- Describe and attach the assessments for each lesson objective.
 - **Journal/writing prompts**
 - Students will go through a classmate's tour of their city to be sure it works out. The city creator will then check to be sure their tour turned out correctly. (No rubric, read over student responses and mark right or wrong and grade accordingly. May just want to grade as a 'participation' grade and check over student responses and address them in the next lesson.)
 - **Tour Worksheet**
 - Give each blank a point value and mark each right or wrong and tally total correct out of total points possible for a grade.
 - **City Map and Tour**
 - Grade each in three parts; first, that all requirements were met (at least 10 streets, at least 15 locations/landmarks, at least 10 stops on their tour), second that their tour was done correctly (look at student journals/exit tickets to be sure), and third on creativity & mathematical content. (If it was created as a brochure, extra credit could be given.)

Extensions and Connections (for all students) Have students research city planning and/or interview a city planner to find out about their strategies, why streets in cities are often parallel, and how they are built. Students could also find another city map like those used in class and create a city tour of this new city.

Strategies for Differentiation

- Students who are having difficulty can work in pairs on city map- an ESL student could create the map and the other could help with writing the tour.
- Students having difficulty with equations of lines may need a few more minutes of explanation from the teacher before beginning their city map.
- Focus on mathematical content and not necessarily grammar.