# Surface Area and Volume

I. **UNIT OVERVIEW & PURPOSE:**
The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems. Students are going to assist Mr. Lee in determining the volume and surface area of an aquarium, ice-cream cones, and of pyramids. Students are going to be helping Mr. Lee decide which type of ice cream cone will be best for his store inside the aquarium and then will help him decide which type of pyramid is bigger while he is planning a vacation. In lesson 1 the students will find the volume and surface area of rectangular prisms by working with aquariums. If there is enough time students can explore the world's largest aquarium and compare the volumes that they found from the lesson from the biggest volume and surface area. In lesson 2 students will help Mr. Lee decide which ice cream cone will have the biggest volume and surface area for his ice cream stand in the aquarium. Students will find and compare the volume and surface area of waffle cone, wafer cone, and a waffle bowl. In lesson 3 students will help Mr. Lee take a vacation. He wants to visit some pyramids in Egypt and China. He needs to figure out which pyramids have the greatest volume and surface area so Mr. Lee can see the best pyramids in the world. By helping Mr. Lee students will learn how to find volume and surface area.

II. **UNIT AUTHOR:**
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III. **COURSE:**
Mathematical Modeling: Capstone Course

IV. **STRAND:**
Geometry

V. **OBJECTIVES:**
The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

VI. **MATHEMATICS PERFORMANCE EXPECTATION(s):**
6) The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

VII. **CONTENT:**
In this unit students will learn to use volume and surface area for rectangular prisms, cones, cylinders and pyramids. Students are going to be an architect building a new aquarium for a tourist attraction. They will have to decide how big a tank should be and how much glass it will take to cover the aquarium. Students will also need find volume and surface area of cones. Students will investigate pyramids and their location.

VIII. **REFERENCE/RESOURCE MATERIALS:**
Students will use Google to research different sized aquariums, paper and pencil, 1 inch cube blocks and different sized small to medium sized aquariums of the teachers choice.

Students will use a calculator to help solve problems.

IX. **PRIMARY ASSESSMENT STRATEGIES:**

Students will complete a set of problems for each lesson.

X. **EVALUATION CRITERIA:**

Will be included in each lesson.

XI. **INSTRUCTIONAL TIME:**

3 -5 days for a 90 minute block or 6-10 days for a 45 minute class.  Some lessons will take more than a day.

# Aquarium

**Strand**
Geometry

**Mathematical Objective(s)**

The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems. Students will use volume to find missing dimensions. Students will use those dimensions to find the surface area of the aquarium.

**Mathematics Performance Expectation(s)**
6) The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

**Related**

8.7 The student will investigate and solve practical problems involving volume and surface area of rectangular solids (prisms), cylinders, cones, and pyramids.

G.13 The student will use formulas for surface area and volume of three-dimensional objects to solve practical problems. Calculators will be used to find decimal approximations for results.

**NCTM Standards**
- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships
- Use visualization, spatial reasoning, and geometric modeling to solve problems
- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others;

**Materials/Resources**
Describe the materials and resources (including instructional technology) you plan to use in each lesson.
- Classroom set of graphing calculators
- Different sized aquariums or fish bowls
- A large number of 1-inch cube blocks

**Assumption of Prior Knowledge**
- Students would have taken Pre-Algebra and Geometry
- Students are expected to know the difference between volume and surface area.
- Students are expected to use visual representations to solve the problems.


# Introduction: Setting Up the Mathematical Task

Modeling Problem:

Mr. Lee is building a new tourist attraction and he needs to know much water he needs to fill the aquarium.  Mr. Lee wants to fill a rectangular prism for his aquarium but he thinks he can start making conjectures using a fish bowl to determine how much water he needs.  How much water will it take to fill a large aquarium and how much glass does he need to go around the tank?
- Have students fill the fish bowl or a small rectangular aquarium with water in small groups and see if they can determine how much water they need.  (5-10 minutes)
- Have students discuss other ways they might be able to find how much water is needed. Such as:
    1) measuring the water using cups (how would you do that for larger aquariums)
    2) using the fish bowl as unit
    3) find a standard unit
- Students will realize that filling a fish bowl will not help them determine how much water or glass they will need for a rectangular aquarium.

You can teach volume and surface area as one lesson or you can do surface area the second day.



**Student/Teacher Actions:**
- Introduce the 1-inch cube blocks and have students fill the fish bowl and count how many blocks it takes to fill the bowl. (10-20 minutes)
- Explain how filling the fish bowl/aquarium with water and blocks is representing the volume.
- Then have students fill a rectangular aquarium in small groups (20-30 minutes)
- See if students can find the relationship between width, length and height to find the volume of the aquarium.
- Students should be able to tell the teacher that Volume = length*width*height.
- Now ask students what they need to do to find how much glass it will take to cover the aquarium.
- Students are to find the surface area of the rectangular prism (include the top).
- Students may count the area of each side and develop the formula or the teacher can give them the formula and have students figure the surface area.
- Students are encouraged to use their calculators to help with calculations.

- Students can start filling in the width, length and height to find the volume instead of filling the entire aquarium.  Use different sizes of aquariums to find volume.
- Revisit the original problem and have students research the world's largest aquarium to find the dimensions and the volume of the aquarium. (30-40 minutes)
- It will be helpful for the teacher to have an answer key depending on the size of the aquariums used in the lesson.

**Monitoring Student Responses**
- Students will communicate their thinking in small and whole class discussions.  Students can also write their ideas on paper to brainstorm before discussing them with classmates and teacher.
- Students will be encouraged to help each other develop ideas and work together to find other ways than filling the aquarium with water.
- For students who are struggling you can use a smaller model or give students hints to help them get the right conclusion.
- To extend this for students that are ready to move on have them find more aquariums with different dimensions and have them research which is most visited.
- To summarize the lesson have students work in their small groups for 5-10 minutes to share their ideas and then have them share with the class.  Encourage students to use terms they have learned in this lesson such as: rectangular prisms, volume, and surface area.

## Assessment
- ○ **Questions**
  - ▪ Find the volume and surface area of the following aquariums:

| Dimensions |
|---|
| 12" x 6" x 8" |
| 30" x 12" x 12" |
| 20" x 10" x 12" |
| 30" x 12" x 18" |
| 20" x 10" x 18" |

  - ▪ A large aquarium tank has a volume of 43200 ft$^3$ with a length of 72 ft and a width of 24 ft.  What is the height?  Find the surface area of the aquarium.
- Rubric:
  - ○ Question 1 total 20 points. Parts a-e 4 points apiece 1 point for the correctly setting up the volume equation and 1 point for correct answer. 1 point for setting up the surface area equation correctly and 1 point for correct answer.

- o Question 2 total 10 points. 5 points for setting the equation up correctly and solving for the height correctly. 5 points for surface area.

## Extensions and Connections (for all students)

- For students who are ready to move on and need a challenge, students can be given the volume and two dimensions and asked to find the third. Discuss how to find the third dimension by using literal equations to solve for the missing variable first.

## Strategies for Differentiation

- Make instruction more concrete, visual, collaborative, and hands-on

- Assign roles to students in collaborative activities. Discover the strengths of ESL students and assign appropriate roles.

- Be aware that there might be some differences in communicating the procedural knowledge of mathematics

- Focus on mathematical content rather than on linguistic form (simplify word problems without changing the math meaning)

- Step by step instructions (orally and in writing). Ask students to repeat aloud for the rest of the class.

# Ice-Cream

**Strand**
Geometry

**Mathematical Objective(s)**

The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems. Students will use volume to find missing dimensions. Students will use those dimensions to find the surface area of ice cream cones.

**Mathematics Performance Expectation(s)**
6) The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

**Related**

8.7     The student will investigate and solve practical problems involving volume and surface area of rectangular solids (prisms), cylinders, cones, and pyramids.

G.13     The student will use formulas for surface area and volume of three-dimensional objects to solve practical problems. Calculators will be used to find decimal approximations for results.

**NCTM Standards**
- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships
- Use visualization, spatial reasoning, and geometric modeling to solve problems
- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others;

**Materials/Resources**
Describe the materials and resources (including instructional technology) you plan to use in each lesson.
- Classroom set of graphing calculators
- Different sized ice cream cones such as waffle cones, wafer cones and waffle bowl.
- A large number of 1-inch cube blocks

**Assumption of Prior Knowledge**
- Students would have taken Pre-Algebra and Geometry
- Students are expected to know the difference between volume and surface area.
- Students are expected to use visual representations to solve the problems.

# Introduction: Setting Up the Mathematical Task

Modeling Problem:

Mr. Lee wants to open an ice cream parlor inside his aquarium. He is trying to decide which ice cream cone will hold the most ice cream and which one has the most surface area to decorate with edible sugar. He has picked a waffle cone, wafer cone and a waffle bowl cone. Which cone will hold the most ice cream? Which cone will show the most decoration?

Waffle cone: height 7 inches and a diameter of 3 ¼ inches.

Wafer cone: height $2 \frac{7}{8}$ inches and a diameter of 2 ¼ inches.

Waffle bowl: height of $1 \frac{13}{16}$ inches and a diameter of 4 ½ inches

- Have students take a guess on which one they think has the greatest volume and which one has the greatest surface area and record their answers in their journal.
- Have students discuss ways they might figure this out
  - Examples:
  - Can fill the cones with ice cream and count the number of scoops
  - Can cover the sides with fabric and then measure.

**Student/Teacher Actions:**
- Introduce the 1-inch cube blocks and explain how filling the cones with ice cream and blocks represent volume. (10-20 minutes)
- Then have students fill the three ice cream cones in small groups (20-30 minutes)
- See if students can find the relationship between diameter, radius and height to find the volume of the aquarium.
- Students should be able to tell the teacher that Volume $= \frac{1}{3} \pi r^2 h$ from previous classes in geometry.
- Students are encouraged to use their calculators to help with calculations.
- Students are to find the surface area of the ice cream cones include the top.
- Students may try to count the sides but they are going to find that $\pi$ is hard to count so they need to know the formula for surface area = area of the cone + area of base (circle)
  SA$= \pi r h + \pi r^2$

- Revisit the original problem and have students answer the question correctly now. (30-40 minutes)
- Answer Key:
    - Waffle Cone: Volume = 19.3568 cubic inches      SA=44.0314 square inches
    - Wafer Cone: Volume=3.8104 cubic inches      SA=14.1372 square inches
    - Waffle Bowl Cone: Volume =9.6089 cubic inches   SA=28.7161square inches

**Monitoring Student Responses**

- Students will communicate their thinking in small and whole class discussions.  Students can also write their ideas on paper to brainstorm before discussing them with classmates and teacher.
- Students will be encouraged to help each other develop ideas and work together to find other ways than filling the cones with water.
- For students who are struggling you can use a smaller model or give students hints to help them get the right conclusion.
- To extend this for students that are ready to move on have them find the dimensions of a push-up ice cream and find the volume and surface area.
- To summarize the lesson have students work in their small groups for 5-10 minutes to share their ideas and then have them share with the class.

## Assessment

- **Questions**
- Mr. Less decided to use the Waffle cone because it has the greatest volume and surface area.  Now Mr. Lee wants to know what size to make a display cone.  He wants to know the volume and surface area of a plastic cone that is 52 inches in height and 20 inches in width.  What is the volume and surface area?
- The formula for the volume of a sphere is $\frac{4}{3}\pi r^3$ and formula for surface area is $4\pi r^2$.

    Find the volume and surface area of a Mr. Lee's waffle cone with a scoop of ice cream with a diameter of 4 inches.

- **Rubric**:
    - Question 1 total 10 points.  2 points for the correctly setting up the volume equation and 3 point for correct answer. 2 point for setting up the surface area equation correctly and 3 point for correct answer.
    - Question 2 total 10 points.  5 points for setting the equation and getting the volume correct.  5 points for setting up the equation and getting the surface area correct.

## Extensions and Connections (for all students)

- For students who are ready to move on and need a challenge, students can be given the volume and asked to find a missing dimension.  Discuss how to find the dimension by using

literal equations to solve for the missing variable first.  You can also expand the lesson to do surface area without including the area of the circle.

## Strategies for Differentiation

- Make instruction more concrete, visual, collaborative, and hands-on

- Assign roles to students in collaborative activities. Discover the strengths of ESL students and assign appropriate roles.

- Be aware that there might be some differences in communicating the procedural knowledge of mathematics

- Focus on mathematical content rather than on linguistic form (simplify word problems without changing the math meaning)

- Step by step instructions (orally and in writing). Ask students to repeat aloud for the rest of the class.

# Pyramids

**Strand**
Geometry

**Mathematical Objective(s)**

The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems. Students will use volume to find missing dimensions. Students will use those dimensions to find the surface area of pyramids.

**Mathematics Performance Expectation(s)**
6) The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

**Related**

8.7 The student will investigate and solve practical problems involving volume and surface area of rectangular solids (prisms), cylinders, cones, and pyramids.

G.13 The student will use formulas for surface area and volume of three-dimensional objects to solve practical problems. Calculators will be used to find decimal approximations for results.

**NCTM Standards**
- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships
- Use visualization, spatial reasoning, and geometric modeling to solve problems
- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others;

**Materials/Resources**
Describe the materials and resources (including instructional technology) you plan to use in each lesson.
- Classroom set of graphing calculators
- Different sized square based pyramids
- A large number of 1-inch cube blocks

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**Assumption of Prior Knowledge**

- Students would have taken Pre-Algebra and Geometry
- Students are expected to know the difference between volume and surface area.
- Students are expected to use visual representations to solve the problems.


# Introduction: Setting Up the Mathematical Task

Modeling Problem:

Mr. Lee wants to take a vacation after he has built a new aquarium with an ice cream shop. During a visit to Xi'an, China Mr. Lee discovered pyramids similar to those in Egypt. When researching the difference between Chinese and Egyptian pyramids, he noticed there was a difference in the sizes. In Xi'an the square base has a width and length of 1,500 ft and a height of 1,000 feet. The Great Pyramid of Giza measures a width and length of 755.9 and a height of 455.8. What is volume and surface area of each pyramid?

Have students fill different size pyramids with blocks to see if they can figure out the formula before solving. (5-10 minutes)

- Have student poor the blocks into a cube to see if they can figure out the formula now. (5 minutes)
- Have students discuss other ways they might be able to find how much water is needed. Such as:
  1) measuring the water using cups
  2) what is the relation between a pyramid and cube?
  3) find a standard unit


**Student/Teacher Actions:**

- Give students the 1-inch cube blocks and have students fill the square based pyramid with the blocks and have them pour the blocks into the cube and keep doing that until the cube is full.

  Once the cube is full students will be able to see that volume = $\frac{1}{3}Bh$ where B=lw (10-20 minutes)
- Explain how filling the pyramid with blocks is representing the volume.
- Students are to find the surface area of the square based pyramid using the same method of volume.
- Students may count the area of each side and develop the formula or the teacher can give them the formula and have students figure the surface area.
- Revisit the original problem and have students research other pyramids to find the volume and surface area. (30-40 minutes)
- Students are encouraged to use their calculators to help with calculations.
- It will be helpful for the teacher to have an answer key depending on the size of the pyramids used in the lesson.

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Developed by Dr. Agida Manizade & Dr. Laura Jacobsen, Radford University MSP project
in collaboration with
Mr. Michael Bolling, Virginia Department of Education

**Monitoring Student Responses**
- Students will communicate their thinking in small and whole class discussions. Students can also write their ideas on paper to brainstorm before discussing them with classmates and teacher.
- Students will be encouraged to help each other develop ideas and work together to find other ways than filling the pyramid with water.
- For students who are struggling you can use a smaller model or give students hints to help them get the right conclusion.
- To extend this for students that are ready to move on have them find more pyramids with different dimensions and have them research which is most visited.
- To summarize the lesson have students work in their small groups for 5-10 minutes to share their ideas and then have them share with the class.

## Assessment
- **Questions**
  - In your journal write a minimal of two page paper on the similarities and differences between Chinese and Egyptian pyramids.

- Rubric:
  - Question 1 total 20 points. 1-5 points for facts on the pyramids. 1-5 points for grammar and spelling. 1-5 points for length of paper. 1-5 points for correct reference in APA format.

## Extensions and Connections (for all students)
- For students who are ready to move on and need a challenge, students can be given the volume and two dimensions and asked to find the third. Discuss how to find the third dimension by using literal equations to solve for the missing variable first.

## Strategies for Differentiation
- Make instruction more concrete, visual, collaborative, and hands-on

- Assign roles to students in collaborative activities. Discover the strengths of ESL students and assign appropriate roles.

- Be aware that there might be some differences in communicating the procedural knowledge of mathematics

- Focus on mathematical content rather than on linguistic form (simplify word problems without changing the math meaning)

- Step by step instructions (orally and in writing). Ask students to repeat aloud for the rest of the class.