Google Earth Trip

I. **UNIT OVERVIEW & PURPOSE:**
Students will use pictorial representations of real life objects to investigate geometric formulas, relationships, symmetry and transformations.

II. **UNIT AUTHOR:**
Hannah Holmes, Falling Creek Middle School, Chesterfield County
Sue Jenkins, St. Catherine's School, Private School

III. **COURSE:**
Mathematical Modeling: Capstone Course

IV. **CONTENT STRAND:**
Geometry

V. **OBJECTIVES:**
The students will use computer software to investigate and analyze the properties of real world objects. They will apply basic formulas of coordinate geometry and investigate symmetry, translations, reflections, rotations, dilations and properties of lines.

VI. **MATHEMATICS PERFORMANCE EXPECTATION(s):**
MPE 3: The student will use pictorial representations, including computer software, constructions, and coordinate methods to solve problems involving symmetry and transformation. This will include:

- Investigating and using formulas for finding distance, midpoint and slope;
- Applying slope to verify and determine whether lines are parallel or perpendicular;
- Investigating symmetry and determining whether a figure is symmetric with respect to a line or a point;
- Determining whether a figure has been translated, reflected, rotated or dilated using coordinate methods.

VII. **CONTENT:**
This unit addresses applications of such notions as symmetry and transformations. Students will use geometric formulas to discover significant geometric realities of real world objects.

VIII. **RESOURCE MATERIALS:**
Students will need a computer with Internet capabilities to access software

programs such as Google Earth and Geogebra or Geometer Sketch Pad. Follow school/county procedures to get software downloaded to computers if necessary.

IX. PRIMARY ASSESSMENT STRATEGIES:

Students will create a portfolio comprised of print outs of sketches from geometric exploration software of geometric explorations of real world objects.

X. EVALUATION CRITERIA:

The portfolio will be graded from a rubric describing expectations of material for the portfolio. The expectation grades will be totaled out of 200 points.

XI. INSTRUCTIONAL TIME:

These lessons will require approximately five extended (90 minute) classes.

Parallels and Perpendiculars

Strand
Geometry

Mathematical Objective(s)
Students will verify the presence of parallel and perpendicular lines in real world objects using slope and angle measurements.

Mathematics Performance Expectation(s)
MPE 3: The student will use pictorial representations, including computer software, constructions, and coordinate methods to solve problems involving symmetry and transformation. This will include:

- Investigating and using formulas for finding distance, midpoint and slope;
- Applying slope to verify and determine whether lines are parallel or perpendicular;
- Investigating symmetry and determining whether a figure is symmetric with respect to a line or a point;
- Determining whether a figure has been translated, reflected, rotated or dilated using coordinate methods.

Expectation(s)
Students will use Geogebra as an analytical tool to determine parallel and perpendicular lines of real world objects by finding the slopes of lines superimposed on digital images. Students will find real world objects' digital images through Google Earth and will transfer images of those objects to Geogebra in order to analyze their properties.

Related SOL
G3a: The student will use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation. This will include applying slope to verify and determine whether lines are parallel or perpendicular.

NCTM Standards

- Create and use representations to organize, record, and communicate mathematical ideas
- Use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations

- Investigate conjectures and solve problems involving two- and three-dimensional objects represented with Cartesian coordinates.
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Materials/Resources

Google Earth and Geogebra should be preloaded on computers; students also need calculators.

Following a tutorial on Geogebra, students will use Google Earth to "travel" to various pre-planned locations around the globe. Students will analyze different real world objects by inserting pictures of these objects into Geogebra. They will discover, explore, and verify parallel and perpendicular lines by using Geogebra to find the slopes of lines on the objects.

Assumption of Prior Knowledge

Students should have knowledge of the differences between parallel and perpendicular lines. Students should be able to determine if lines are parallel or perpendicular by comparing slopes of the lines. Students should have knowledge of basic right triangle trigonometry techniques.

Introduction: Setting Up the Mathematical Task

- In this lesson, students will become familiar with Geogebra software as they practice constructing points, lines, parallel lines, perpendicular lines, and angles. Students will use the software to measure lengths of line segments, slopes of lines, and measures of angles.
- Introduction to task utilizing prior knowledge, 10 minutes; introduction to Geogebra software, 30 minutes; Google Earth tour and analysis of digital images, 30 minutes; extension, 20 minutes.
- Begin by having students brainstorm their knowledge of parallel and perpendicular lines in the context of slope. Have them recall specific theorems concerning the construction of a line through a point that is parallel to a another line, the number of lines that can be constructed perpendicular to a line through a point, the formula for slope of a line, and degree measures of angles.
- Students will work through Student Exploration 1 Sheet, Practice With Geogebra.
- Students will work through Student Exploration 2 Sheet, Google Earth Tour.

Student Exploration 1:

Student/Teacher Actions:

- Teachers may wish to project Geogebra and demonstrate various elements of the program before students attempt to use it on their own.
- Each student works through the Student Exploration 1 Sheet using his/her own computer, but regularly compares results with a partner.
- Teachers circulate through the room to keep students on task and help students work through any problems they encounter with Geogebra software.
- Pictorial results of the exploration will vary from student to student. Slope computations by hand should match computations by the software.

Monitoring Student Responses

- Students will return to a classroom group format to discuss the software. Questions such as:
    - What, if anything, did you find confusing about using this software?
    - Are commands logically located?
    - What are the two ways in which you hand-computed the slope of the line?
- Have students print their Geogebra window, or if a printer is not available, have them save the Geogebra window to a folder on their computer. They can email the folder to the teacher upon completion of the lesson.

Student Exploration 2:

Student/Teacher Actions:

- Teachers, or their IT departments if necessary, should load the file "Parallel and Perpendicular" on students' computers in Google Earth.
- Each student works through the Student Exploration 2 Sheet using his/her own computer, but regularly compares results with a partner.
- Teachers circulate through the room to keep students on task and help students work through any problems they encounter with Google Earth or Geogebra software. It is likely that students will encounter difficulties when they find the angle of "lean" of the Leaning Tower of Pisa.

Monitoring Student Responses

- Students will return to a classroom group format to discuss the results of their tours.
- Have students print their Geogebra window, or if a printer is not available, have them save the Geogebra window to the previously created folder on their computers. They can email the entire folder to the teacher upon completion of Lesson 1.

Assessment

- Students will either print all constructions or email them to their teacher for printing.
- Each construction is graded for accuracy using the given rubrics.

Extensions and Connections (for all students)

- Teachers may initiate a classroom discussion concerning the ways in which graphing software can help students validate components of digital photos.
- Students may brainstorm other areas in which digital analysis using Geogebra might be interesting, such as analyzing building movements in Japan during the hurricane, building movements in the wind, etc. An excellent video that students could adapt to this process can be found at http://www.youtube.com/watch?v=sHgIzIE2HrQ.

Strategies for Differentiation

- The use of the computer is an advantage for students with processing or memory issues.
- As a hands-on device, the use of computers helps with the kinesthetic learning style of many students.
- English language learners (ELLs): materials may be provided in other languages.
- High-ability students may research online to discover the rule of thumb for "sway allowance" of tall buildings, i.e. how is the horizontal distance that a building may safely sway computed?

Exploration Sheet 1: Practice with Geogebra

Open Geogebra on your computer, and have your calculator close by.

1. Practice making points:

Click on the point icon, labeled "A," at the top of the window and then click on the graph. Note that the label assigned to the point, along with its coordinates, appears in the window on the left, which is called the "Algebra View." Make four points, A, B, C, and D scattered about the graph.

2. Practice deleting items:

Click on the pointer (arrow button) at left. Click on point D, and delete the point by hitting "delete."

3. Practice making lines:

Click on the icon to the right of the point icon. Recalling that two points are sufficient to construct a line, click on A and B to form a line. (Note that after you select a command icon, instructions for using that command appear on the right of the window.)

** Find the slope of the line using the slope formula. You may use your calculator to calculate the value. Write the formula for slope and show your calculations in the following space.**

4. Practice making line segments:

Click on the small arrow on the lower right-hand corner of the line icon and choose "Segment between Two Points." Then click on points B and C to form a line segment.

5. Practice constructing a line through a point that is perpendicular to a given line: Click on the icon to the right of the line drawing icon. Choose "Perpendicular Line." Following the instructions at right in the window, select point C and line ??. The perpendicular line should appear.

6. Practice constructing a line through a point that is parallel to a given line:

Click on the small arrow on the lower right-hand corner of the same icon as in #5, and select "Parallel Line." Following the instructions at right in the window, select point C and ??. The parallel line should appear.

7. Practice measuring the length of a line segment:

Click on the lower arrow of the icon that features an angle. Choose "Distance or Length." Click on ?? to find its length.

8. Practice measuring angles:

Click on the lower arrow of the same icon as in #7. Choose "Angle." To measure the interior of an angle, click in a clockwise direction on the three points that lie on the rays that form the angle. The measure of the angle, along with a colored angle marker, should appear. (If you click in a counter-clockwise direction, the exterior angle will be measured.)

9. Practice finding slope of a line:

Click on the lower arrow of the same icon as in #7. Choose "Slope." Click on ??. Its slope should appear. Also measure the slope of the line you computed in #6. (Note that placing the pointer over the line will highlight the line you need to use.) Since the lines are parallel, what would you expect their slopes to be? Can you think of one more way to calculate the slope, using the equation of the line that appears in the Algebra View? Compare your results with your partner.

Next, find the slope of the line you constructed in #5 that is perpendicular to ??. Compare their slopes. Since the lines are perpendicular, what would you expect their slopes to be? Discuss with your partner.

10. Save your Geogebra sketch as first initial last name_geogebra_practice (ex. Jsmith_geogebra_practice)

Exploration Sheet 2: Google Earth Tour

1. Open Google Earth on your computer.

2. You should see the tour "Parallel and Perpendicular" under My Places.

3. Visit each site on the tour by double clicking on the blue square with a star next to the name of each place.

4. Return to the Leaning Tower of Pisa. Click on the Blue label at the site to see a picture of the tower. Save the image to your desktop, labeled "Pisa."

5. Open Geogebra. Go to "View," and choose grid, axes, and input bar. You should have a window with a grid, but no visible axes.

6. Click on the small arrow in the corner of the icon that is third from the right, labeled "ABC," on the tool bar. Choose "Insert Image." Following the instructions that appear to the right, click on the grid, and a box should appear with an option to search your computer for a file. Select the Pisa image you saved on your desktop. The digital picture should import to your grid.

7. Select the pointer icon to the left of the toolbar, which will disengage all commands. Drag the picture to a location where any grid line corresponds with the horizontal base of the Leaning Tower, which provides you a horizontal frame of reference.

8. Right-click (or control-click on a Mac) on the picture to find "Object Properties." Click "Background Image" and close. The image should then appear <u>behind</u> the grid lines.

9. Measure the angle of "lean" by following the steps below:

- Construct point A at the base of the Tower, where it meets the grass, on its "taller" side. (Check with your partner, then with your teacher if this is unclear.)
- Construct a horizontal line □□ right at the base of the Tower, where it meets the grass. Use your grid if necessary to adjust points A and B such that line □□ is horizontal.
- Construct a line, b, through point A that is perpendicular to □□. Construct another point C along □□. This will provide a vertical line for comparison.
- Construct two points, D and E, along the left edge of the Tower.
- Construct the line through those two points.

- Find the point of intersection of the two lines by choosing the point icon and dragging to “Intersect Two Objects.” Choose the two lines.
- You now have an angle that you can measure from the vertical perspective. Measure the angle, being sure to click in a clockwise direction.
- Now consider the angle that is made by the Tower and the ground on its “shorter” side. Measure that angle. What is the relationship between that angle and the angle of “lean,” angle EAC?

10. Print your construction. If no printer is available, save the construction to your file.

11. On your own paper, use the angle measurement you computed and right triangle trigonometry to answer the following question: How far (horizontally) does the Tower lean if its taller side’s height is 186.02 ft?

12. Check your answer by constructing a segment on your Geogebra window that completes the triangle between the vertical line segment and the leaning tower and measure its length.

13. Print or save these two constructions. At this point, you have a total of four constructions from Lesson 1. Save file as first initial last name_leaning_tower (ex. Jsmith_leaning_tower)

Student Exploration 1 Rubric

	2 points	1 point	0 points
Line through A, B	Appears on sketch	Partially completed or inaccurately done	Not done/missing
Ling segment through B, C	Appears on sketch	Partially completed or inaccurately done	Not done/missing
Perpendicular line	Appears on sketch	Partially completed or inaccurately done	Not done/missing
Parallel line	Appears on sketch	Partially completed or inaccurately done	Not done/missing
Measure of □□	Appears on sketch	Partially completed or inaccurately done	Not done/missing
Measure of angle	Appears on sketch	Partially completed or inaccurately done	Not done/missing
Slope of □□	Appears on sketch	Partially completed or inaccurately done	Not done/missing
Slope of perpendicular to □□	Appears on sketch	Partially completed or inaccurately done	Not done/missing

Student Exploration 1 answer key

1 and 2. Points A, B, and C should be scattered around the graph.

3. A line should be constructed through A and B.

4. A line segment should be constructed through B and C.

5. A perpendicular line should be constructed through C and perpendicular to □□.

6. A line parallel to □□ should be constructed through point C.

7. A length should be given for □□.

8. Any angle's measurement should be computed.

9. The slope of □□, as well as the slope of the parallel line in #6 should be computed. These values should be equivalent. Second, the slope of the line perpendicular to □□ should be computed. These two values should be negative reciprocals of each other.

Student Exploration 2 rubric

Tower of Pisa, questions 1 – 10

	3 points	2 points	1 points	0 points
Image	Accurately imported	Imported, but not used	Inaccurately imported	Not imported
Lines	All three are constructed accurately	Two of three are constructed accurately	Once accurately constructed	Not constructed or all inaccurate
Angular Measure	Correct angle is chosen and accurately measured	Correct angle, but inaccurate measure	Incorrect angle, but measured accurately	Incorrect angle or not angle or inaccurately measured
Horizontal Measure	Correctly set up and computed accurately	Correctly computed but inaccurately set up	Correctly set up, but inaccurately computed	Not completed

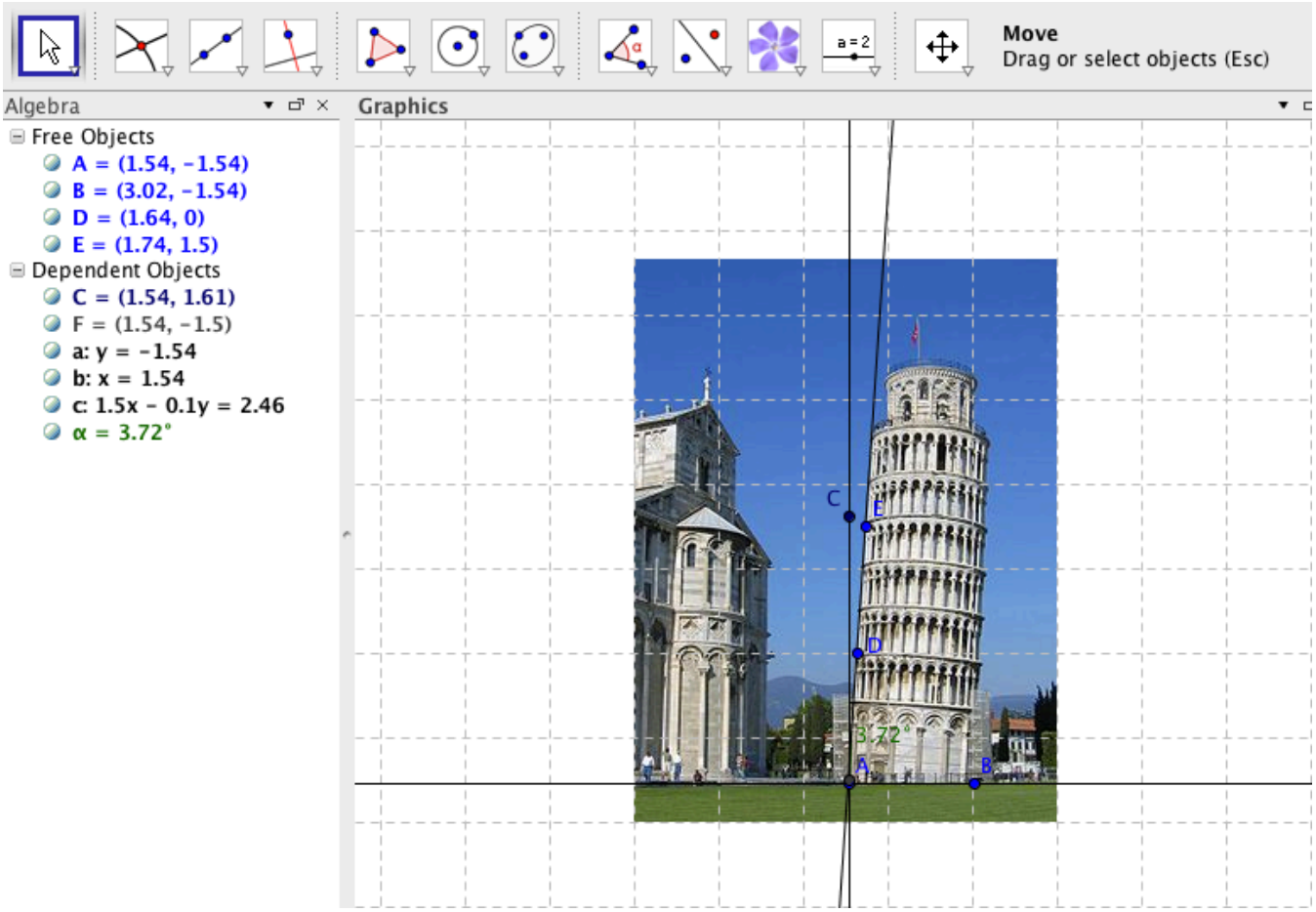
Final two sketches (same rubric for each)

	3 points	2 points	1 point	0 points
Image	Accurately imported	Imported, but not used	Inaccurately imported	Not imported
Parallel/Perpendicular lines	Four are constructed accurately	Two out of three are constructed accurately	One constructed accurately	Not constructed, or all are inaccurate

Student Exploration 2 answer key

The first ten steps of the exploration sheet will result in a sketch that resembles the one below. The actual angle of "lean" at this time is 3.99˚, and students' answers should be between 3.5˚ and 4.5˚.

Question 9, last bullet: The two angles are complementary.



11. Using the correct value for the angle of "lean:" tan 3.99˚ = x/186.02'. x = 12.975 feet.

12. Answers will vary. Geogebra sketches should include pictures in which parallel and perpendicular lines have been identified and their slopes identified.

Take a Little Trip

Strand
Geometry

Mathematical Objective(s)
Students will use the distance and midpoint formulas to find the distances between two places on Google Earth using the coordinates of the locations. Students will use the distance they have found to determine the cost of flying from one location to another. Students will learn to calculate distance in degrees rather than miles or kilometers.

Mathematics Performance Expectation(s)
MPE 3: The student will use pictorial representations, including computer software, constructions, and coordinate methods to solve problems involving symmetry and transformation. This will include:

- Investigating and using formulas for finding distance, midpoint and slope;
- Applying slope to verify and determine whether lines are parallel or perpendicular;
- Investigating symmetry and determining whether a figure is symmetric with respect to a line or a point;
- Determining whether a figure has been translated, reflected, rotated or dilated using coordinate methods.

Expectation(s)
Students will us Google Earth to find the coordinates of different real world locations and use these coordinates to find the distances and midpoints of these locations. Students will use information on airplanes and fuel prices to determine the best route for a trip to each given location on Google Earth.

Related SOL
G3a: The student will use pictorial representations, including computer software, constructions, and coordinate methods to solve problems involving symmetry and transformation. This will include investigating and using formulas for finding distance, midpoint and slope.

NCTM Standards

- Create and use representations to organize, record, and communicate mathematical ideas

- Use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems to analyze geometric situations
- Investigate conjectures and solve problems involving two- and three-dimensional objects represented with Cartesian coordinates
- Investigate conjectures and solve problems involving two- and three-dimensional objects represented with Cartesian coordinates

Materials/Resources

Google Earth, Calculators, Trip planning instructions
Students will use the coordinates of the pre-planned destinations in Google Earth to find the distances and midpoints between each location.

Assumption of Prior Knowledge

Students should have knowledge of the distance and midpoint formulas and how to use these formulas given the coordinates of two points.

Introduction: Setting Up the Mathematical Task

- In this lesson, students will use the distance and midpoint formulas to find the distances between different locations and plan a trip at the lowest cost to the given locations.
- Introduction to utilizing prior knowledge, 10 minutes; description of task, 10 minutes; student exploration and completion of task, 50 minutes; extension, 20 minutes.
- Have students review the distance and midpoint formulas. Have a discussion on how to use the given coordinates on Google Earth in the formulas for distance and midpoint.
- Students will work through Student Exploration 1 Sheet: Take a little trip
- Students will work in pairs. The teacher will assign pairs based on student skills pairing a higher student with a lower student.

Student Exploration 1:

Student/Teacher Actions:

- Students will use the coordinates for each location on Google Earth and given information about fuel prices and airplanes to plan a trip to each location for the lowest cost.
- The teacher will circulate throughout the room to keep students on task and help students work through any problems using Google Earth or with the formulas.

- The teacher can ask students if the price of trips would be lower by flying directly from one location to another, or by stopping halfway to refuel.
- Trip routes and airplanes used will be different for each pair of students.

Monitoring Student Responses

- Students will return to a classroom group format to discuss the different routes discovered and which pair came up with the best price. The teacher will facilitate classroom discussion with such questions as:
    - Which airplanes were most economical when used to fly directly from one location to another? (use any locations from Google Earth trip)
    - Were there any trips that cost less by stopping halfway to refuel?
    - What order of routes were most cost efficient?
- Have students write out the route to their trip giving the cost of each leg of the trip and the total cost of the trip. Students should also demonstrate the use of the distance and midpoint formulas in planning the trip.

Assessment

- Students will turn in the description of their trip showing how they used the distance and midpoint formulas, the route they planned, how much it costs to fly each leg of the trip and how much the trip costs in total.
- Each trip will be graded for correct use of the distance and midpoint formulas using the given rubric.

Extensions and Connections (for all students)

- The teacher will lead the class in a discussion on how airlines use similar information to plan flights across the country and across the world and how this information also affects ticket prices.
- Students will convert latitude and longitude to miles as an interdisciplinary connection to scientific conversions.

Strategies for Differentiation

- Students with processing, memory or motor issues may use Geogebra to help calculate the distance and midpoints between different locations given the coordinates.
- All students may use Geogebra to check their calculations which will help kinesthetic and visual learners.
- High-ability students can look up information on other aircrafts not listed with the given information to see if they can find a better aircraft for different portions of the trip.

- Teachers may choose to pair students based on their skill levels. Pairing lower level students with higher level students may be beneficial to both students.

Take a little trip

A group of friends are interested in taking a trip to go sightseeing around the world. They have contacted you to help them plan the best trip for the best price. Your job is to plan the route the friends will take and which planes are best to take with the lowest cost to the friends. You will be given the price of fuel per gallon, the number of gallons each plane will hold and how far each plane will travel per gallon.

A few things to remember while planning the trip:

- Planes must have a full tank to fly.
- You may stop halfway to the final destination to refuel.
- Planes cannot land on water.
- Assume you are flying directly from one destination to another (don't worry about real airports).
- You will compute the distance from one location to another by converting degrees to miles. There are 69.17 miles per latitude and longitude degree.

Destinations:

- Empire State Building
- Stonehenge
- Louvre
- Leaning Tower of Pisa
- Parthenon

Price of Gas per Gallon = $5.91		
Airplane	Gallons	Miles per gallon
Airbus 300	54,023	6.84
Boeing 747	60,125	7.52
Gulfstream 6	4214	0.91
Challenger 600	2242	1.93
Pilatus PC12	402	4.6
Hawker 4000	2085	1.76
Lear Jet	931	2.75

Questions to help you plan the trip:

- Calculate the distance and midpoints by hand; check your work through Geogebra.
- What is the route the friends will be taking?
- Which planes are used for each section of the trip?
- How much does it cost for fuel for each section of the trip?
- How much is the total cost of the trip?
- Why does your route have the best price?

Rubric for Take a Little Trip Assignment

	10 – 9 points	8 – 7 points	6 – 5 points	4 – 3 points	2 – 1 points	0 points
Use of Formulas	Students have used distance and midpoint formulas correctly for all destinations	Students have used distance and midpoint formulas correctly for most of the destinations	Students have used the distance and midpoint formulas for four of the destinations	Students have used the distance and midpoint formulas for two or three destinations	Students have used the distance and midpoint formulas for one destination	Students have not used distance and midpoint formulas
Destinations	Students have included all destination in their trip	Students have used four or five destinations in their trip	Students have used three destinations in their trip	Students have used two destinations in their trip	Students have used one destination in their trip	Students did not use any destinations
Answered questions	Students have correctly answered all questions	Students have correctly answered five questions	Students have correctly answered three or four questions	Students have correctly answered one or two questions	Students have attempted to answer all questions, but incorrectly	Students do not have a price for their trip
Use of given guidelines	Students have followed all of the given guidelines to plan their trip	Students have followed most of the given guidelines to plan their trip	Students have followed some of the given guidelines to plan their trip	Students have followed few of the given guidelines to plan their trip	Students have followed one of the given guidelines to plan their trip	Students have not followed any of the given guidelines to plan their trip

Correct coordinates for trip

Students should use the following coordinates in the distance and midpoint formulas for each destination. However, each group may have different responses for the questions depending on their specific trip. The coordinates are written, in order, as degrees latitude and longitude.

- Empire State Building – (-73, 40)
- Stonehenge – (-1, 51)
- Louvre – (2, 48)
- Leaning Tower of Pisa – (10, 43)
- Parthenon – (23, 37)