Land Development

I. UNIT OVERVIEW & PURPOSE:

Adding, Subtracting, Multiplying, and Dividing Rational Polynomials. In this lesson, students will develop a mathematical model using a given Land Development Situation. They will use their existing knowledge of operations with polynomials to create this mathematical model and solve the problem. In the mathematical model, they will list simplifying assumptions, discuss how sensitive/robust is the model and will discuss how the operations and resultant functions change if the assumptions change.

II. UNIT AUTHOR: Scott Larimer

III. COURSE:

Mathematical Modeling: Capstone Course

IV. CONTENT STRAND:

Algebra

V. OBJECTIVES:

Students will:
add, subtract, multiply, and divide polynomial expressions.
Generate and interpret polynomial functions for a given context
Transfer between and analyze multiple representations of functions including graphs, tables, formulas, and words.
Find and describe in context the local maxima and minima of a quadratic function.
Solve quadratic functions using a variety of techniques including graphing, quadratic formula, and completing the square.

VI. MATHEMATICS PERFORMANCE EXPECTATION(s):

College and Career Ready Mathematics Performance Expectation(s)

The student, given rational, radical, or polynomial expressions, will: a) add, subtract, multiply, divide, and simplify rational algebraic expressions; b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents; c) write radical expressions as expressions containing rational exponents and vice versa; and d) factor polynomials completely.

The student, given rational, radical, or polynomial expressions, will: a) add, subtract, multiply, divide, and simplify rational algebraic expressions; b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents; c) write radical expressions as expressions containing rational exponents and vice versa; and d) factor polynomials completely.

The student will transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Students will select and use appropriate representations for analysis, interpretation, and prediction

The student will investigate and analyze functions (linear, quadratic, exponential, and logarithmic families) algebraically and graphically. Key concepts include:

b) local and absolute maxima and minima;

The student will solve, algebraically and graphically,

b) quadratic equations over the set of complex numbers

VII. CONTENT:

Adding, subtracting, multiplying, and dividing polynomial expressions
Graphing Quadratic Equations
Solving Quadratic Equations
Analyzing Quadratic Equations

VIII. REFERENCE/RESOURCE MATERIALS:

Classroom set of graphing calculators
Access to computers and internet

IX. PRIMARY ASSESSMENT STRATEGIES:

Journal Entries

X. EVALUATION CRITERIA:

Journal Entries

XI. INSTRUCTIONAL TIME:

Four 45-minute class periods

Lesson 1: Land Development Setup

Strand

Algebra

Mathematical Objective(s)

Adding, Subtracting, Multiplying, and Dividing Rational Polynomials. In this lesson, students will develop a mathematical model using a given Land

Development Situation. They will use their existing knowledge of operations with polynomials to create this mathematical model and solve the problem. In the mathematical model, they will list simplifying assumptions, discuss how sensitive/robust is the model and will discuss how the operations and resultant functions change if the assumptions change.

Mathematics Performance Expectation(s)

The student, given rational, radical, or polynomial expressions, will: a) add, subtract, multiply, divide, and simplify rational algebraic expressions; b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents; c) write radical expressions as expressions containing rational exponents and vice versa; and d) factor polynomials completely.

Related SOL

A.11

AII.2

NCTM Standards

- Understand relations and functions and select, convert flexibly among, and use various representations for them
- Apply and adapt a variety of appropriate strategies to solve problems
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others

Additional Objectives for Student Learning (include if relevant; may not be math-related):

N/A

Materials/Resources

N/A

Assumption of Prior Knowledge

- The typical student would have already taken Algebra 1 class.
- To succeed in this lesson students should have understanding of polynomial functions specifically, quadratic, and rational functions.
- The relevant real life context in this problem is Land Development.

Introduction: Setting Up the Mathematical Task

Introduce and clarify the modeling problem (5 minutes)

Modeling Problem:

You have just purchased a large chunk of land from a farmer and intend to subdivide it into a housing development of single family homes. You will be paying the cost of developing the land (building roads, sewers, house, etc). Find a function that will give the minimum price should you sell each piece of land to insure that you at least break even with your original purchase price.

- State your simplifying assumptions. (small groups 10 min; followed by whole class 10 min)
- Identify dependent and independent variables in your mathematical model. (small groups 10 min; followed by whole class 5 min)
- Identify additional information that you have to gather (in order to solve the problem) using technology. (whole class 5 min)

Student/Teacher Actions:

- Students should state their simplifying assumptions in a small group discussion. The teacher should provide minimal input, only supplying clarifying suggestions when groups have completely stalled out.
- Teacher should have a class discussion of the model and ask for student input to create a whole-class model.

It would be helpful to have a discussion on what are some possible simplifying assumptions, such as:

1. Square land.
2. Each property is clear and flat.
3. There is no existing road frontage on any property.
4. Uniform property sizes.
5. There is a minimum distance from the property line
6. Uniform 1-story house construction (using all of the buildable land on the property)
7. Uniform property prices.
8. Each property will be accessible by road once developed.
9. Each property will be accessible by utilities (electricity, water, sewer, cable, etc).

Possible routes for discussion: To insure that we break even with our original purchase price, one can think of A) Each individual property's price containing of their share of the road, utilities, and home construction or B) The original purchase price of the land having an additional cost of road, utility, and home construction. For the purposes of this problem, we will be focusing on **A) Each individual property's price containing of their share of the road, utilities, and home construction**

- Students should then identify dependent and independent variables in the whole-class mathematical model.
- Teacher will have a class-discussion relevant to the model
- Identify additional information that you have to gather (in order to solve the problem) using technology. (whole class 5 min)
    - Additional information needed would include the cost of building the road, connecting the utilities, and cost per square feet for the average home in their area.
- Homework: Students will journal about their ideas about today's class discussion. Journals entries should include:
    - Ideas discussed in class
    - Any additional unspoken ideas for tomorrow's class
    - Personal reflections about the project being presented

Lesson 2 - Land Development information gathering

Strand
Algebra

Mathematical Objective(s)

Adding, Subtracting, Multiplying, and Dividing Rational Polynomials. In this lesson, students will develop a mathematical model using a given Land Development Situation. They will use their existing knowledge of operations with polynomials to create this mathematical model and solve the problem. IN the mathematical model, they will list simplifying assumptions, discuss how sensitive/robust is the model and will discuss how the operations and resultant functions change if the assumptions change.

Mathematics Performance Expectation(s)

The student, given rational, radical, or polynomial expressions, will: a) add, subtract, multiply, divide, and simplify rational algebraic expressions; b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents; c) write radical expressions as expressions containing rational exponents and vice versa; and d) factor polynomials completely.

Related SOL

A.11

AII.2

NCTM Standards

- Understand relations and functions and select, convert flexibly among, and use various representations for them
- Apply and adapt a variety of appropriate strategies to solve problems
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others

Additional Objectives for Student Learning (include if relevant; may not be math-related):
N/A

Materials/Resources

- Computers with internet access

Assumption of Prior Knowledge

- The typical student would have already taken Algebra 1 class.
- To succeed in this lesson students should have understanding of polynomial functions specifically, quadratic, and rational functions.
- The relevant real life context in this problem is Land Development.

Researching the Mathematical Task

Modeling Problem:

You have just purchased a large chunk of land from a farmer and intend to subdivide it into a housing development of single family homes. You will be paying the cost of developing the land (building roads, sewers, house, etc). Find a function that will give the minimum price should you sell each piece of land for to insure that you at least break even with your original purchase price.

Student/Teacher Actions:

- Reread the modeling problem to the students and remind students of their whole-class simplifying assumptions, dependent and independent variables, and additional information they need to gather. (5 minutes)
- Have students spend time on classroom computers, laptops from a cart, or in the computer lab researching the additional information that they need to collect (30 minutes)
    - Additional information needed would include the cost of building the road, connecting the utilities, and cost per square feet for the average home in their area.
- Discuss results of student searches with students and come to a class consensus for the cost of each additional piece of information.

Assessment:

- Students will record the costs of each topic and their source for each figure
- Students will also record the class consensus for the costs
- Students will write a journal entry of why they believe their costs were different from other student responses and the class consensus. They should include:
    - What source gave them their costs
    - What other classmates found for their costs
    - What factors might affect their costs
    - Why they believe their costs would be different

Lesson 3 - Land Development calculations

Strand
Algebra

Mathematical Objective(s)

Adding, Subtracting, Multiplying, and Dividing Rational Polynomials. In this lesson, students will develop a mathematical model using a given Land Development Situation. They will use their existing knowledge of operations with polynomials to create this mathematical model and solve the problem. IN the mathematical model, they will list simplifying assumptions, discuss how sensitive/robust is the model and will discuss how the operations and resultant functions change if the assumptions change. They will transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. The student will investigate and analyze functions (specifically quadratic) algebraically and graphically and will locate and describe in context the local and absolute maxima and minima. They student will solve, algebraically and graphically, quadratic equations over the set of complex numbers using factoring and quadratic formula or completing the square.

Mathematics Performance Expectation(s)

The student, given rational, radical, or polynomial expressions, will: a) add, subtract, multiply, divide, and simplify rational algebraic expressions; b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents; c) write radical expressions as expressions containing rational exponents and vice versa; and d) factor polynomials completely.

The student will transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Students will select and use appropriate representations for analysis, interpretation, and prediction

The student will investigate and analyze functions (linear, quadratic, exponential, and logarithmic families) algebraically and graphically. Key concepts include:

b) local and absolute maxima and minima;

The student will solve, algebraically and graphically,

b) quadratic equations over the set of complex numbers

Related SOL

A.11, AII.2, AII.8, AFDA.4

NCTM Standards

- Understand relations and functions and select, convert flexibly among, and use various representations for them
- Apply and adapt a variety of appropriate strategies to solve problems
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others

Additional Objectives for Student Learning (include if relevant; may not be math-related):
N/A

Materials/Resources
Classroom set of graphing calculators

Assumption of Prior Knowledge

- The typical student would have already taken Algebra 1 class.
- To succeed in this lesson students should have understanding of polynomial functions specifically, quadratic, and rational functions.
- The relevant real life context in this problem is Land Development.
- Graphing Quadratic Equations.
- Solving Quadratic equations graphically and algebraically.

Working with the Mathematical Task:

- **Modeling Problem:**
  You have just purchased a large chunk of land from a farmer and intend to subdivide it into a housing development of single family homes. You will be paying the cost of developing the land (building roads, sewers, house, etc). Find a function that will give the minimum price should you sell each piece of land for to insure that you at least break even with your original purchase price.

  - Reread the modeling problem to the students and remind students of their whole-class simplifying assumptions, dependent and independent variables, and additional information they needed to gather. (5 minutes)
    - Additional information needed included the cost of building the road, connecting the utilities, and cost per square feet for the average home in their area.
  - Have students work to develop polynomial expressions to represent cost for each property that include cost of road per square foot of property, cost of utilities per square foot of property and cost of home per square foot of property. (15 minutes followed by 5 minute teacher discussion)
  - Have students work in groups to develop a combined and simplified polynomial equation to represent the total cost of property per square foot (10 minutes followed by 5 minute teacher discussion)
  - Have students use their equation to answer questions related to the mathematical model. (10 minutes)

Student/Teacher Actions:

To explore and develop this model the following are important points of discussions:

- The land is divided into plots that are x by x ft.
- Therefore, each property will be x^2 square feet
- According to Google, it costs approximately $115 per foot paved of road to be constructed.
- Therefore, each property will need to include $\frac{115x}{2}$ (since each section of road will be shared by 2 properties that are across the street from each other)
- According to neighborhood home owners association, the minimum setback for a single family residence is 15 ft from the property line and 20 ft from a road side.

- Therefore, each house can be a maximum of (x – 20 – 15) (x – 15 – 15) which is equal to (x – 35)(x – 30) sq ft.
- According to Google, the average house costs $75 per sq ft. to build.
- Therefore each house will cost 75(x – 35)(x – 30) dollars
- According to selfbuild.co.uk, the cost of connect utilities can be between $2000 and $3000, we'll say $2500 on average
- When all said and done, each property will cost $\frac{115x}{2} + 75(x - 35)(x - 30) + 2500$ dollars to develop which gives the equation $75x^2 - 4817.5x + 81250$

Monitoring Student Responses

- Students make their mathematical thinking and understanding public by responding to the aforementioned questions in their small groups as well as during the whole class discussions. Students could also use different assumptions for reevaluate and modify their model) allow students to share their models and modifications. Allow time to share students' models and their reasoning behind each mathematical model.
- Students are likely to forget/miss the subtracting the setback from both sides of property (ex: front and back, not left and right)
- Simplify this model even more for a group of student who are experiencing difficulty.
- To summarize your lesson allow 5 -10 minutes for each small group of students to share their mathematical models and their reasoning behind each mathematical model with the whole class.

Assessment

Sample Questions

- Given the previous mathematical model with the cost function C = $75x^2 - 4817.5x + 81250$, what is the minimum size property that you could have? Why?
- Graph the function and describe the minimum and it's importance in context.
- Would this minimum be a feasible property size? Why or why not?
- What is the price of development for a piece of property that is 40 x 40? 50x50? 100x 100? What is the size of the house you could build on each of these size properties?
- Discuss with your group and come to a consensus about the minimum size house that you could tolerate.

- Write a function using our restrictions (20 ft from road and 15 ft from property line) and use it to find the size property that would allow you to build a house of that size.
- Calculate the cost of the whole property (including utilities, road, and house) of the property size that you
- How would your model be changed if you required 2 story homes?
- How would your model be changed if the properties were not square (rectangles for instance)?
- Describe the model presented by another group. How is this model different from yours? Is this model more robust/sensitive).

Extensions/Modifications

It is possible to use this unit over a long period of time while talking about multiple topics in the middle of the unit. For example, the teacher could

- Introduce the modeling problem
- Talk about simplifying assumptions
- Research the information
- Talk about expressions and functions
- Have students write expressions and functions for the unit
- Talk about graphing and analyzing quadratics
- Have students graph and analyze the quadratic in the unit
- Etc

Possible modifications of the lessons include (but are not limited to):

- Creating a PowerPoint presentation to go along with project. Sample PowerPoint is attached.
- For ESL students:
    a. Reword questions – make them more simple to read and shorter.
    b. Add pictures – to clarify what was being talked about (adding context)
    c. Bolded key words – make questions easier to read
    d. Had lists appear one item at a time – less confusing, not as much text on screen at once.
    e. Add diagrams for better visualization
- Have students work with partners
- Extended time
- Use student's accommodations