Unit: Are Kitchen Cabinets at the Correct Height?

I. UNIT OVERVIEW & PURPOSE:

During this unit students will investigate the many aspects of the Normal Distribution using height data collected from females in the class. In the following six lesson plans students will compare normal data to data that is not normal; use percentiles and the Empirical Rule to make inferences about the height data; normalize the data using z scores; find various probabilities, regarding female height, using the area under the normal curve; discuss the need for the Central Limit Theorem; and apply the normal distribution to confidence intervals and hypothesis testing. By the end of the unit students will understand the "normal" range of female heights and then compare this to the height of kitchen cabinets. Finally, students will use the analysis of heights, and any other form of persuasion, to argue if kitchen cabinets should or should not be placed at a different height.

II. UNIT AUTHORS:

Lauren LaVenture and Brandon Taylor
Lord Botetourt High School
Botetourt County Public Schools

III. COURSE:

Mathematical Modeling: Capstone Course

IV. CONTENT STRAND:

Data Analysis and Probability

V. OBJECTIVES:

The student will:

- study the characteristics of normally distributed data
- analyze percentiles
- normalize data using z-scores
- use the area under the normal curve to find probabilities
- understand the Central Limit Theorem
- apply normal distributions to confidence intervals and hypothesis testing

VI. MATHEMATICS PERFORMANCE EXPECTATION(s):

- MPE 23: The student will analyze the normal distribution. Key concepts include:
  - a) characteristics of normally distributed data;
  - b) percentiles;
  - c) normalizing data, using z-scores; and
  - d) area under the standard normal curve and probability
- MPE 9: The student will design and conduct an experiment/survey. Key concepts include
  - a) sample size;
  - b) sampling technique;

c) controlling sources of bias and experimental error;
d) data collection; and
e) data analysis and reporting

- MPE 22: The student will analyze graphical displays of univariate data, including dotplots, stemplots, and histograms, to identify and describe patterns and departures from patterns, using central tendency, spread, clusters, gaps, and outliers. Appropriate technology will be used to create graphical displays.

VII. CONTENT:

Throughout this unit students will analyze what it means to be of "normal" female height. Students will collect their own data, discuss the data, and eventually find an acceptable range of female heights. Finally, students will use the results of analysis to debate whether the average height placement of kitchen cabinets should be changed.

VIII. REFERENCE/RESOURCE MATERIALS:

- Data collected from students: height, weight and shoe size
- Handouts for investigations (Follows each respective lesson as mentioned for the appropriate lessons)
- Class set of graphing calculators
- Optional: Microsoft Excel

IX. PRIMARY ASSESSMENT STRATEGIES:

Assessments will be in the form of:

- Short quizzes
- Writing – short answer and essay
- Final debate

All specific questions for the assessments are listed under the assessment section of each respective lesson

X. EVALUATION CRITERIA:

- Quizzes are graded on correctness – keys provided for each in the lessons
- Short answer and essay questions are graded on mathematical correctness and the use of certain information in the explanations – keys provided with suggested explanations
- The debate will be assessed on the preparation and argument presented by each group of students. There is no right or wrong side students simply need to use all available information to make and defend their case

XI. INSTRUCTIONAL TIME:

Regular Schedule: about 14 days
Block Schedule: about 7 days

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Lesson 1: Normally Distributed Data

Strand
Data Analysis and Probability

Mathematical Objective(s)
During this lesson students will determine if a data set is a normal distribution by:

- Creating histograms of data sets
- Comparing the shape of the histograms
- Discussing the characteristics of a normal distribution
- Analyzing if a given data set is normally distributed

The specific skill taught in the lesson is recognizing a normal distribution visually from its histogram. The strategy used is comparison between the visual representation of a normal distribution and a non-normal distribution.

Mathematics Performance Expectation(s)
- MPE 23: The student will analyze the normal distribution. Key concepts include:
  a) characteristics of normally distributed data
- MPE 9: The student will design and conduct an experiment/survey. Key concepts include
  d) data collection
- MPE 22: The student will analyze graphical displays of univariate data, including dotplots, stemplots, and **histograms**, to identify and describe patterns and departures from patterns, using central tendency, spread, clusters, gaps, and outliers. Appropriate technology will be used to create graphical displays.

Related SOL

- AII.11 (normal distribution)
- AFDA.7a (characteristics of the normal distribution)
- PS.1 (analyze histograms)
- PS.16 (normal distribution)

All of these SOLs will be addressed in other lessons during this unit except for PS.1. The main focus of this particular lesson is AFDA.7a which is also mentioned in AII.11 and PS.16.

NCTM Standards
- understand histograms, parallel box plots, and scatterplots and use them to display data
- for univariate measurement data, be able to display the distribution, describe its shape, and select and calculate summary statistics
- understand how sample statistics reflect the values of population parameters and use sampling distributions as the basis for informal inference

- organize and consolidate their mathematical thinking through communication
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- recognize and use connections among mathematical ideas
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- recognize and apply mathematics in contexts outside of mathematics
- create and use representations to organize, record, and communicate mathematical ideas
- use representations to model and interpret physical, social, and mathematical phenomena

Materials/Resources

- Classroom set of graphing calculators or software like Microsoft Excel that can create a histogram of given data
- 2 Venn Diagrams per student – attached, following the lesson, in landscape orientation
- Data collected from students

Assumption of Prior Knowledge

- Students should be able to construct a histogram of given data by hand or with a graphing calculator
- Students should be able to compare and contrast the histograms with a focus on shape – this will be expanded to determine if the data set is normally distributed
- Students should be able to discuss the mean, median and mode of the data sets and how these characteristics are shown on the histograms
- Depending on the data sets received from the class, it may be difficult for students to see the differences between the histograms. This can be avoided by providing students with a pre-determined data set that will clearly show the differences between a normal distribution and a non-normal distribution.
- This lesson builds on a student's ability to analyze a histogram and then to use the shape to determine if a data set is normally distributed
- The relevant context is height, weight and shoe size

Introduction: Setting Up the Mathematical Task

- "In this lesson, you will investigate how to determine if a data set is normally distributed by analyzing a histogram of the data"
- Time outline:
  Introduction – 10 min

Exploration – 40 min
Assessment – 20 min

- Introduce the task: Begin by introducing the unit. Ask how many students have trouble reaching items in the kitchen cabinet and discuss briefly.
- How do you think the standard kitchen cabinet height was determined? What considerations should be made?
- How do we determine "average"? Does everyone meet "average" exactly? How can we allow for the variation in a population? What does data distribution mean to you? What is one such of distribution?
- Instructional technique: Think/Pair/Share – students will compare histograms on their own, then discuss their thoughts with a partner and then come together as a class for a final discussion on what makes a data set normally distributed
- To move towards the goal of the lesson students will make independent comparisons and then discuss their thoughts with others
- While making comparisons between the histograms, students will make use of their prior knowledge of graphical shape and the location of the mean, median and mode on the chart
- While students are working through the pair portion of the exploration the teacher should walk around and ask leading questions of students who have not yet considered shape or mean, median and mode
- Students will record the comparisons on a Venn Diagram and will also test their understanding of a normal distribution by determining and justifying if a third set of data is normal or non-normal

Student Exploration:

To begin: ask all female students to record anonymously their height, weight and shoe size on three separate pieces of paper. If the population size is too small to be workable then collect the data from several classes the day before. There is no need to pair the three bits of data to each individual – it is best if there is no connection so students will be comfortable providing the information. If a student does not want to give this personal information then it is ok and this should not be revealed to the class – these students can simply turn in a blank piece of paper.

After gathering the data write the height and weight data sets so students can enter them into their calculator to create the histograms. The teacher may opt for using technology such as TI-Navigators or Microsoft Excel to distribute the data sets to the students by sending to each student's calculator or sharing the Excel document.

Individual Work: Students must first use the graphing calculator, or chosen software, to create the height histogram and weight histogram separately. Students will then begin comparing the two histograms and record their observations in the first Venn diagram (following the end of the lesson – the diagrams are in landscape orientation). As students think they know which data set is normally distributed, they can begin filling out the second Venn diagram.

Small Group Work: After a short time allow students to discuss their comparisons with a partner and they can add to their Venn diagrams.

Whole Class Sharing/Discussion: Discuss the comparisons the students found and create a final Venn diagram for both the comparison of the histograms and the characteristics of a normal versus non-normal distribution.

Student/Teacher Actions:

- Students will be participating in the Think/Pair/Share and completing the Venn diagrams
- Teacher should be roaming the room and asking leading questions to help students complete their comparisons. Also the teacher will need to lead the whole class discussion and make sure the Venn diagrams are completed
- Solutions to Venn diagrams:
  Histograms – height is symmetrical, mean, median, mode are all at or near the peak, equal
  - weight is skewed right, mean > median > mode
  - both have a mounded shape
  Normal vs. Non-normal – normal is bell-shaped/symmetrical, mean = median = mode
  - non-normal is not bell shaped, measures of center are not equal
- Questions: How would you describe the shape of the histogram? Where do you suppose the mean is of this data set? Can you find exactly what the mean, median and mode are of the data sets? Plot these on the histograms and see how they are the same or different.
- Misconceptions or errors: Students are likely to struggle with which way the histogram is skewed – ask which direction the tail is in; this is the direction of the skew.
- The graphing calculators will produce useable histograms, but with computer software it will be easier for students to compare the histograms side by side and they will have more options in blocking the data.

Monitoring Student Responses

- Expectations:
  - Students will record their thoughts and new knowledge on the Venn diagrams
  - Students will listen to each other's thoughts and be willing to discuss their comparisons – students do not need to agree just discuss openly
  - Teacher and students will discuss the differences in the histograms to determine the characteristics of normal

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- Teacher will keep asking leading/probing questions until all students understand the look of a normally distributed data set
    - Teacher can have more advanced students predict how other visual representations of a data set will look for both normal and non-normal cases
- Summarization:
    - The closure of the lesson should be a review of the normal distribution Venn diagram and specifically pinpointing the characteristics of being a normally distributed data set versus those that are not normal. This should be done at the conclusion of the whole group discussion
    - Students will show their understanding of the characteristics of the normal distribution by analyzing the shoe size data and explaining in their own words why they think the data is or is not normally distributed.

Assessment

- Question 1: Using the shoe size data, create a histogram using the graphing calculator and provide a sketch of the graph
- Question 2: Describe the shape of the histogram
- Question 3: Discuss the values of the mean, median and mode and each ones location on the histogram
- Question 4: Is shoe size normally distributed? Explain your reasoning
- Evaluation:
    - Question 1: sketch should show a symmetrical graph
    - Question 2: bell-shaped, mounded, symmetrical (all acceptable)
    - Question 3: mean, median and mode should be approximately equal and should be approximately at the peak of the mound
    - Question 4: Normal because of being symmetrical meaning mean, median and mode are approximately equal

Extensions and Connections (for all students)

- How would a stem-and-leaf plot or box-plot look for a normal and non-normal distribution?
- Is real-life data going to be perfectly normally distributed? Is this ok? What is the line for being non-normal?
- Does the sample size affect the "closeness" to the normal distribution? Central Limit Theorem
- How can you check normality without creating a histogram?

Strategies for Differentiation

- Some students may struggle with viewing the histograms on the calculator or computer – encourage these students to sketch the histograms on paper or graph paper to be more

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accurate. They can also plot the mean, median and mode on the graph to help decipher the relationship

- Some students may struggle with the concept that to be considered a normal distribution the data does not need to be perfectly symmetrical – real life data with rarely fit this definition. To help, discuss the ideas approximation and that statistics is not an exact science and is open to interpretation. This is why being able to clearly justify your reasoning is necessary.

***Venn Diagram Charts located on following 2 pages**

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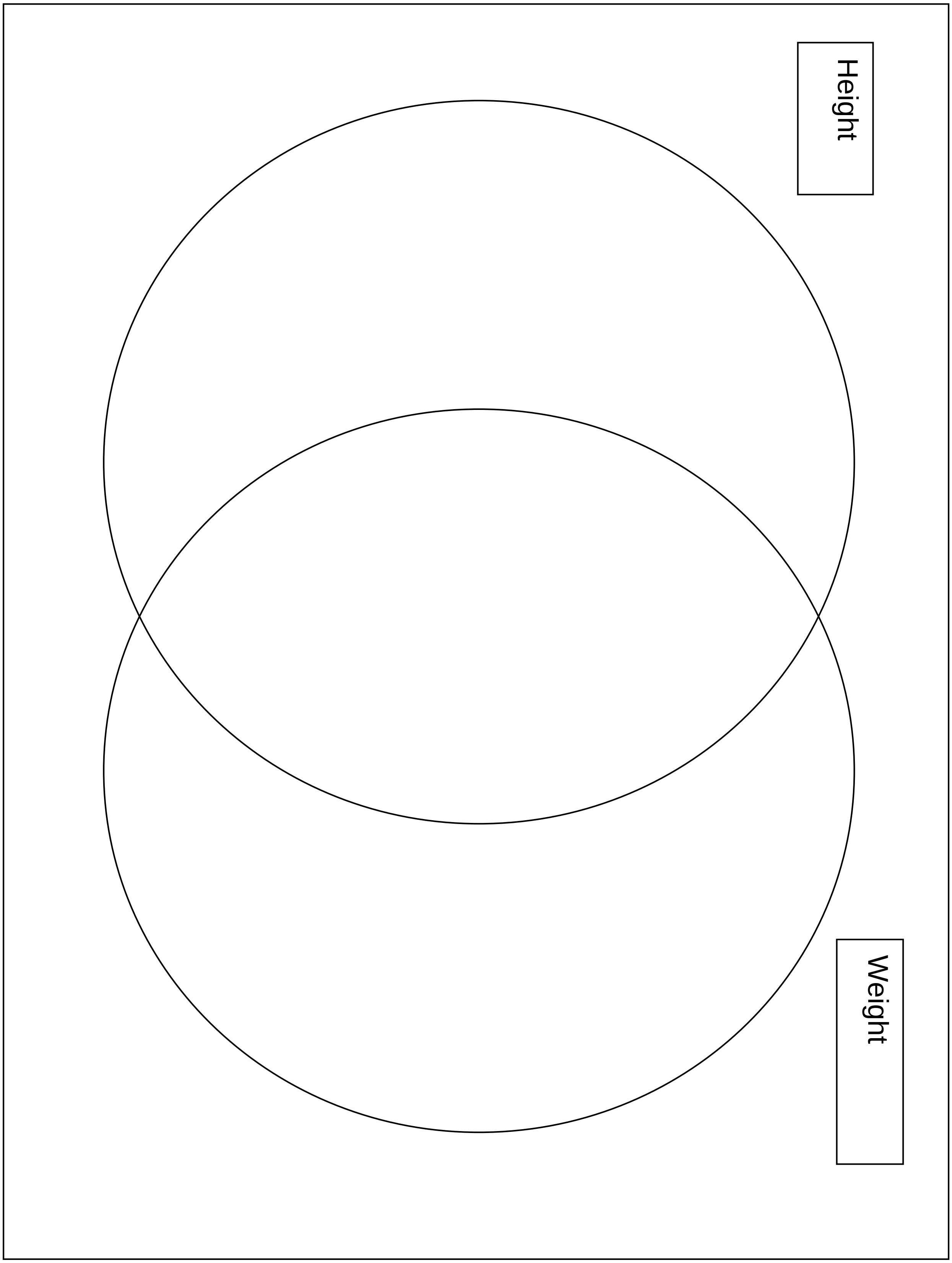
Comparing Height and Weight Histograms



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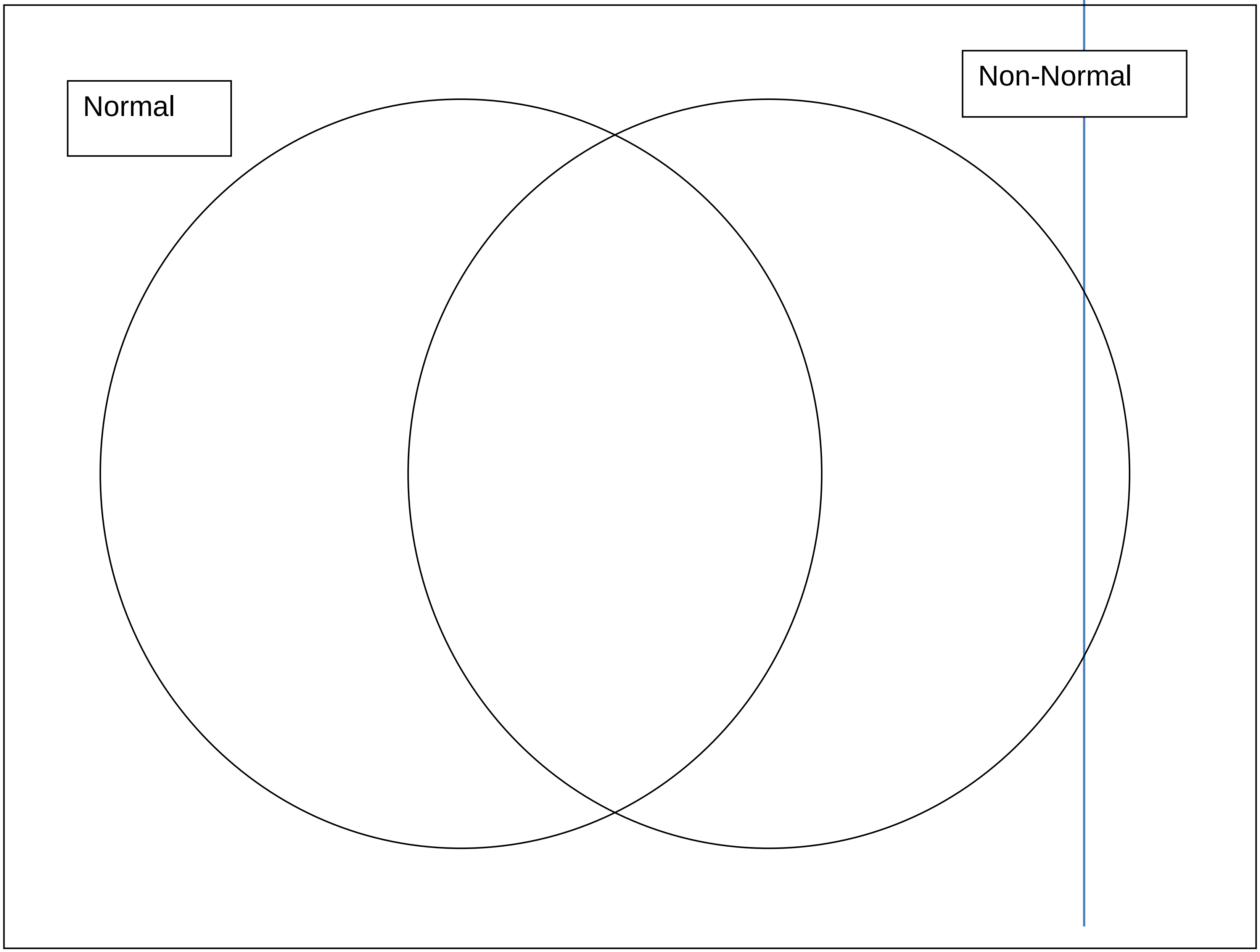
Comparing Normal and Non-Normal Distributions



Lesson 2: Percentiles and Using the Empirical Rule

Strand
Data Analysis and Probability

Mathematical Objective(s)
During this lesson students will determine, given a set of data, the empirical rule for normal curves. In doing so, the student will:

- Create a histogram of the given data
- Use previous knowledge to determine the percentage of data to the left and to the right of the mean
- Given a point in the data, determine the percentile that particular data point lies in and use that to determine the percentage above and below that data point.
- Verify that the empirical rule holds (approximately) for the set of data being discussed.

The specific skill set here is to utilize the empirical rule to make approximations of area and probabilities about women's heights and other real-world data sets. We will also look at the empirical rule as a way to roughly estimate the percentile ranking of a particular point within a given set of data.

Mathematics Performance Expectation(s)

- MPE 23: The student will analyze the normal distribution. Key concepts include:
  a) characteristics of normally distributed data
  b) percentiles
- MPE 22: The student will analyze graphical displays of univariate data, including dotplots, stemplots, and **histograms**, to identify and describe patterns and departures from patterns, using central tendency, spread, clusters, gaps, and outliers. Appropriate technology will be used to create graphical displays.

Related SOL

- AII.11 (normal distribution)
- AFDA.7a (characteristics of the normal distribution)
- PS.1 (analyze histograms)
- PS.16 (normal distribution)

All of these SOLs will be addressed in other lessons during this unit except for PS.1. The main focus of this particular lesson is AFDA.7a which is also mentioned in AII.11 and PS.16.

NCTM Standards

- understand histograms, parallel box plots, and scatterplots and use them to display data

- for univariate measurement data, be able to display the distribution, describe its shape, and select and calculate summary statistics
- understand how sample statistics reflect the values of population parameters and use sampling distributions as the basis for informal inference
- build new mathematical knowledge through problem solving
- solve problems that arise in mathematics and in other contexts
- recognize and use connections among mathematical ideas
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- recognize and apply mathematics in contexts outside of mathematics

Materials/Resources

- Classroom set of graphing calculators or software like Microsoft Excel that can create a histogram of given data
- Data collected from students
- Colored pencils

Assumption of Prior Knowledge

- This lesson assumes all of the prior knowledge in the previous lesson.
- The student should understand properties of the normal distribution to include how the mean and standard deviation determines the shape of the distribution.
- Student knows what a standard deviation is and how to find it numerically and using technology.

Introduction: Setting Up the Mathematical Task

- Looking at the data that we discussed in the last class, what is the expected value of the data set? (i.e. what is the mean of the data set?)
- What is the standard deviation of the data set?
- Assuming that the data is representative of a normal population model, draw a normal curve representing the data that was collected in class.
- Given the mean of this normal curve and picture, what does the mean appear to do to the picture of the distribution itself?
- What does this say about the "amount" of data that is to the left and to the right of the mean?

Student Exploration

To begin: Ask students to list all of the data they have collect on student heights (in inches) and weights (in pounds) in order from least to greatest, vertically on a sheet of paper. Students should know already from yesterday that the heights are approximately normally distributed and weight not so much. Have students draw a line in the data set that represents where the center (in this case we will assume that the center is determined by the mean) is.

Individual Work: After students have completed the above tasks, have student find the standard deviation of the data involving women's heights numerically and verify this result using their TI graphing calculator. Using this result, have students determine the values that would represent one, two, and three standard deviations away from the mean in both directions. Once this has been completed, in the same manner as above, have students mark these places in the data set.

Small Group Work: Have students compare their results for the standard deviation with others in their group and their respective placements of the standard deviations on the data list. Now, as a small group, students should use their data to find the percentage of data points that lie to the right (in our case here, below) one, two , and three standard deviations from the mean. Similarly, they should do the same thing to the right (in our case here, above) one, two, and three standard deviations from the mean. Have students, within their group, discuss their results. Students should find that less than one percent of the data lies above and below the three standard deviation mark, approximately two and half percent lie above and below the two standard deviation mark, and roughly sixteen percent lie above and below one standard deviation of the mean.

Whole Group Discussion: Have small groups give their results to the entire class. The teacher should facilitate this discussion to make sure that all small groups have contributed their answers. Ask students what type of distribution we are assuming that female heights represent. Does it make sense that we obtained roughly the same results on both sides of the mean? Why is this the case? What key concept explored yesterday lends itself to this answer?

Student/Teacher Actions: (Reminder: for this activity we are assuming that the sample mean gathered from the data of female heights is the population mean and that the sample standard deviation derived in this lesson represents the population standard deviation.)

- Have students make a normal distribution based on the "parameters" given above.

At this time it is important to remind students how to draw the standard deviations on their normal curve (please note that this is where the curve changes concavity and can be found relatively easily on their picture). They should label three standard deviations to the left and right of the mean on their paper.

- Now, have students label the various percentages that they found corresponding to one, two and three standard deviations above and below the mean and color the area under the normal curve that corresponds to these percentages.

Questions to ask:

- What is the apparent area between the mean and plus or minus one standard deviation? Represent this value as a percentage.
- Based on this answer, what is the apparent percentage or area between plus and minus one standard deviation from the mean?
- What should the percentage or area to the left and right of the mean be? Why, again, is this the case?
- What should the entire area under the curve be and what is its corresponding percentage?

- Now, have students calculate the area or percentage that lies between plus and minus two and plus and minus three standard deviations. Have students share their results with the class.

Now, this is the teaching moment. Explain to students that what they have found is not odd or abnormal, but actually quite normal for data sets that follow a normal distribution. What they have derived from this data set is the empirical rule for normal curves. That is, approximately 68% of the data lies between plus and minus one standard deviation from the mean, 95% lies within plus or minus two standard deviations from the mean, and 99.7% lies within plus or minus three standard deviations from the mean and that there should be a total of 100% under the entire curve.

- Misconceptions and errors: This empirical rule is not an exact art, it is merely an approximation for data sets that are approximately normally distributed. The percentages achieved by women's heights may not be exactly 68-95-99.7% but should be "close" to those values. This may be a very good time to indicate to them that as the sample gets larger, the more likely you are to get percentages closer to that of the empirical rule.
- At this point it is very relevant to explain the idea of percentiles to your students. That is, using the empirical rule, if you are one standard deviation above the mean, then you are in the 84^{th} percentile. This means that your height is approximately equal to or greater than roughly 84 percent of the individuals in this population (again, we have transformed our sample data into something representative of a population with the

same parameters). Hence there are approximately 16% of the individuals in this populations that are greater that your height.

Monitoring Student Responses: Students will now complete a similar activity with two more data sets: the weights and shoe sizes that were collected in yesterday's lesson.

- Teacher should ask the class whether or not the empirical rule will apply to the distribution of weights for students. Why or why not? Can this be converted into percentiles like we did together for the heights of women?
- Teacher should then ask whether the same could be said for the distribution of shoe sizes. Why or why not? Students may need to be reminded that we concluded that shoe sizes exhibit, roughly, a normal distribution and hence the empirical rule may be applied. Have student calculate the mean and standard deviation of this data set and draw a normal curve that depicts this data. Students should label their axes and indicate what the empirical rule states about this distribution as far as percentages go. Have students verify their calculations numerically by going through the same process for heights that was done earlier.

Assessment

- Question 1: What was the mean and standard deviation of the shoe size data?
- Question 2: Were there any difficulties or issues in drawing a normal curve that depicts that data calculated?
- Question 3: What were the values for one-two-and three standard deviations away from the mean on either side?
- Question 4: Do the numerical calculations computed for areas to the left and right of every standard deviation yield similar results to that of the height distribution?
- Question 5: Does is appear that the empirical rule applies to this distribution (again, assuming a population with equivalent parameters)? Should it?
- Question 6: If you are in the 75 percentile for shoe size, what does this say, specifically? Be as specific as possible.
- Question 7: Think about the height of the cabinets in your kitchen. Do you think that the majority of women are above or below *that* height?

Extensions and Connections (for all students)

- Have you taken the SAT or Pre-SAT? Do you remember your percentile ranking that was given on the score report? If not, make one up. What does that number mean? Does that percentile rank indicate that you are average, above average, or below average? How does this make you feel?

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- Looking at your own shoe size, where are you with respect to the mean? Generally speaking, guys should be significantly higher than the mean for women, so in order to make legitimate comparisons, what might we have to do?
- Do we have enough information, as of right now, to determine which shoes size corresponds to the 75th percentile for women? (The answer to this is yes, but students do not have the skills yet to do the calculation).
- Would it be appropriate to find a percentile for your weight? Why or why not?

Strategies for Differentiation

- If students are having difficulty drawing their own normal distribution to represent their data, have them utilize their histogram that was constructed in class yesterday. Have students mark where the standard deviations would fall on that histogram and then sketch a normal curve about this information.
- Some students may struggle with the concept that to be considered a normal distribution the data does not need to be perfectly symmetrical – real life data with rarely fit this definition. To help, discuss the ideas approximation and that statistics is not an exact science and is open to interpretation. This is why being able to clearly justify your reasoning is necessary.
- If students were unable to calculate percentages from the data, it may be necessary to show them how to calculate those percentages.
- If students do not remember how to calculate a standard deviation by hand, it may be very appropriate to show them the formula, what the symbols mean, and how to derive that computation; of course, emphasis will continue to be on the graphing calculator where appropriate.

Lesson 3: Normalizing Data

Strand
Data Analysis and Probability

Mathematical Objective(s)
Students will:

- Know the characteristics of the Standard Normal Distribution
- Understand the z statistic and its use in normalizing data
- Convert a value from a normally distributed data set to a standard normal z score

The specific goal of this lesson is for students to be able to take a value from a normal distribution and find its corresponding z score in the standard normal distribution. Students will practice this conversion using the height data set collected during lesson one.

Mathematics Performance Expectation(s)
- MPE 23: The student will analyze the normal distribution. Key concepts include:
  c) normalizing data, using z-scores

Related SOL

- AII.11 (normal distribution)
- AFDA.7c (z scores)
- PS.16 (normal distribution)

All of these SOLs will be addressed in other lessons during this unit. The main focus of this particular lesson is AFDA.7c which is also mentioned in AII.11 and PS.16.

NCTM Standards
- understand how sample statistics reflect the values of population parameters and use sampling distributions as the basis for informal inference
- build new mathematical knowledge through problem solving
- solve problems that arise in mathematics and in other contexts
- apply and adapt a variety of appropriate strategies to solve problems
- monitor and reflect on the process of mathematical problem solving
- make and investigate mathematical conjectures
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- analyze and evaluate the mathematical thinking and strategies of others

- use the language of mathematics to express mathematical ideas precisely
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- use representations to model and interpret physical, social, and mathematical phenomena

Materials/Resources

- Classroom set of graphing calculators
- For each student Investigation handout: Z-score Formula Investigation – attached after lesson
- Female height data collected during lesson 1

Assumption of Prior Knowledge

- Going into this lesson students should have a working knowledge of mean and standard deviation and recall the characteristics of the normal distribution and the Empirical Rule from the previous lessons
- Students should have an introductory level understanding of descriptive statistics and of distributions
- When deriving their formula for finding z-scores, students should discuss the shifting of the mean to 0 and scaling of the standard deviation to 1
- Students will have difficulty understanding what a z-score is and how it is related to a normal distribution and the standard normal distribution
- Prior to this lesson students should be familiar with mean and standard deviation, the Empirical Rule and the normal distribution
- This lesson builds on the normal distribution and provides the conversion between the standard normal distribution and any normal distribution
- The relevant context for this lesson is a continuation of the height data collected from female students at the start of the unit. This data will be used to answer the unit question of should the hanging height of kitchen cabinets be changed?

Introduction: Setting Up the Mathematical Task

- "In this lesson, you will investigate the relationship between a normal distribution and the standard normal distribution by finding a formula for conversion"
- Introduction: 10 min
  Exploration 20 min
  Assessment: 10 min
- To introduce the lesson and get students thinking about the task, ask the following questions and discuss as a class:
  When looking at a normal distribution curve, what are some questions a statistician would

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like to answer? Is it possible to compare two normal distributions that have different means and different standard deviations? What does "standardized" mean to you? Where have you heard this word used?

- Students will work in pairs to answer questions on the handout and derive a formula for calculating z-scores
- Students will move towards the objectives by deriving a formula for finding z-scores and then using this formula to convert values from the normally distributed height data to z-scores.
- Students will use their knowledge of standard deviation, the Empirical Rule and the normal distribution to help understand what a z-score is and in deriving their formula
- The teacher will help students understand by walking around the room and talking with the pairs of students to help them work out their formula
- Students will complete a handout during the investigation and will also complete a short quiz on calculating z-scores from the height data

Student Exploration 1:

Small Group Work: Students will work through the investigation handout: Z-score Formula Investigation (attached at end of lesson) in pairs

Student/Teacher Actions:

- Students should be working in pairs answering the questions on the handout and then deriving a formula for converting values in a normal distribution to a z-score
- The teacher should be talking with each pair of students as they work on the handout. The teacher should check the students responses for accuracy and clear up any misunderstandings
- Handout answers: 1) 2 and -2
  2) $-1 \leq z \leq 1$
  3) the student performed better than 99.7% of the students
  4) TBD by teacher – depends on class data.
  5) 0
  6) 1
  7) subtract the height data mean
  8) divide by the height data standard deviation
  9) Formula: $z = \frac{\bar{y}-\mu}{\sigma}$ where $\bar{y}$ is the value being converted, μ is mean of the data set, and σ is the standard deviation of the data set
- To change a normal curve into the standard normal curve, where does the mean need to be? How can you mathematically adjust the mean to be this value? What value should the

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standard deviation be for the standard normal curve? How can you mathematically adjust the standard deviation to this value?

- Students will struggle with finding their formula. Students can think of this formula as a shift of the mean and a scaling of the standard deviation. Other students may want to think in terms of how to get the mean to 0 and standard deviation to 1 – for these students remind them that 0 is the additive identity and 1 is the multiplicative identity.
- Students should be able to use the graphing calculator to find the mean and standard deviation of the height data collected in class during lesson 1.

Monitoring Student Responses

- Expectations:
  - Students will record their answers and formula on the handout
  - Students will listen to each other's thoughts and be willing to discuss their ideas for deriving the z-score formula
  - Teacher and students will discuss the relationship between a normal distribution and the standard normal distribution to determine the z-score formula
  - Teacher will keep asking leading/probing questions until all students have the ability to derive the z-score formula
  - Teacher can have more advanced students work with other student pairs to help them think through the needed formula
- Summarization:
  - The closure of the lesson should be a verification of the z-score formula – all student pairs should have come up with their own formula and these can be written on the board and discussed as a class
  - Students will show their understanding of the purpose of the formula by converting values from the height data to z-scores and then discussing what the z-scores mean

Assessment

- **Questions**
  - 1) What approximate z-score would you expect for a female height of 61"? Explain.
  - 2) Calculate the z-score for 61". Does this meat your expectation?
  - 3) Would you consider a female who stands at a height of 5 ft 8 in as being shorter than normal, normal or taller than normal? Justify your response.
- **Evaluation: answers will vary based on the data set collected**
  - 1) z-score should be between -1 and 1 because 61" is close to the mean and therefore should be less than 1 standard deviation away.

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- 2) $z = \frac{61-\mu}{\sigma}$; plug in values for μ and σ as determined by the data set
- 3) Students should calculate the z-score for 68”. Likely 68” will fall above 2 standard deviations and would be considered taller than normal (taller than more than 95% of the population by using the Empirical Rule)

Extensions and Connections (for all students)

- How are z-scores related to percentiles and the Empirical Rule?
- The Empirical Rule gives the percent of expected values that fall within 1, 2 or 3 standard deviations of the mean. How can z-scores allow you to find these percentages for values that are not at precisely 1, 2 or 3 standard deviations

Strategies for Differentiation

- Students that are visual learners can compare the look of the standard normal distribution to the histogram or normal curve for the height data to help see how to shift and scale to the standard normal curve and thus derive the formula for calculating z-scores.

*** Handout: Z-score Formula Investigation**
Located on the following page

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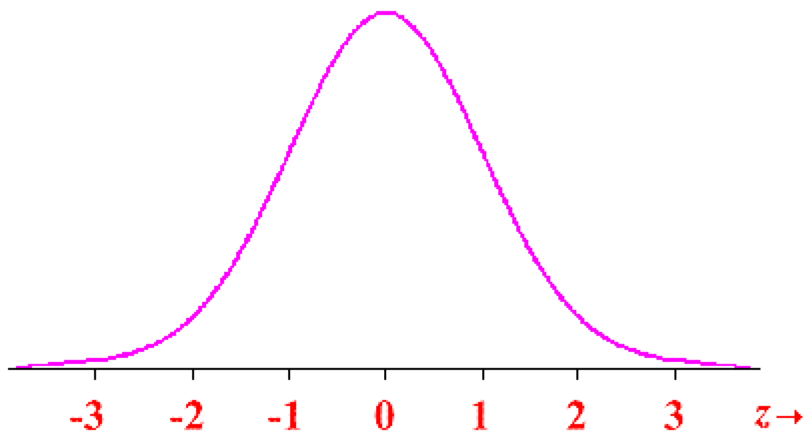
Z-score Formula Investigation

Vocabulary:

Standard Normal Distribution: a normal distribution with a mean of 0 and standard deviation of 1
Z-score: for a standard normal distribution, *z* is the number of standard deviations a value is away from the mean



In the previous lesson you used the Empirical Rule and found that for a normal distribution 68% of the data is within 1 standard deviation of the mean, 95% is within 2 standard deviations and 99.7% is within 3 standard deviations.

Assuming you have a Standard Normal Distribution, answer the following questions:

1. What is the z-score when a value is 2 standard deviations away from the mean?
2. What range of z-scores would you expect for a value that is part of the 68% of the data within 1 standard deviation of the mean?
3. If a student's SAT score has a z-score of 3, how would you describe this student's performance?

How can you convert a value in a normal distribution to a z-score?

With a partner determine the formula for a z-score by considering the following questions:

4. What was the mean and standard deviation of the height data collected in class?
5. To convert to the standard normal distribution, what value must the mean become?
6. To convert to the standard normal distribution, what value must the standard deviation become?
7. How can you mathematically adjust the mean to the needed value?
8. How can you mathematically adjust the standard deviation to the needed value?
9. What is the formula for finding a z-score?

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Lesson 4: Area Under the Curve and Probabilities

Strand

Data Analysis and Probability

Mathematical Objective(s)

During this lesson students will find probabilities associated with the normal distribution by:

- Using diagrams of a normal distribution to help setup and solve probability problems
- Finding the probability that a given value from a normal distribution is below, above or between other given values.
- Calculating a cut score that has a given probability above, below or between the cut score value(s).

The specific skills taught in the lesson are finding probabilities and cut scores for the normal distribution. The strategy used is using the visual representation of the normal distribution to set up the problem and then using graphing calculators to find the values.

Mathematics Performance Expectation(s)

- MPE 23: The student will analyze the normal distribution. Key concepts include:
  d) area under the standard normal curve and probability

Related SOL

- AII.11 (normal distribution)
- AFDA.7d (probabilities and the normal distribution)
- PS.16 (normal distribution)

All of these SOLs will be addressed in other lessons during this unit. The main focus of this particular lesson is AFDA.7d which is also mentioned in AII.11 and PS.16.

NCTM Standards

- understand how sample statistics reflect the values of population parameters and use sampling distributions as the basis for informal inference

Materials/Resources

- Classroom set of graphing calculators
- For each student - Handout: Finding Probabilities and Cut-Scores of a Normal Distribution – attached at end of lesson
- Height data collected from students

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Assumption of Prior Knowledge

- Going into this lesson students should have a working knowledge of probabilities, the characteristics of the normal and standard normal distributions, and z-scores
- Students should have an introductory level understanding of probabilities, including compound probabilities, and of distributions
- Students should discuss if the probability they are referring to is on the right side, left side or between values.
- Students will have difficulty determining if they are looking for an upper-bound, lower-bound or 2 cut-scores for given probabilities. Students will also struggle with which calculator function they need to use: normalcdf or invNorm
- Prior to this lesson students should be familiar with the normal distribution and probabilities
- This lesson builds on the normal distribution and allows students to use the distribution to find probabilities and cut scores
- The relevant context for this lesson is a continuation of the height data collected from female students at the start of the unit. This data will be used to answer the unit question of should the hanging height of kitchen cabinets be changed?

Introduction: Setting Up the Mathematical Task

- "In this lesson, you will investigate how to use the normal distribution to find probabilities and cut-scores"
- Introduction: 10 min
Investigation: 30 min
Assessment: 20 min
- To introduce the lesson and get students thinking about the task, ask the following questions and discuss as a class:
To answer our unit question about the height of kitchen cabinets, what do we need to know? Why did we only collect height data on females? What would happen to our data if it included an equal number of heights from males and females? How would this affect the normal distribution? How can we use our normal distribution to find the normal or average range of heights for females?
- Students will work in pairs to answer questions on the handout and use functions on the graphing calculator to find probabilities and cut-scores
- Students will move towards the objectives by visually representing the problems they are solving with a sketch of the normal distribution, and practicing the calculator functions of normalcdf and invNorm
- Students will use their knowledge of probabilities, percentiles and the Empirical Rule, and the normal distribution to help setup and solve the probability questions

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- The teacher will help students understand by walking around the room and talking with the pairs of students to help them setup the problems, determine which of the two functions they need to use, and checking answers
- Students will complete a handout during the investigation and will also answer 3 short answer questions on calculating probabilities and cut-scores in relation to kitchen cabinet height

Student Exploration 1:

Small Group Work: In pairs students will answer the questions on the attached handout:

Finding Probabilities and Cut-Scores of a Normal Distribution

Student/Teacher Actions:

- Students should be working in pairs answering the questions on the handout by drawing sketches of the normal distribution and using the calculator functions normalcdf and invNorm
- The teacher should be talking with each pair of students as they work on the handout. The teacher should check the students responses for accuracy and clear up any misunderstandings
- Handout answers: 1) .726
  2) .274
  3) There sum is 1 which makes sense since a value must be either more or less than 10 (remember cumulative function will include 10)
  4) Empirical Rule says 68% is within 1 standard deviation which would be between 2 and 12, between 6 and 10 will be less than half of 68%
  5) .305
  6) 3.628
  7) 190.25
  8) between 169.75 and 190.25

- For students who are struggling with the handout, the teacher may want to ask the following questions to help students through their struggle:
  - What are you given, value or probability?
  - Which do you want to find?
  - Where on your sketch is your given value?
  - Which side of this value do you want to find the probability for?
  - If you are looking for a probability, do you want to use the normal cdf function or the inverse norm function?
  - Where on your sketch do you expect your cutoff value to be?

    - Shade the area represented by the given probability. Is this probability above, below or between the value(s) you need to find?
- Students will struggle with understanding which side of the normal distribution they are looking at (lower or upper) – to help with this students should draw a sketch of the distribution, shade the area they are working with and then answer the question, also students should check that their answer makes sense in the context of the data set and their sketch. Students will also struggle with which of the two calculator functions they should use – to help with this students should first consider what information they are given and what they are looking for; ex) the normal cdf will give a probability answer if this is not what the question is looking for then students should be able to recognize that the calculator decimal answer is not the value they need.
- Students should be able to use the graphing calculator to find both probabilities and cut-scores. It is possible to use the z-table instead of the calculator and if there is time or if a student wants a more in depth understanding, the table method can also be taught.

Monitoring Student Responses

- Expectations:
    - Students will record their answers a on the handout
    - Students will listen to each other's thoughts and be willing to discuss their ideas for setting up the problems and deciding which function to use
    - Teacher and students will discuss how to make the sketch for each situation and which function should be used
    - Teacher will keep asking leading/probing questions until all students are able to successfully answer the questions on the handout
    - Teacher can have more advanced students try some problems by using the z-table instead of the calculator. This will require the students to use the z-score conversion formula and will give a more in-depth understanding of what they are finding in the problems.
- Summarization:
    - The closure of the lesson should be a check of the answers to the handout and revisit the introductory discussion questions to lead into the assessment short answer questions
    - Students will show their understanding of calculating probabilities and cut-scores by answering three short answer questions. These questions will also get students thinking about the overall unit question of whether the height kitchen cabinets should be changed.

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Assessment

- **Journal/writing prompts**
  - What is the probability that a female stands at a height of 6 feet or taller? How would you find the lower bound cut-score that yields this same probability? What is this cut-score?
  - If we define normal height as the 50% of female heights centered at the mean, what is the range of normal female height? Explain your process.
  - The standard hanging height for kitchen cabinets is 60" above the floor to the bottom of the cabinets and most cabinets are 30" to 42" tall. Assume the average female can reach 2 feet above her head. Do you believe the hanging height of kitchen cabinets is acceptable? Explain. Is there other information you need to make your decision? Why do you suppose kitchen cabinets are hung at this standard height?
- Evaluation:
  - Probability should be quite small – exact answer depends on the data. To find the lower bound score students should draw a normal distribution, shade the lower tail and label this as the percentage found previously, finally calculate the cut-score using the calculator: invNorm(area, μ, σ)
  - First draw a normal distribution, shade the middle and label as 50%. Students should note that they need to find two cut scores with the first being the bottom 25% and the second being at 75%. The calculator functions are: invNorm(.25, μ, σ) and invNorm(.75, μ, σ)
  - Students should use the range found in the previous question as part of their explanation for why the cabinets are at a correct height or not. Students may also make arguments based on counter height and the amount of available space in a kitchen. Students may also mention that if male heights are also considered, then the normal human height range will be taller.

Extensions and Connections (for all students)

- Have students find the probabilities and cut scores using the z-table instead of the graphing calculator functions. This will give students a more in depth understanding of how the probabilities are found and will emphasize the importance of the standard normal distribution.
- Discuss the assumption made in question 2 of the assessment that "normal" height is the middle 50% of the data. Lead this into a conversation about confidence intervals.

Strategies for Differentiation

- Teacher can have more advanced students try some problems by using the z-table instead of the calculator. This will require the students to use the z-score conversion formula and will give a more in-depth understanding of what they are finding in the problems.

***Handout: Finding Probabilities and Cut-Scores of a Normal Distribution**
on following page

Finding Probabilities and Cut-Scores of a Normal Distribution:

Probabilities: you can find a lower or upper probability for a given value in the distribution, or the probability of being between two given values.

Process:

1. Sketch a normal curve. Approximate where your given value is and shade the area under the curve that represents the probability you are looking for.
2. Use the normal cdf function on the calculator to find the probability.
   Enter: lower bound, upper bound, μ, σ
   If the lower bound is negative infinity, then enter -10^{99}
   If the upper bound is positive infinity, then enter 10^{99}

Practice Problems:

1. Find the probability that a value in a normal distribution is less than 10 with , $\mu = 7, \sigma = 5$
2. Find the probability that a value in a normal distribution is more than 10 with $\mu = 7, \sigma = 5$
3. Do your answers for questions 1 and 2 make sense? Explain.
4. Predict the probability that a value in this same distribution is between 6 and 10. Use the Empirical Rule to help. Explain your reasoning.
5. Find the probability that a value in a normal distribution is between 6 and 10 with $\mu = 7, \sigma = 5$. How accurate was your prediction? If your prediction was off, can you now explain this probability?

Cut-scores: When you are given a probability you can find the value in the distribution that yields that probability. These problems are essentially backwards or inverses of the previous questions.

Process:

1. Sketch a normal curve. Approximate where your cut score is and shade the area under the curve that represents the given probability
2. Use the invNorm function on the calculator to find the cut score.
   Enter: area (probability), μ, σ
   The area entered must be to the left of the cut score

Practice Problems:

6. Find the cut score in a normal distribution with $\mu = 7, \sigma = 5$ that gives a 25% lower probability
7. In a normal distribution with $\mu = 180, \sigma = 8$, find the cut score to be in the top 10%
8. Using the same distribution as question 7, what are the cut-scores to yield an 80% probability centered at the mean?

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Lesson 5: Confidence and Tests for Inference

Strand
Data Analysis and Probability

Mathematical Objective(s)
During this lesson students will…

- Estimate an interval for a given parameter (namely the mean for a population).
- Discuss and understand the meaning of the p-value of a test of significance.
- Use a confidence interval to determine whether sample is statistically significant.
- Interpret the meaning of a test of significance in the context of the original problem.

The specific skills taught in the lesson are finding the endpoints of a confidence interval for a given parameter using the calculator, determining the p-value of a test by finding the critical value for a set of data and finding the tail area under the curve, and being able to state conclusions of the test in the context of the original problem.

Mathematics Performance Expectation(s)

- MPE 23: The student will analyze the normal distribution. Key concepts include:
  a) characteristics of normally distributed data;
  b) percentiles;
  c) normalizing data, using z-scores; and
  d) area under the standard normal curve and probability

Related SOL

- AII.11 (normal distribution)
- AFDA.7d (probabilities and the normal distribution)
- PS.16 (normal distribution)
- PS.18 (confidence intervals)
- PS.19 (test of significance)
- PS.20 (sampling distribution and the Central Limit Theorem)

All of these SOLs will be addressed in other lessons during this unit. The main focus of this particular lesson is AFDA.7d which is also mentioned in AII.11 and PS.16, 18, 19, and 20.

NCTM Standards

- understand how sample statistics reflect the values of population parameters and use sampling distributions as the basis for informal inference

Materials/Resources

- Classroom set of graphing calculators
- Height data collected from students

Assumption of Prior Knowledge

- Going into this lesson students should have a working knowledge of probabilities, the characteristics of the normal and standard normal distributions, and z-scores
- Students should have an introductory level understanding of probabilities, including compound probabilities, and of distributions
- Prior to this lesson students should be familiar with the normal distribution and probabilities
- This lesson builds on the normal distribution and allows students to use the distribution to find probabilities and cut scores
- Students should have some prior knowledge on the basic concept behind a confidence interval and what it means to measure with confidence
- Students should have some idea of what a test of significance does and what it can tell you
- Students will have difficulty determining which calculator function to use and when to use it
- The relevant context for this lesson is a continuation of the height data collected from female students at the start of the unit. This data will be used to answer the unit question of should the hanging height of kitchen cabinets be changed?

Introduction: Setting Up the Mathematical Task

- What does it mean to measure with confidence? When we say that we have confidence in our answer or our response to a question, what does that mean?
- If you were to take an average of everyone's age in this class, between what two values would you say that that average lies? How confident are you in those endpoints?
- If we were to ask a similar question about the heights of female students in this class, between what two heights would you say that the average likely lies in?
- Let's say that we know the average height for cabinet placement in American houses is 72 inches tall. Let's say that the average height for a sample of 25 women is 64 inches. Do you think that this sample data is significant enough to suggest that the cabinet placement in American homes should be changed to something other than 72 inches? Why or why not?

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Student Exploration

To begin: Ask students to get the data out that was collected at the beginning of the unit involving weight. If they do not have it already, have students find the sample mean of that data.

Discussion Questions:

- Based on previous work in this area, what does it mean to be a sample?
- Being that we found a sample mean, to what population parameter is this sample statistic estimating?
- Does this sample mean have to be the same as the population parameter in question?
- Does this statistic give us a chance to estimate with confidence what the true population parameter may be?

At this time, we will construct a confidence interval for the population parameter that we are interested in (the mean weight of American high schoolers). There are two ways to do this using the TI-83/84 series calculators. The first is to input the actual numeric data into a list and then doing the T-Interval (we are assuming that the standard deviation is unknown) and the second is by simply inputting the data directly (the sample mean and sample standard deviation) into the T-Interval function in the calculator.

This would be a fantastic time to discuss what confidence means. To what degree of confidence do we want to report our response with? What does this say about the size of the interval? If I want to be 100% confident that the true mean weight of high schoolers is between *a* and *b*, then how large would our interval be? If we wanted to be less confident in our interval estimate, what would that conceivably do to the size of the interval?

Now, have students test these ideas by using different confidence levels for this set of data.

Compute the 80% CI, 90% CI, 95% CI, 99% CI, and a 100% CI. This would also be a good time to explain why we are using the T-Interval as opposed to the Z-Interval. Since this is not a formal probability and statistics class, we will ignore the robust nature of the test and make the assumption that we are, in fact, sampling from a population that carries a normal distribution.

Individual Work: After students have completed the above tasks, have students complete a series of similar tests using the heights of female students in class. Remind them once again of what the confidence interval is estimating. It is very important that students really grasp an understanding for the difference between a statistic and a parameter. Again, have students construct the different confidence intervals as they did in the above activity. Are their similar results obtained in terms of the size of the interval when the confidence level is changed?

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- Consider once again that the average cabinet height in American homes is 72 inches. Did any of the confidence intervals in the above activity contain this value? If so, what does this indicate? If not, what could this tell you? Have students make a hypothesis about what it means to have the parameter value given to us contained in the confidence interval.

Small Group Work: We will now work briefly on a test of significance for the height of women.

- What does it mean, in statistics, to be significant?
- Remind students that a test of significance uses a sample from a normally distributed population and, assuming that the value of the null hypothesis is true, determines the probability of getting data as extreme as what we obtained in the sample.
- For purposes of this class, we will not formally define what a null and alternative hypothesis is.
- Remind students that since we do not know what the population standard deviation is, then we will have to use a T-Test (the second option on the Stat-Tests menu.
- Have students go through and input the appropriate data for this test as requested by the calculator. This is a good time to show them how to once again input data and just the statistics for the test. Please indicate to students that we will use the value of the null hypothesis ($\mu_0 = 72$) and we will conduct a two sided test in this case so they need to select the option $\mu: \neq \mu_0$.
- Now, have students state what the p-value of the test is and what that means. This may require some work on the teacher's part to make this leap. I stress that this value is, given what the mean is assumed to be, the probability of data such as ours occurring. If the probability is small (we shall say less than or equal to 0.05), then the likelihood of this event occurring is very slim…and it did! What does this say about our hypothesized value of the parameter? Are we off? Should it change?

Assessment

Whole Group Discussion:

As a class, culminate today's discussion with the following questions.

- Was our data statistically significant? Why or why not?
- What did you notice about our confidence intervals with respect to the parameter? Did any of the intervals contain our hypothesized value? What relationship can be drawn from a confidence interval that does not contain our hypothesized value and a test of significance?

- Suppose the size of our sample changed. What would this do to the test of significance? Is a larger or smaller sample better when conducting a test of significance and/or a confidence interval? Why do you think this?

Extensions and Connections

- Have students collect data on the heights of men in the classroom and then construct the different confidence intervals as done above for women. Have student compare the respective sizes of the confidence intervals (their lengths) as well as note the differences in their magnitude.
- Now, have students conduct a similar test of significance for this data using the null hypothesis once again as $\mu_0 = 72$. Is this test significant? Why or why not? What was the p-value of the test? What does this value mean in the context of this problem? Compare this result to the confidence intervals that were constructed. What do you notice about the hypothesized value with respect to those intervals?

Strategies for Differentiation

- If you have more advanced students, the teacher can certainly discuss the differences between two-tailed and one-tailed tests of significance and where the p-value is in relation to these tests.
- It may be appropriate, depending on the level of the student, to FIND the p-value of the test using the test statistic provided in the data output and using the t-cdf function in the calculator to see if the same results could be obtained.

Lesson 6: Culminating Activity and Wrap-Up

Strand
Data Analysis and Probability

Mathematical Objective(s)
During this lesson students will…

- estimate an interval for a given parameter (namely the mean for a population).
- discuss and understand the meaning of the p-value of a test of significance.
- use a confidence interval to determine whether sample is statistically significant.
- interpret the meaning of a test of significance in the context of the original problem.

Mathematics Performance Expectation(s)

- MPE 23: The student will analyze the normal distribution. Key concepts include:
  - a) characteristics of normally distributed data;
  - b) percentiles;
  - c) normalizing data, using z-scores; and
  - d) area under the standard normal curve and probability

Related SOL

- AII.11 (normal distribution)
- AFDA.7d (probabilities and the normal distribution)
- PS.16 (normal distribution)
- PS.18 (confidence intervals)
- PS.19 (test of significance)
- PS.20 (sampling distribution and the Central Limit Theorem)

All of these SOLs will be addressed in other lessons during this unit. The main focus of this particular lesson is AFDA.7d which is also mentioned in AII.11 and PS.16, 18, 19, and 20.

NCTM Standards

- understand how sample statistics reflect the values of population parameters and use sampling distributions as the basis for informal inference

Materials/Resources

- Classroom set of graphing calculators
- Height data collected from students

Assumption of Prior Knowledge

- It should be assumed that all students have a thorough knowledge and comprehensive understanding of the lessons that have been covered in this unit, how to conduct the various tests that have been described, and state generalized conclusions based on those test results in the context of the original problem.

Introduction: Setting Up the Mathematical Task

Reflective Journal Entry

- To set up a background for this activity have students take out a piece of paper and answer the following in a narrative form:
- What were the results for the tests of significance for males and females?
- Explain these results in terms of the confidence intervals that you constructed earlier.
- Why do you feel that the average American cabinet height of 72 inches was chosen?
- Based on you alone, do you feel this height is appropriate?
- Consider someone else around you. Do you think this is an appropriate height? Why or why not?

Assessment

Whole Group Discussion:

Now, after students have written down their responses to the above journal entry, have the class separate themselves based on whether they think the chosen height of 72 inches is or is not appropriate.

- Using mathematical evidence, have each side argue why they feel the chosen height for cabinets is or is not appropriate. Students should justify their answers based on sample size, test of significance, etc.

Allow this activity to go back and forth so long as the discussion is relevant and well defined.

- Ask students why this is probably not the BEST way to conduct this test and consider a better way.
- When 72 inches was chosen as the average height for kitchen cabinets, what things were considered? Do you feel that they separated men and women when determining this number? What do you think the likely scenario was in this case?
- So, should the heights of kitchen cabinets be changed? Is this a different answer than what you wrote in your reflective journal entry?

Extensions and Connections

- Consider something else in our world or society in which an average is used as a baseline. Where do you think this number comes from? How do you think they got the number they did? If students have difficulty with this, you may include something like par on a golf course or the number of laps for a NASCAR race depending on the track.

Strategies for Differentiation

- For students who do not participate actively in whole group discussions or debates, they should be able or allowed to justify their positions on the topic in a written fashion as opposed to the open debate.
- As an alternative to the in-class debate, students could create a PowerPoint presentation as a multistage process to comment on each section and create their own assessment material. It should be noted that students who know how to formulate and ask appropriate questions about the material likely know the material really well. Hence, a "self-absorbed" evaluation such as this may be a wonderful alternative.