Google Earth Trip

I. UNIT OVERVIEW & PURPOSE:
Students will use pictorial representations of real life objects to investigate geometric formulas, relationships, symmetry and transformations.

II. UNIT AUTHOR:
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III. COURSE:
Mathematical Modeling: Capstone Course

IV. CONTENT STRAND:
Geometry

V. OBJECTIVES:
The students will use computer software to investigate and analyze the properties of real world objects. They will apply basic formulas of coordinate geometry and investigate symmetry, translations, reflections, rotations, dilations and properties of lines.

VI. MATHEMATICS PERFORMANCE EXPECTATION(s):
MPE 3: The student will use pictorial representations, including computer software, constructions, and coordinate methods to solve problems involving symmetry and transformation. This will include:

- Investigating and using formulas for finding distance, midpoint and slope;
- Applying slope to verify and determine whether lines are parallel or perpendicular;
- Investigating symmetry and determining whether a figure is symmetric with respect to a line or a point;
- Determining whether a figure has been translated, reflected, rotated or dilated using coordinate methods.

VII. CONTENT:
This unit addresses applications of such notions as symmetry and transformations. Students will use geometric formulas to discover significant geometric realities of real world objects.

VIII. RESOURCE MATERIALS:
Students will need a computer with Internet capabilities to access software

programs such as Google Earth and Geogebra or Geometer Sketch Pad. Follow school/county procedures to get software downloaded to computers if necessary.

IX. **PRIMARY ASSESSMENT STRATEGIES:**
Students will create a portfolio comprised of print outs of sketches from geometric exploration software of geometric explorations of real world objects.

X. **EVALUATION CRITERIA:**
The portfolio will be graded from a rubric describing expectations of material for the portfolio. The expectation grades will be totaled out of 200 points.

XI. **INSTRUCTIONAL TIME:**
These lessons will require approximately five extended (90 minute) classes.

Symmetry

Strand
Geometry

Mathematical Objective(s)
Students will examine symmetry in the architecture of buildings found through Google Earth. They will examine bilateral and rotational symmetry.

Mathematics Performance Expectation(s)
MPE 3: The student will use pictorial representations, including computer software, constructions, and coordinate methods to solve problems involving symmetry and transformation. This will include:

- Investigating and using formulas for finding distance, midpoint and slope;
- Applying slope to verify and determine whether lines are parallel or perpendicular;
- Investigating symmetry and determining whether a figure is symmetric with respect to a line or a point;
- Determining whether a figure has been translated, reflected, rotated or dilated using coordinate methods.

Related SOL
G3b: The student will use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation. This will include applying slope to verify and determine whether lines are parallel or perpendicular.

NCTM Standards

- Create and use representations to organize, record, and communicate mathematical ideas
- Use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations
- Investigate conjectures and solve problems involving two- and three-dimensional objects represented with Cartesian coordinates.
- Investigate conjectures and solve problems involving two- and three-dimensional objects represented with Cartesian coordinates.

Materials/Resources

Google Earth, Geogebra 4.0, Calculators, access to the Internet
Students will analyze different real-world objects by inserting pictures of these objects into Geogebra. They will discover, explore and verify types of symmetry by using Geogebra to find axes of symmetry. They will create figures in Geogebra that demonstrate rotational and translational symmetry.

Assumption of Prior Knowledge

Students should have completed Lesson 1 in this strand or have previous experience with Geogebra 4.0. They should have a basic understanding of geometry, including parallel lines, congruency, line segment bisection and regular polygons.

Introduction: Setting Up the Mathematical Task

- In this lesson, students will use Geogebra 4.0 to analyze the symmetry of various real-world objects. Using the software, they will construct points, lines, parallel lines, angles and line segments. They will measure line segments and angles using Geogebra. They will construct and translate objects using Geogebra.
- Introduction to terms of symmetry, 10 minutes; brief review of Geogebra, 10 minutes; Student Exploration 1, 30 minutes; Student Exploration 2, 30 minutes; group discussion and sharing of results, 10 minutes.
- Begin by discussing what the term "symmetry" means in a geometric context. Define (if necessary) different types of symmetry: bilateral (reflectional), rotational and translational.
- Have students discuss their understanding of symmetry and then brainstorm various objects that, to them, exhibit symmetry. Encourage students to consider types of symmetry other than bilateral, including rotational and translational.
- Students will work through Student Exploration 1 Sheet, Bilateral Symmetry.
- Students will work through Student Exploration 2 Sheet, Rotational Symmetry.

Student Exploration 1:

Student/Teacher Actions:

- Each student works through Student Exploration 1 Sheet using his/her own computer but regularly compares results with a partner.
- Teachers circulate through the room to keep students on task and help students work through any problems they encounter with Geogebra software.
- Pictorial results of the exploration will vary from student to student. Slope computations by hand should match computations by the software.

Monitoring Student Responses

- Students will return to a classroom group format to discuss their results. Questions such as:
  - Why do you think architecture is so often constructed with bilateral symmetry?
  - What other examples of bilateral symmetry did you find?
  - How did you determine if an object exhibited bilateral symmetry?
- Have students print their Geogebra window, or if a printer is not available, have them save the Geogebra window to a folder on their computer. They can email the folder to the teacher upon completion of the lesson.

Student Exploration 2:

Student/Teacher Actions:

- Each student works through Student Exploration 2 Sheet using his/her own computer, but regularly compares results with a partner.
- Teachers circulate through the room to keep students on task and help students work through any problems they encounter with Google Earth or Geogebra software. There are likely to be difficulties encountered when students translate polygons to create tessellations, particularly when colors are applied.

Monitoring Student Responses

- Students will return to a classroom group format to discuss their results.
- Have students print their Geogebra window, or if a printer is not available, have them save the Geogebra window to the previously created folder on their computers. They can email the entire folder to the teacher upon completion of Lesson 2.

Assessment

- Students will either print all constructions or email them to their teacher for printing.
- Each construction is graded for accuracy using the given rubric.

Extensions and Connections (for all students)

- Teachers may initiate a classroom discussion concerning the ways in which graphing software can help students validate components of digital photos.

- Students may research ways in which symmetry does and does not appear in nature and in architecture. They may use Geogebra to reflect a cropped photo (one which has been divided in half along its axis of symmetry) over its axis of symmetry, to investigate the appearance of such a shape. Faces present particularly interesting examples for this process, so students may wish to bring in digital photos of themselves for this activity.

Strategies for Differentiation

- The use of the computer is an advantage for students with processing or memory issues. It also addresses the kinesthetic learning style of many students.

- For kinesthetic learners, you could bring a cheap full body-length mirror to class. Hold the mirror vertically down the center of their body. Have students look at their reflection and determine if their entire body is symmetrical.
- English language learners (ELLs): materials may be provided in other languages.
- High-ability students may research online to investigate the use of symmetry in art, architecture, and science.

Exploration Sheet 1: Symmetry in Architecture around the World

"What does the seventeenth-century Rundetarn (Round Tower) of Copenhagen have in common with the thirteenth-century Leaning Tower of Pisa? Or Houston's Astrodome, the first indoor baseball stadium built in the United States, with the vast dome of the Pantheon in Rome? Or a Chinese pagoda with the Sydney Opera house? A first response might be 'shape' but a more accurate answer would be 'symmetry.' Each of these strange couples of buildings shares a different kind of symmetry that links them, in spite of their temporal and cultural differences. As Magdolna and István Hargittai have noted, "symmetry, in architecture as in other arts, is 'a unifying concept.' " –Kim Williams, Architect, http://vismath.tripod.com/kim/

1. Open Geogebra 4.0 on your computer. Go to "View," and choose grid, axes, and input bar. You should have a window with a grid, but no visible axes.

2. Open Google Earth and select the Eiffel Tower from the file. Drag the image into the Geogebra window or import it using the "Insert Image" tool.

3. Determine if the image has bilateral (reflection) symmetry:

Construct a point at the very top of the Tower, and construct another point in the middle of either level 1 or level 2 of the Tower. (You can determine the middle by counting the struts on the level.) Construct a line through the two points. We will refer to this line as the axis of symmetry.

4. Construct a point at each end of each level and construct the midpoint between the two points. Does the midpoint lie on the axis? If so, the Eiffel Tower has bilateral symmetry.

5. Save your construction to your folder and repeat the process with another building of your choice from Google Earth.

6. Answer the following questions:

   a. Why do you think buildings are constructed with symmetry?

   b. How would asymmetry affect a structure as tall as the Eiffel Tower?

c. Would the Eiffel Tower be as appealing without bilateral symmetry? Why/why not?

7. Go to Chartres, France, and examine photos of the Cathedral. Find a photo that shows the front of the Cathedral. Answer the following questions:

a. Does the building exhibit bilateral symmetry? Why/why not? Be very specific in answering this question.

b. If there is not complete bilateral symmetry, is any part of the front of the building symmetric? If so, what parts?

c. Why do you think this cathedral was constructed in this manner?

8. Find a photo of one of the rose stained glass windows in Chartres Cathedral. Does it exhibit bilateral symmetry? Do you know any other type of symmetry it exhibits?

Just for fun:

Import (into Geogebra) a cropped photograph of a face or object that has bilateral symmetry, cropped along its axis of symmetry. Drag its axis of symmetry so that it lies along the y-axis. Use the “Reflect Object about Line” tool. Click on the photo, then click on the y-axis. Your photo will be reflected about the y-axis.

a. Compare your original picture to your reflected photo. How do you like the new picture?

b. If you used a human face, does perfect symmetry improve the attractiveness of the person in the photo? Explain.

Exploration Sheet 2: Rotational Symmetry

1. Open Geogebra, go to View, and select Axes, which will remove the axes from the window. Construct an equilateral triangle by clicking on the "Polygon" tool arrow, and selecting Regular Polygon. Click two points in the plane; when the window appears, choose "3." Your triangle should appear.

2. Explore the symmetry of the triangle:

a. Does the triangle have an axis of symmetry? Construct the perpendicular bisector that extends from a vertex and is perpendicular to the opposite side. Is this line the axis of symmetry of the triangle? Does it have any others? If so, construct them.

b. Using the pointer tool, grab a vertex point of the triangle and rotate it. How many axes of symmetry could you create within this triangle? An equilateral triangle exhibits both bilateral symmetry and "rotational" symmetry, which means that after rotating the figure, it still looks the same.

3. In the same window, use the directions above to construct a regular hexagon. Using the pointer tool, grab a vertex of the figure and rotate it.

a. Does this figure have bilateral symmetry? If so, how many axes of symmetry can you find? Construct each one, ensuring that they are accurately constructed.

b. Does this figure have rotational symmetry? How do you know?

c. Save this sketch to your folder.

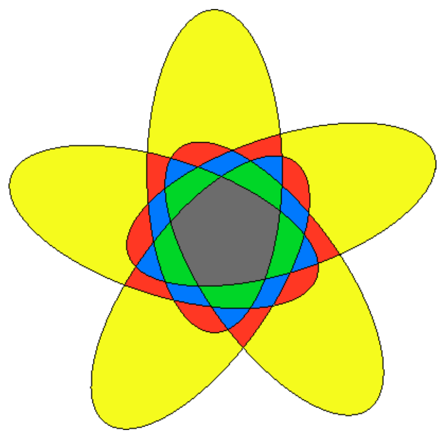
4. Consider the figure below. If you have it on your desktop, import it into Geogebra.

a. Does this figure have an axis of symmetry?

b. Rotate the figure about its center point by 75˚. What do you observe?

c. Rotate it twice more, about its center point by 75˚ each time. What do you observe? Does this figure exhibit rotational symmetry? If so, how many times may you rotate it so that it looks the same, and then its original shape returns to view?

d. If you were able to import the picture, save your Geogebra sketch in your folder.



5. Open a new window in Geogebra. Can you construct a figure that has rotational symmetry, but no axis of symmetry? Save your sketch in your folder.

Student Exploration 1 rubric

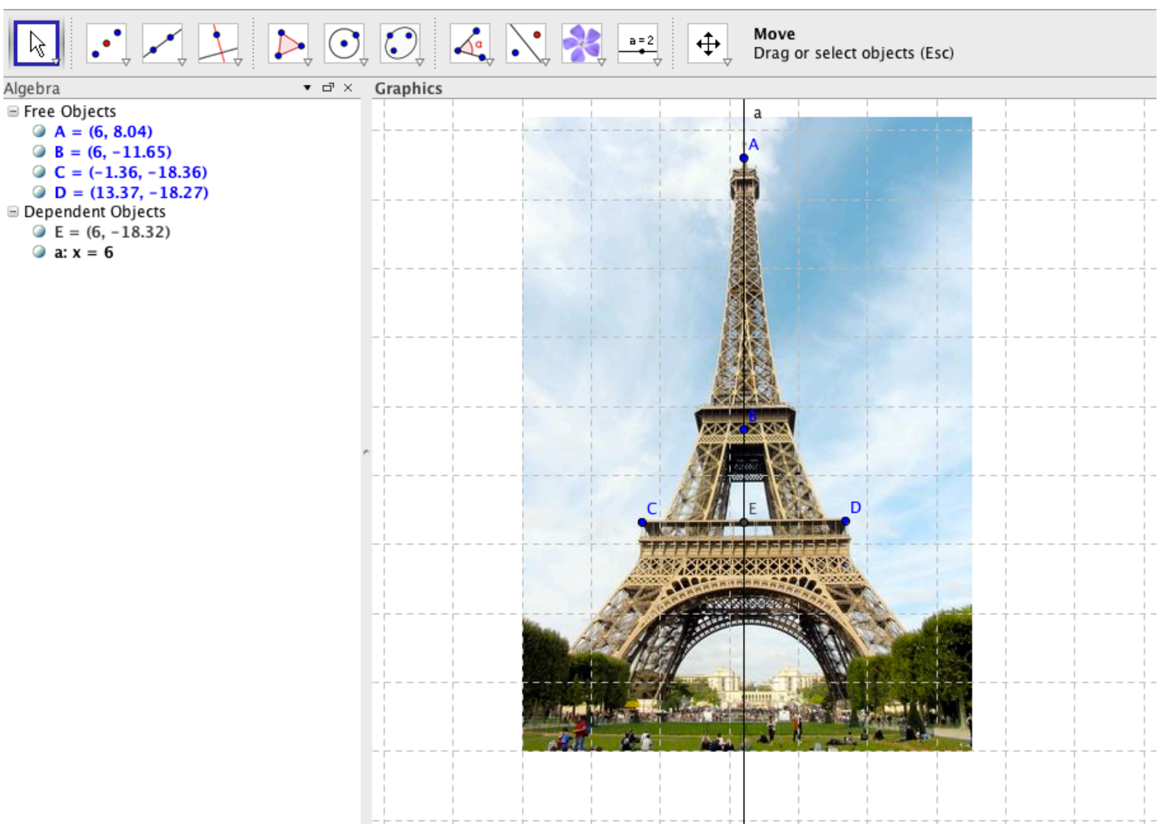
	3 points	2 points	1 points	0 points
Image	Accurately imported	Imported, but not used	Inaccurately imported	Not imported
Lines	Two are constructed accurately	Two which are constructed inaccurately	One constructed	Not constructed
Midpoint	Accurately constructed	Somewhat accurately constructed	Inaccurately constructed	Not constructed
Question on Bilateral Symmetry	Correct, based on sketch	Correct, based on inaccurate sketch	Inaccurate response	No answer

Answers to questions:

	2 points	1 points	0 points
6a.	Complete answer	Partial answer	No answer
6b.	Complete answer	Partial answer	No answer
6c.	Complete answer	Partial answer	No answer
7a.	Complete answer	Partial answer	No answer
7b.	Complete answer	Partial answer	No answer
7c.	Complete answer	Partial answer	No answer
8.	Complete answer	Partial answer	No answer

Student Exploration 1 answer key

Questions 1 – 5: The construction on the Eiffel Tower should resemble the following image. Note that the structure does exhibit bilateral symmetry.

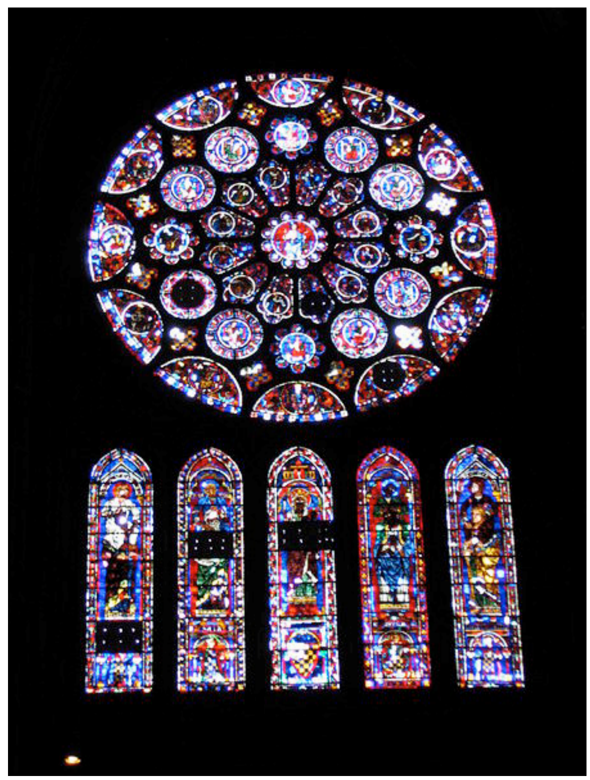


6. Answers may vary, but some are listed below.

a) Aesthetics, intent to impress, structural strength
b) Asymmetry in a structure such as the Eiffel Tower would most likely threaten the integrity of its structural strength.
c) Probably not as impressive, less in tune with the architecture of France, less classical.

7. (see photo below)

a) The building's spires are not symmetric, but the base of the building is symmetric.
b) The base of the building, excluding its spires, exhibits bilateral symmetry. (Note the photo below.) If an axis of symmetry were constructed through the center of the base of the building, it would have perfect bilateral symmetry.
c) The spires were added at different times and represented different perspectives.



8. (Answers will vary) The wall of stained glass windows above exhibits bilateral symmetry, in the sense that a central axis of symmetry would divide the wall into symmetric parts. The circular (rose) portion of the window exhibits rotational symmetry, not in its pictorial detail, but in its basic geometric shapes.

Student Exploration 2 rubric

	2 points	1 points	0 points
2a. Axis of Symmetry	Complete answer	Partial answer	No answer
Perpendicular Bisector	Complete answer	Partial answer	No answer
Construction of other axes of symmetry	Accurate answer	Partial or inaccurate answer	No answer
Number of axes of symmetry	Accurate answer	Partial or inaccurate answer	No answer
3a.	Complete answer	Partial answer	No answer
3b.	Complete answer	Partial answer	No answer
4a.	Complete answer	Partial answer	No answer
4b, c.	Complete answer	Partial answer	No answer

Student Exploration 2 answer key

1. A triangle should be visible in the Geogebra sketch.

2. a. The triangle has an axis of symmetry through each of its vertices.

   b. Three axes of bilateral symmetry, and rotational symmetry.

3. a. It does have bilateral symmetry. There are six axes of symmetry, three through opposite vertices and three through the midpoint of opposite sides.

   b. It does have rotational symmetry. If it is rotated 60˚, the shape of the hexagon is retained. This process can be repeated 6 times.

4. a. It does not have an axis of symmetry.
   b. The figure looks exactly like it began.
   c. It continues to look the way it began and has rotational symmetry. Five times.

5. Answers will vary.

Tessellations

Strand
Geometry

Mathematical Objective(s)
Students will examine tessellations in art and in the architecture of buildings found through Google Earth.

Mathematics Performance Expectation(s)
MPE 3: The student will use pictorial representations, including computer software, constructions, and coordinate methods to solve problems involving symmetry and transformation. This will include:

- Investigating symmetry and determining whether a figure is symmetric with respect to a line or a point;
- Determining whether a figure has been translated, reflected, rotated or dilated using coordinate methods.

Related SOL
G3b: The student will use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation. This will include applying slope to verify and determine whether lines are parallel or perpendicular.

NCTM Standards

- Use various representations to help understand the effects of simple transformations and their composition
- Understand and represent translations, reflections, rotations, and dilations of objects in the plane by using sketches, coordinates, vectors, function notation, and matrices

- Create and use representations to organize, record, and communicate mathematical ideas
- Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture

Materials/Resources

Google Earth, Geogebra 4.0, access to the Internet

Students will learn the meaning of tiling the plane. They will examine various regular polygons to discover which completely tile the plane in a regular tessellation. They will then create tessellations in Geogebra.

Assumption of Prior Knowledge

Students should have completed Lesson 1 in this strand or have previous experience with Geogebra 4.0. They should have a basic understanding of geometry, including parallel lines, congruency, line segment bisection and regular polygons.

Introduction: Setting Up the Mathematical Task

- In this lesson, students will use Geogebra 4.0 to create original tessellations.
- Introduction to tessellations in art, science, and architecture, 10 minutes; brief review of Geogebra, 10 minutes; Student Exploration 1, 30 minutes; Student Exploration 2, 30 minutes; group debrief, 10 minutes.
- Begin by having students review formulas for measuring the interior and exterior angles of regular polygons. Use Student Exploration Sheet 1 to generate the concept of tiling the plane with regular triangles, squares, and hexagons.
- Students will work through Student Exploration Sheet 2, creating a Tessellation.

Student Exploration 1:

Student/Teacher Actions:

- Each student works through Student Exploration 1 Sheet with a partner.
- Teachers circulate through the room to keep students on task, ensure that their sketches satisfy the required directives and help students work through any problems.

Monitoring Student Responses

- Have students print their Geogebra window, or if a printer is not available, have them save the Geogebra window to a folder on their computer. They can email the folder to the teacher upon completion of the lesson.

Student Exploration 2:

Student/Teacher Actions:

- Teachers initiate a classroom discussion concerning ways in which tessellations are found in nature as well as in art and architecture. Teachers may wish to project examples of tessellations in nature, such as pineapples, beehives, and crystal structures, or bring in pictures or even samples of tessellated objects. Examples of tessellations in architecture may be found at http://www.thelck.com/patterns/tenPointStar.html.
  Examples of tessellations may also be found in MC Escher's work.
- Each student works through Student Exploration 2 Sheet using his/her own computer, but regularly compares results with a partner.
- Teachers circulate through the room to keep students on task and help students work through any problems they encounter with Geogebra software. There are likely to be difficulties encountered when students translate polygons to create tessellations, particularly when colors are applied.

Monitoring Student Responses.

- Before students begin the exploration, have them discuss, as a group, what figure is being tessellated in the optical illusion picture. Responses should differ and should range from rhombuses to hexagons divided into three congruent parts.
- Following the exploration, students will return to a classroom group format to discuss their results. Questions such as:
  - Now that you've tiled the plane with regular tessellations, how might you tile the plane with two or more different types of regular polygons?
  - Could you tile the plane with rectangles? Non-regular triangles?
  - How could you know in advance if you can tile the plane with any specific shape?
- Have students print their Geogebra window, or if a printer is not available, have them save the Geogebra window to the previously created folder on their computers. They can email the entire folder to the teacher upon completion of Lesson 2.

Assessment

- Students will either print all constructions or email them to their teacher for printing.
- Each construction is graded for accuracy using the given rubric.

Extensions and Connections (for all students)

- Students may individually research ways in which tessellations appear in nature, art, and architecture.
- Just for fun, have students visit the well-designed NCTM Tessellation generator at http://www.shodor.org/interactivate/activities/Tessellate/. They can create their own tessellations that begin with the three basic regular polygons, and then change them into interesting shapes and colors.

Strategies for Differentiation

- The use of the computer is an advantage for students with processing or memory issues. It also addresses the kinesthetic learning style of many students.

- English language learners (ELLs): materials may be provided in other languages.
- High-ability students may research online to investigate the use of tessellations in art, architecture and science.

Exploration Sheet 1:

Experimenting with Tiling the Plane

A tessellation occurs when regular polygons are repeatedly reflected over a plane to completely cover it with **no gaps or overlaps**. Think of the shapes of honeycombs made by bees, or the skin of a pineapple. The question we will address is: Which regular polygons will tile the plane? Which will not?

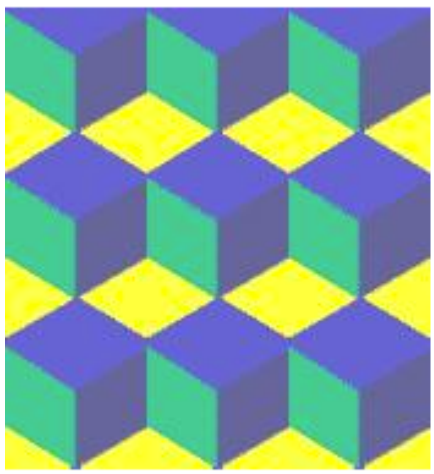
1. Open Geogebra, go to View, and select Axes, which will remove the axes from the window. Construct an equilateral triangle by clicking on the “Polygon” tool arrow, and selecting Regular Polygon. Click two points in the plane; when the window appears, choose “3.” Your triangle should appear.
2. Now, reflect the triangle over each of its sides. Choose the “Reflect Object about Line” icon (4th from the right) in the Geogebra window. Then click on the triangle, choose one of its sides and the triangle will reflect over that side. Repeat with all three sides.
3. Now, reflect the triangle over each of its vertices. Choose, under the same icon as in #2, “Reflect Object about Point.” Click on the triangle, choose one of its vertices and the triangle will reflect over that point. Repeat the reflection process with the newly reflected triangles about the original triangle’s vertices. What do you notice about the tiling? Does it completely fill the plane?
4. Repeat the process above with a square. Does it completely fill the plane?
5. Repeat the same process of reflection over a side with a pentagon, hexagon, heptagon, and octagon, only there is no need to reflect over the vertices in these cases. Which figures tile the plane? Which figures leave “gaps” in the plane? Which figures overlap when they’re reflected about their sides?
6. Create a table with three columns, labeled as follows:

No. of Sides	Measure of each Interior Angle	Does the Polygon Tile the Plane?	How many Polygons are at each Vertex?
3			
4			

5			
6			
7			
8			

7. Refer both to your table and your Geogebra sketch as you complete the table. Examine the columns of those polygons that do tile the plane. What seems to be the connecting factor for those polygons? What is the sum of their interior angles at each vertex?

Exploration Sheet 2: **Now You See It…**



In this exploration, you will use tessellations to make an optical illusion.

The easiest way to create tessellations in Geogebra is to create polygons that translate by a vector, which is a ray that has both length and direction. Follow the instructions below to create the shape of the optical illusion shown above, coloring the various shapes the same or similar colors to the picture shown above.

1. Use the Point tool to create three points, A, B and C. Imagine that you are creating them on the face of a clock, and locate A where the "6" would be located, B where the "8" would be located, and C where "10" would be located.
2. Select the tool "Vector between Two Points," which is under the line tool (3^{rd} from left).

3. Create a vector (in a different section on the screen) by clicking twice, horizontally, left to right. The labels should read D and E. Create a second vector that is perpendicular to the first by clicking on D, and then clicking directly vertically above D. You should have constructed two vectors that form an angle that resembles a right angle. By clicking on the pointer tool, you can move the vector around.
4. Select the tool that is labeled: Translate Object by Vector. Click on point A, and then click on vector DF. Point A' should appear directly above point A. Immediately click on point B, then on vector DE. Repeat with point C. The array of points should resemble the vertices of a hexagon.
5. Select the Polygon tool, which is the 5th icon from the left. Create a hexagon by connecting the six points you created in number 4 above.
6. To create the center of the hexagon, select the Point tool, but scroll down to the Midpoint or Center command. Click on points A and A' to create their midpoint, G.
7. Create three polygons in your figure. Click on the polygon icon and connect A, B', C' & G; A, B, C, & G; and A', C, G, & C'.
8. If you wish, at this time you may color the "block" by double clicking (right click on a PC) on the polygon and selecting Object Properties. Go to the Color tab and select the color you wish. Then go to the Style tab and slide the Opacity bar up past 75%; otherwise, the color will be transparent.
9. Translate the polygons as follows: Reselect the Translate Object Vector tool. Click on polygon AB'C'G, and then click on the horizontal vector DE. Repeat with each individual polygon. Then repeat by using the vertical vector DF. Your tessellation should be complete.
10. Color it as you wish, using the instructions in #8 above.
11. Print or save your sketch in the folder on your desktop.
12. Play around with your tessellation by dragging on either of the ends of your vectors. Does the optical illusion depend on the shapes more than the colors? Or does it depend more on the colors?
13. Move the vectors' ends around to distort your tessellation, and make a completely new tessellation. Is it still an optical illusion, or just a design?
14. For more information on tessellations in art, architecture, or science, refer to the information below.

    ART: M.C. Escher, and Robert Penrose

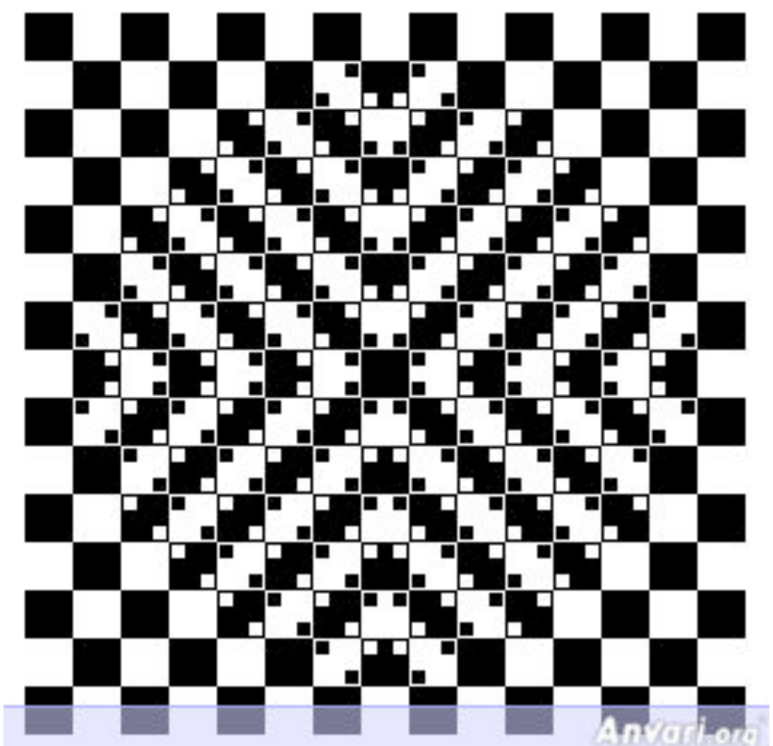
    ARCHITECTURE: Search Google Earth for Buckminster Fuller's geodesic dome building in the Missouri Botanic Garden, the Pyramid in front of the Louvre in

Paris, and tessellations	the MAA headquarters building's foyer in Washington, DC. What kinds of are in each? What other types of tessellations can you find?

SCIENCE: Virology, crystallography, biology.

Just for fun:

Can you reconstruct this optical illusion using tessellations in Geogebra?



Student Exploration 1 rubric

	2 points	1 points	0 points
Triangles	Successful sketch	Attempted	No answer
Squares	Successful sketch	Attempted	No answer
Pentagons	Successful sketch	Attempted	No answer
Hexagons	Successful sketch	Attempted	No answer
Heptagons	Successful sketch	Attempted	No answer
Octagons	Successful sketch	Attempted	No answer

Question 6: Table: .5 points for each blank

Question 7	Accurate answers to both questions: 3 pts	Accurate answer to one questions: 2 pts	Inaccurately answered: 1 pt	No answer: 0 pts

Student Exploration 1 answer key

Questions 1 – 3. Equilateral triangles completely tile the plane. There are no gaps.

Question 4. Squares completely tile the plane. There are no gaps.

Question 5. Regular hexagons can tile the plane, but the others cannot. They either leave large gaps or they overlap.

No. of Sides	Measure of each Interior Angle	Does the Polygon Tile the Plane?	How many Polygons are at each Vertex?

3	60˚	Yes	6
4	90˚	Yes	4
5	108˚	No	-
6	120˚	Yes	3
7	128.57˚	No	-
8	135	No	-

Question 7. Any regular polygon whose interior angles divide evenly into 360˚ will tile the plane. The sum at each set of vertices is 360˚.

<u>Student Exploration 2 rubric</u>

	2 points	1 points	0 points
Hexagon construction	Accurate construction	Partially accurate construction	No construction
Vector construction	Accurate construction	Partially accurate construction	No construction
Successful translation by vector	Accurate construction	Partially accurate construction	No construction
Color Selection	Accurate construction	Partially accurate construction	No construction
Tiling of the plane	Accurate construction	Partially accurate construction	No construction
12a.	Accurate answers	Partial Answers	No answer
12b.	Accurate answers	Partial Answers	No answer
13.	Accurate answers	Partial Answers	No answer

Student Exploration 2 answer key

Questions 1 – 11. The tessellation should resemble the image on the worksheet.

12. The optical illusion will change, depending on how far students drag the vectors. At times the illusion depends on the shapes; at other times, on the colors.

13. It becomes “just a design.”

Guess the change

Strand
Geometry

Mathematical Objective(s)
Students will be given a set of objects from the Google Earth trip and determine how the objects have been translated, reflected, rotated or dilated. Students will use objects from the Google Earth trip and create their own translations, reflections, rotation and dilations for a partner to determine which has been done.

Mathematics Performance Expectation(s)
MPE 3: The student will use pictorial representations, including computer software, constructions, and coordinate methods to solve problems involving symmetry and transformation. This will include:

- Investigating and using formulas for finding distance, midpoint and slope;
- Applying slope to verify and determine whether lines are parallel or perpendicular;
- Investigating symmetry and determining whether a figure is symmetric with respect to a line or a point;
- Determining whether a figure has been translated, reflected, rotated or dilated using coordinate methods.

Expectation(s)
Students will determine if a translation, reflection, rotation or dilation has occurred in different given situations and state how the object has been translated, reflected, rotated or dilated. Students will create their own translations, reflections, rotations and dilations for a partner to determine which has occurred.

Related SOL
G3d: The student will use pictorial representations, including computer software, constructions, and coordinate methods to solve problems involving symmetry and transformation. This will include determining whether a figure has been translated, reflected, rotated or dilated.

NCTM Standards

- Create and use representations to organize, record, and communicate mathematical ideas

- Use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems to analyze geometric situations
- Investigate conjectures and solve problems involving two- and three-dimensional objects represented with Cartesian coordinates
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Materials/Resources
Google Earth trip, access to Geogebra, Student Exploration 1 worksheet
Students will determine if a translation, reflection, rotation or dilation has occurred to the objects given on Student Exploration 1 worksheet.

Assumption of Prior Knowledge
Students should have knowledge of what it means to translate, reflect, rotate or dilate and object and if these things are done around a point or line.

Introduction: Setting Up the Mathematical Task

- In this lesson students will determine if a translation, reflection, rotation or dilation has occurred to given objects. Students will also create their own translations, reflections, rotations and dilations.
- Introduction to utilizing prior knowledge, 10 minutes; description of task, 5 minutes; Student Exploration sheet 1, 35 minutes; Creating own, 30; extension, 10 minutes.
- Have students review translation, reflection, rotation and dilation and which of these occur on a point or on a lines.
- Students will work through Student Exploration sheet 1.
- Students will create their own translations, reflections, rotations and dilations to trade with a partner who will determine which has been done to the objects.

Student Exploration 1:

Student/Teacher Actions:

- Students will use the Student Exploration 1 worksheet and Geogebra to determine if a translation, reflection, rotation or dilation has occurred to the objects and how they have been translated, reflected, rotated or dilated.
- Students will create their own translations, reflections, rotations and dilations and switch with a partner for each other to determine which has been done to the objects.
- The teacher will circulate around the room to guide and observe students as students work through the problems.

- The teacher will give helpful hints and suggestions of how students can create their own translations, reflections, rotations and dilations.
- Students should have the same answers for Students Exploration 1.
- Students will have different creations for their own translations, reflections, rotations and dilations.

Monitoring Student Responses

- Students will return to a classroom group format to discuss the answers for Student Exploration 1 worksheet. The teacher will ask different students to give their answers and explain why they got their answer.
- Students may share their creations and see if the class can determine what was done to the object.

Assessment

- Students will turn in their answers to Student Exploration 1 worksheet.
- Students will turn in the answer key to their own creations of translations, reflections, rotations and dilations.
- Partners will turn in their answers to the created translations, reflections, rotations and dilations as a homework assignment.
- Student Exploration 1 worksheet will be graded for accuracy of student answers using the given answer key.
- Student creations of translations, reflections, rotations and dilations will be grade from given rubric.

Extensions and Connections (for all students)

- The teacher will lead a discussion on translations, reflections, rotations and dilations that can be found in the real world.
- Class can discuss translations, reflections, rotations and dilations with parent functions such as $y = x^2$, $y = x^3$, y = ▯, y = *3*▯, y = ▯▯, y = ln(x) and y = ▯.

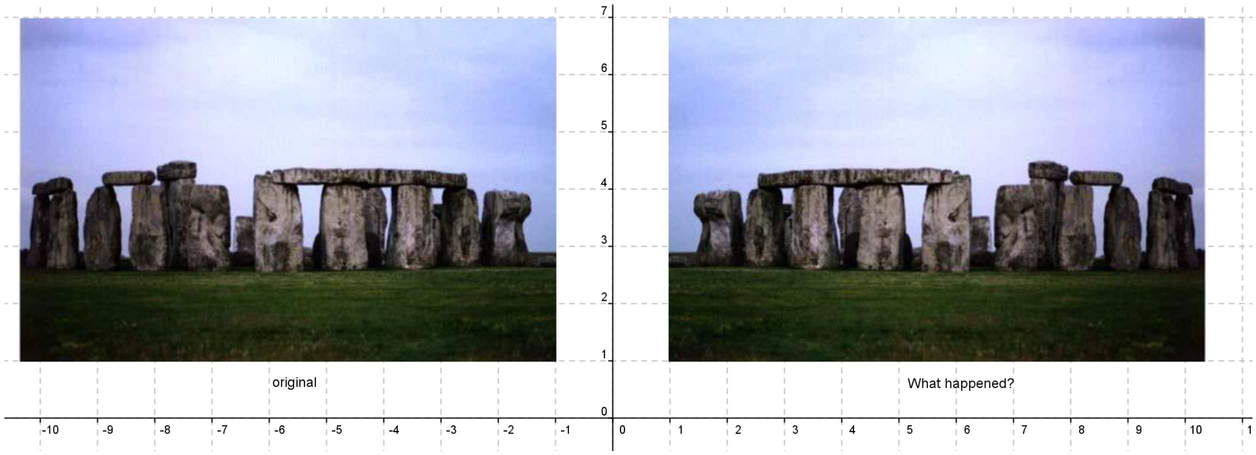
Strategies for Differentiation

- The use of the computer is an advantage for students with processing or memory issues. It also addresses the kinesthetic learning style of many students.

- English language learners (ELLs): materials may be provided in other languages.
- High-ability students will make creations which have at least two of the following: translation, reflection, rotation or dilation.

Student Exploration Sheet 1

1)



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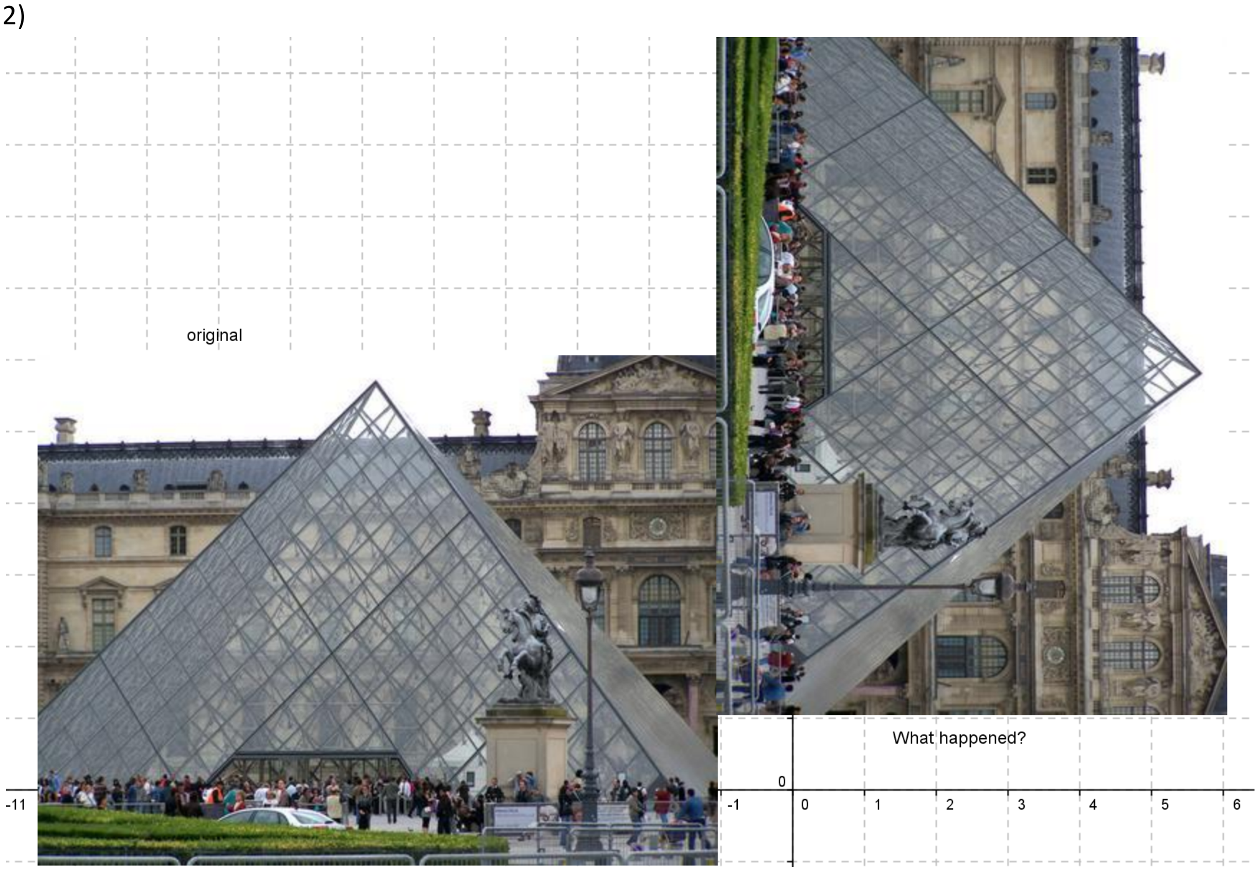

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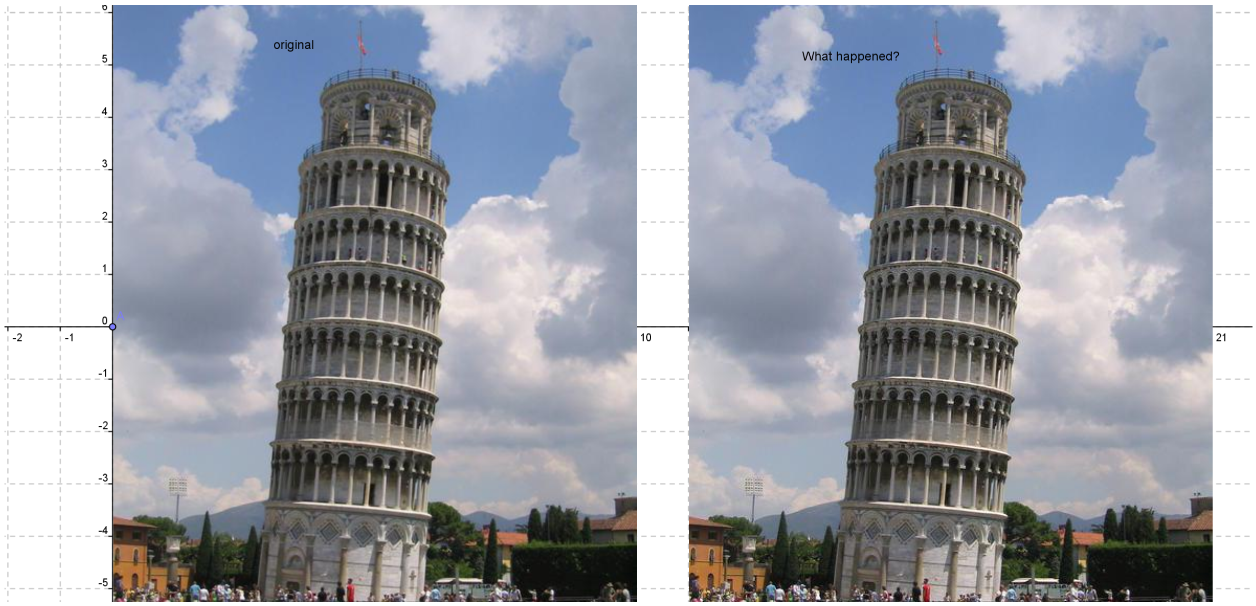
original
What happened?
6
5
4
3
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21

4)


3
original
What happened?
A
-5
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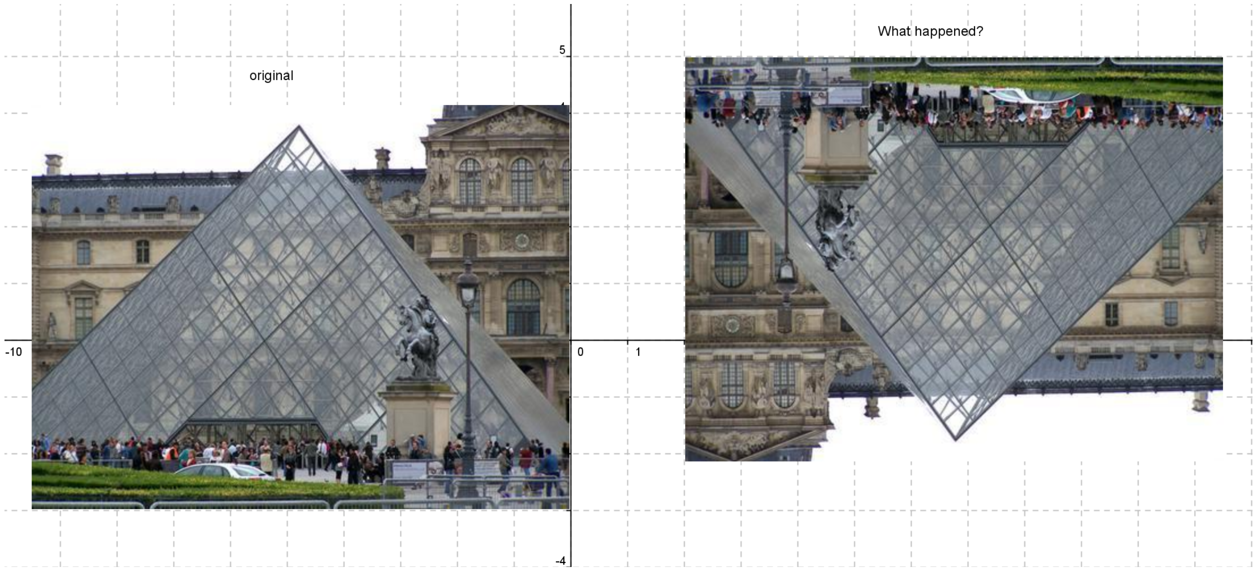
5)


original
What happened?
5
-10
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6)


What happened?
Original
5
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7)


original
What happened?
7
6
5
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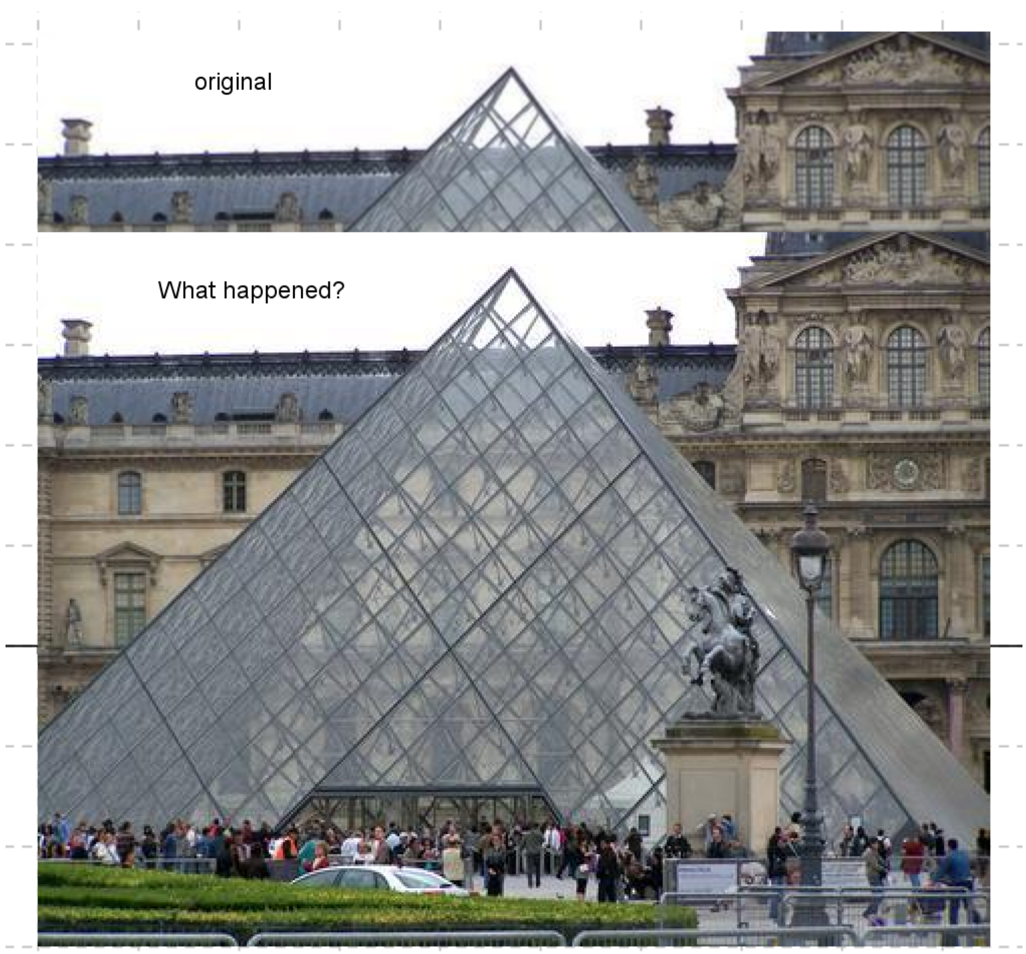
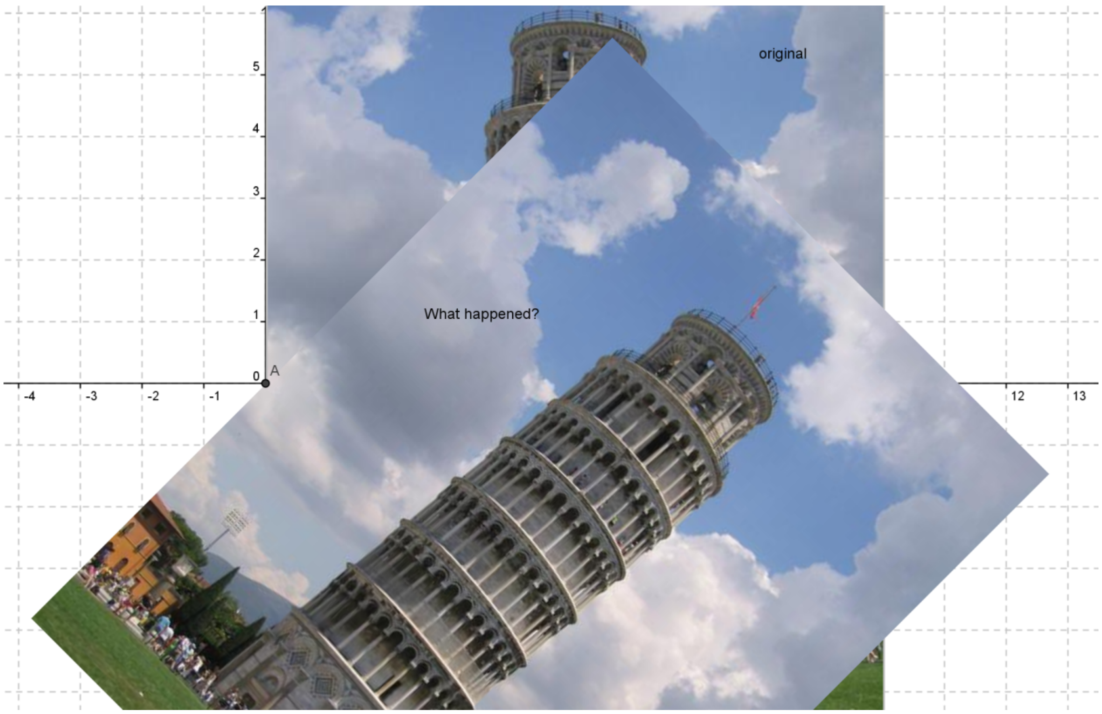
8)


original
What happened?

9)


original
What happened?
A
0
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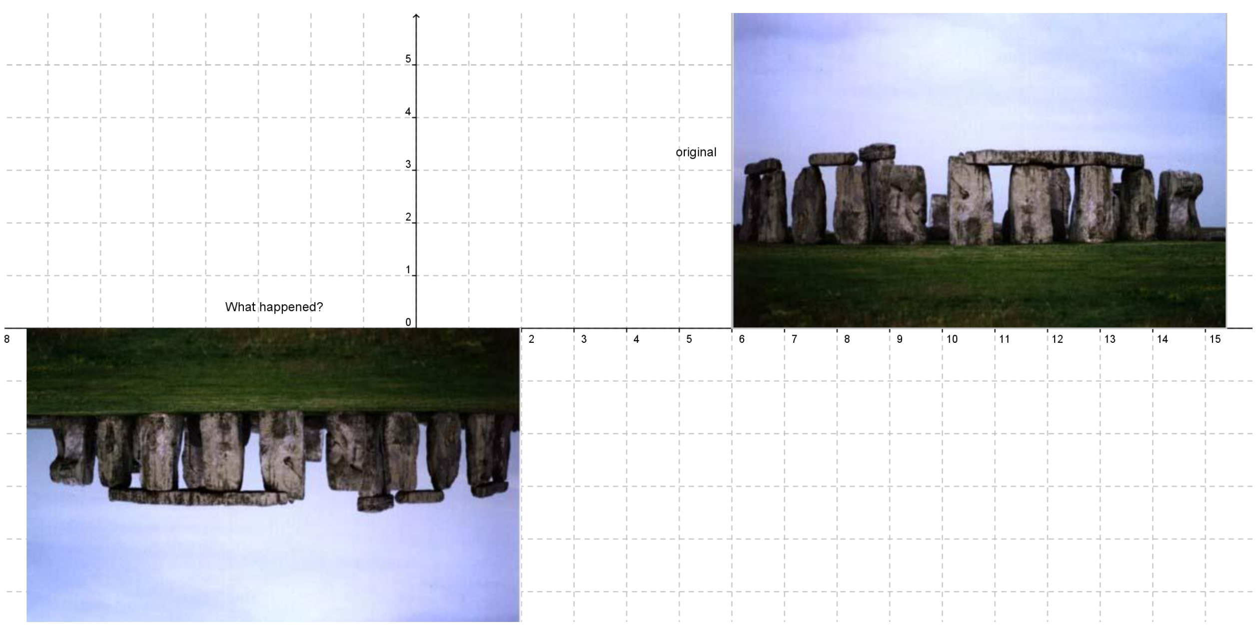
13

10)


original
What happened?
0
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15

Answer Key to Student Exploration 1

1) Object has been reflected about the y-axis
2) Object has been rotated clockwise 90° around the point (0,0)
3) Object has been translated to the right 11 units
4) Object has been dilated by the factor of ½
5) Object has been reflected about the point (1,1)
6) Object has been dilated by a factor of 2
7) Object has been reflected about the point (0,3)
8) Object has been translated down 2 units
9) Object has been rotated clockwise 45° around the point (0,0)
10) Object has been rotated about the point (6,0) then translated 3 units to the left

Each questions from Student Exploration 1 is worth 2 points for a total of 20 points

Rubric for Student creations

	5 – 4 points	3 – 2 points	1 – 0 points
Translation	Students have used all or most of the following: translations, reflections, rotations and dilations in their creations	Students have used some of the following: translations, reflections, rotations and dilations in their creations	Students have used few or none of the following: translations, reflections, rotations and dilations in their creations
Correct answers	Students have given correct answers for all or most of their creations	Students have given correct answers for some of their creations	Students have given correct answers for few or none of their creations
Geogebra use	Students have used Geogebra to correctly make all or most of their pictures	Students have used Geogebra to correctly make some of their pictures	Students have used Geogebra to correctly make few or none of their pictures
Pictures	Students have correctly made at least five pictures	Students have correctly made two or three pictures	Students have correctly made one or no pictures