Earthquake Problem

I. UNIT OVERVIEW & PURPOSE:

The purpose of this unit is to use Earthquakes as a starting point to connect some geometry and algebra concepts to real-world applications. These concepts include distance, circle equations, logarithms, and regression equations.

II. UNIT AUTHOR: Gayle Jamison, Salem High School

III. COURSE:

Mathematical Modeling: Capstone Course (the course title might change)

IV. CONTENT STRAND:

Geometry/Algebra

V. OBJECTIVES:

--The student, given the coordinates of the center of a circle and a point on the circle, will write the equation of the circle.

---The student will find the equation of best fit for given data

--The student will calculate distance and intensity given formulas

VI. MATHEMATICS PERFORMANCE EXPECTATION(s):

MPE.21 The student, given the coordinates of the center of a circle and a point on the circle, will write the equation of the circle.

MPE.2 The student will collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

VII. CONTENT:

If an earthquake has its center in Richmond, Va, depending on the magnitude, it will affect the underground utilities in Richmond and surrounding cities. How will the gas company determine where it needs to check its lines? Students will use their knowledge of geometry to calculate distance from Richmond, equations of circles and the conversion of units (degrees to radians and km to miles). They will also investigate the Richter Scale and intensity of earthquakes using logarithms and regression equations.

VIII. REFERENCE/RESOURCE MATERIALS:

--Classroom set of graphing calculators

--access to latitude/longitude converter(http://itouchmap.com/latlong.html)

-- map of Virginia/Eastern United States (maps.google.com)

--distance between two gps coordinates applying the 'Haversine' formula:

$a = \sin^2(\Delta lat/2) + \cos(lat1).\cos(lat2).\sin^2(\Delta long/2)$

$c = 2.\text{atan2}(\sqrt{a}, \sqrt{(1-a)})$

Distance = (radius of earth) * c

radius of the Earth (approximately 6371km)

IX. PRIMARY ASSESSMENT STRATEGIES:

--written paragraph about discussion of the problem
--calculate some actual distances between cities (convert to miles)
--write equations based on given information

X. EVALUATION CRITERIA:
--a rubric will be used to include the main points brought up in the discussion
--look for the use of the distance formula and check gps coordinates
--check equations for given information

XI. INSTRUCTIONAL TIME:
4 days

Earthquake Lesson 1

Strand
Geometry

Mathematical Objective(s)
--Converting degrees to radians

--converting kilometers to miles

--distance formula

--writing the equation of a circle

Mathematics Performance Expectation(s)
MPE.21 The student, given the coordinates of the center of a circle and a point on the circle, will write the equation of the circle.

Related SOL: SOL# G.12. (The student, given the coordinates of the center of a circle and a point on the circle, will write the equation of the circle.)

NCTM Standards

- analyze properties and determine attributes of two- and three-dimensional objects;
- identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships;
- use symbolic expressions, including iterative and recursive forms, to represent relationships arising from various contexts;

- draw reasonable conclusions about a situation being modeled.
- communicate mathematical thinking coherently and clearly to peers, teachers, and others

Materials/Resources

- Classroom set of graphing calculators
- Access to map of Virginia and Eastern United States (maps.google.com)
- Access to latitude/longitude converter(http://itouchmap.com/latlong.html)
- Radius of the Earth is approximately 6371km.

Assumption of Prior Knowledge

- Pythagorean Theorem---distance formula.
- Unit conversion
- Student should have completed geometry
- Students will talk about Earthquakes---magnitude, aftershocks, how far the quake can be felt.
- Students may have problems converting degrees to radians and understanding why it is necessary.
- Distance formula should already be known.
- Circle vocabulary----radius, center, etc, should already be known
- General knowledge of earthquakes

Introduction: Setting Up the Mathematical Task

Suppose an Earthquake occurred whose center was in Richmond, VA. The gas company needs to determine how far out it needs to check for leaks in the lines due to the shift in the earth. If the earthquake has a magnitude of 3.0, then it can be felt for 100km. In this lesson, you will calculate the distances from Richmond, VA to other cities surrounding it. You will also determine if certain cities are within a given radius of the city by using an equation of a circle. Based on where you live, would you have felt this earthquake?

- Small group discussion of earthquakes and what calculations you will need to make (10 minutes)---whole class discussion (5 minutes)
- More whole class discussion on what additional resources needed to solve the problem and where to find those resources (10 minutes)

- How many miles is 100km? What are the coordinates of the latitude and longitude for Richmond?
- Students should work in groups of 3-4 on this activity.
- What cities felt the earthquake? How did you determine this? Mathematically? Geometrically?
- Did the city located at $38°2'$ N $78°29'$W feel the quake?

Student/Teacher Actions:

- Students are calculating the distance between the sets of GPS coordinates
- The teacher is monitoring work and listening for opportunities for discussion about different methods for finding solutions.
- Students can use the distance formula (convert degrees to radians and multiply by the radius of the earth). The given city is 79 miles away so it did not feel the quake.
- To calculate the GPS coordinates that would be within the limits you can write the equation of a circle using Richmond coordinates as the center: $(x-37°32')^2+(y-77°26')^2=100^2\,km$

Monitoring Student Responses

  - students will write what their group did and show work.
  - students will volunteer to share the work of their group
  - teachers will look for frustrated groups and look for math errors such as not converting degrees to radians and forgetting to take the square root in the distance formula.

  An extension would be to change the magnitude of the earthquake to make it necessary to incorporate the curve of the earth in the calculations. Apply the 'Haversine' formula:

  a = $\sin^2(\Delta lat/2) + \cos(lat1).\cos(lat2).\sin^2(\Delta long/2)$

  c = 2.atan2(√a, √(1−a))

  Distance = (radius of earth) * c

  radius of the Earth (approximately 6371km)

- How do you plan to summarize your lesson? Describe it here.
  - Closure is in the student presentations of their solutions.

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Assessment

- Describe and attach the assessments for each lesson objective.
  - **Questions**

- Give GPS coordinates and have students find the equation of the circle where one coordinate is the center and the other is on the circle.
- Give GPS coordinates and have students find the equation of the circle if both coordinates are on a diameter of the circle.
- Explain why it is necessary to change the GPS degree coordinates into radians for this activity.

Strategies for Differentiation

The differentiation strategies might include but are not limited to the following list created specifically for ESL students. Feel free to adopt these to your lesson:

- Make instruction more concrete, visual, collaborative, and hands-on
- Assign roles to students in collaborative activities. Discover the strengths of EOL students and assign appropriate roles.
- Be aware that there might be some differences in communicating the procedural knowledge of mathematics
- Focus on mathematical content rather than on linguistic form (simplify word problems without changing the math meaning)
- Language and content should be presented simultaneously
- Seating (near teacher or next to a buddy, native language if possible)
- Write legibly and in print
- Step by step instructions (orally and in writing) Ask students to repeat aloud for the rest of the class.
- Give EOL student more time for questions and answers.
- Let them discuss in pairs first
- Use of Dictionaries
- Enunciate clearly and slowly without speaking louder.
- Pause between sentences or thought groups.
- Use gestures and visuals to help clarify the message.
- Avoid using idioms and slang words.
- Repeat, rephrase, and paraphrase.
- Simplify the language used rather than the mathematical concepts taught (use known vocabulary and simple sentence constructions).

- When students speak, focus on their message rather than their grammatical skills and accuracy. Respond using the proper grammatical form rather than overtly correcting their mistakes.
- Observe and record students' participation in small group activities.
- Give LEP students (especially beginners) alternate ways to participate in whole-class discussions and respond to questions (think/pair/share, flashcards to raise over head, hand and/or body movements, individual chalkboards for solving computations).
- Assess whether LEP students have mastered mathematical concepts rather than their English grammar and fluency.

The accommodations are adopted from the following source.
http://www.doe.virginia.gov/VDOE/Instruction/ESL/LEPmathResource.pdf

Lesson 2 Earthquake Intensity

Strand
Algebra

Mathematical Objective(s)
--Compare intensities of earthquakes using logarithms

Mathematics Performance Expectation(s)
MPE.2 The student will collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

Related SOL.
SOL # AII.9 The student will collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models.
Mathematical models will include polynomial, exponential, and logarithmic functions.

NCTM Standards

- identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships;
- use symbolic expressions, including iterative and recursive forms, to represent relationships arising from various contexts;
- draw reasonable conclusions about a situation being modeled.
- communicate mathematical thinking coherently and clearly to peers, teachers, and others

Materials/Resources

- Classroom set of graphing calculators

Assumption of Prior Knowledge

- Student should have completed AFDA
- Students will talk about Earthquakes---magnitude, aftershocks, how far the quake can be felt.
- General knowledge of logarithms
- General knowledge of earthquakes

Introduction: Setting Up the Mathematical Task

Suppose an Earthquake occurred whose center was in Richmond, VA. . If the earthquake has a magnitude of 3.0, then it can be felt for 100km. An earthquake has to reach a magnitude of 5 on the Richter scale, how much greater in intensity is the second earthquake compared to the first one? In this lesson, you will compare the intensity of earthquakes.

- Small group discussion of earthquakes and what does intensity mean?
- The Richter Scale is a measurement that can be used to compare earthquake magnitudes based on the formula:

$R = \log\left(\frac{x}{0.001}\right)$ where R = the Richter scale value

x = the intensity (strength) of an earthquake as registered on a seismograph

Note that the Richter value, R, is a logarithm which means that it is an exponent (where the base is 10).

Student/Teacher Actions:

- Students are using the formula to calculate the difference in intensities of the two earthquakes.
- The teacher is monitoring work and listening for opportunities for discussion about different methods for finding solutions (there are 2 methods---a long one and a short one)

The larger one was 100 times more intense.

Monitoring Student Responses

- Describe how you expect:
  - Did a group use the shorter way?
  - Does everyone understand both methods?
  - Were you surprised by your answer? Why or why not?

- How do you plan to summarize your lesson? Describe it here.
  - Closure is in the student presentations of their solutions.

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Assessment

- Describe and attach the assessments for each lesson objective.
  - **Questions**
    - If an earthquake is 25 times more intense, how much larger is the magnitude on the Richter Scale? (1.4)
    - If an earthquake felt in Richmond measures 3.1 on the Richter Scale and the same earthquake measures 3.2 on the Richter Scale in Charlottesville, what is the difference in the intensity felt between the two cities?
  - **Journal/writing prompts**
    - When working with logarithms, do you prefer changing them to exponents or using the laws of logarithms? Explain the difference between these two methods.

Strategies for Differentiation

The differentiation strategies might include but are not limited to the following list created specifically for ESL students. Feel free to adopt these to your lesson:

- Make instruction more concrete, visual, collaborative, and hands-on
- Assign roles to students in collaborative activities. Discover the strengths of EOL students and assign appropriate roles.

- Be aware that there might be some differences in communicating the procedural knowledge of mathematics
- Focus on mathematical content rather than on linguistic form (simplify word problems without changing the math meaning)
- Language and content should be presented simultaneously
- Seating (near teacher or next to a buddy, native language if possible)
- Write legibly and in print
- Step by step instructions (orally and in writing) Ask students to repeat aloud for the rest of the class.
- Give EOL student more time for questions and answers.
- Let them discuss in pairs first
- Use of Dictionaries
- Enunciate clearly and slowly without speaking louder.
- Pause between sentences or thought groups.
- Use gestures and visuals to help clarify the message.
- Avoid using idioms and slang words.
- Repeat, rephrase, and paraphrase.
- Simplify the language used rather than the mathematical concepts taught (use known vocabulary and simple sentence constructions).
- When students speak, focus on their message rather than their grammatical skills and accuracy. Respond using the proper grammatical form rather than overtly correcting their mistakes.
- Observe and record students' participation in small group activities.
- Give LEP students (especially beginners) alternate ways to participate in whole-class discussions and respond to questions (think/pair/share, flashcards to raise over head, hand and/or body movements, individual chalkboards for solving computations).
- Assess whether LEP students have mastered mathematical concepts rather than their English grammar and fluency.

The accommodations are adopted from the following source.
http://www.doe.virginia.gov/VDOE/Instruction/ESL/LEPmathResource.pdf

Lesson 3 Graphing Earthquake Data

Strand
Algebra

Mathematical Objective(s)
--To determine the best regression model for earthquake data

Mathematics Performance Expectation(s)
MPE.2 The student will collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

Related SOL.
SOL # AII.9 The student will collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models.
Mathematical models will include polynomial, exponential, and logarithmic functions.

NCTM Standards

- identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships;
- use symbolic expressions, including iterative and recursive forms, to represent relationships arising from various contexts;
- draw reasonable conclusions about a situation being modeled.
- communicate mathematical thinking coherently and clearly to peers, teachers, and others

Additional Objectives for Student Learning (include if relevant; may not be math-related):
N/A

Materials/Resources

- Classroom set of graphing calculators

Assumption of Prior Knowledge

- Student should have completed AFDA
- Students should know how to use the regression function on the TI graphing calculators

Introduction: Setting Up the Mathematical Task

An earthquake happened nearby and now we are curious about earthquakes.

Here is some data to analyze.

Richter Number	Increase in Magnitude	Approximate Energy Released
1	1	0.00017 metric ton
2	10	0.006 metric ton
3	100	0.179 metric ton
4	1,000	5 metric tons
5	10,000	170 metric tons
6	100,000	5,643 metric tons
7	1,000,000	179,100 metric tons
8	10,000,000	5,643,000 metric ton

Source: http://www.backbenchmedia.com/richter-scale-equation-earthquakes/ converted to metric ton

Student/Teacher Actions:

- Using the Richter scale as the independent variable and the increase in magnitude as the dependent variable, find the regression equations for linear, quadratic, exponential and power. Which is the best fit?
- Using the Richter scale as the independent variable and the energy released as the dependent variable, find the same regression equations as before. Which is the best fit?
- Using the increase in magnitude as the independent variable and the energy released as the dependent variable find the same regression equations. Which is the best fit?
- The teacher will monitor student progress, especially listening for the justification of the line of best fit.

Monitoring Student Responses

- Students should enter the equations and look at the table to find which equation most closely matches the data points.

Closure is in the student presentations of their solutions.

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Assessment

- **Questions**
  - Answers to the first 3 questions.
  - Students can be given other data and find the regression equations.
  - Students can find other data themselves and find the regression equations.

- **Journal/writing prompts**
  - What other real-world data would be good for finding best-fit equations?
  - Explain how you used the graphing calculator to determine which regression equation was the "best" fit. Do you think there could be more than one correct answer?

Strategies for Differentiation

The differentiation strategies might include but are not limited to the following list created specifically for ESL students. Feel free to adopt these to your lesson:

- Make instruction more concrete, visual, collaborative, and hands-on
- Assign roles to students in collaborative activities. Discover the strengths of EOL students and assign appropriate roles.
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- Let them discuss in pairs first
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