

Natural Disaster Recovery and Quadrilaterals

I. UNIT OVERVIEW & PURPOSE:

In this unit, students will apply their knowledge of quadrilaterals to solve mathematics problems concerning a tornado that struck a small town in Southwest Virginia. Students will verify the properties of quadrilaterals, incorporate the Pythagorean Theorem, and utilize area and perimeter formulas to solve real-world problems.

II. UNIT AUTHOR:

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III. COURSE:

Mathematical Modeling: Capstone Course

IV. CONTENT STRAND:

Geometry

Measurement

V. OBJECTIVES:

Quadrilaterals, Right Triangles, Perimeter, Area. In this unit, students will determine and apply the basic properties of quadrilaterals. Students will apply the perimeter and area formulas of quadrilaterals. The students will verify and prove the area formula for a trapezoid. The students will solve real-world problems using the Pythagorean Theorem and properties of right triangles. The students will determine and apply units of measurement to solve and describe the solutions to math problems.

VI. MATHEMATICS PERFORMANCE EXPECTATION(s):

MPE.4 Verify characteristics of quadrilaterals and use properties of quadrilaterals to solve real-world problems

MPE.5 Solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry

VII. CONTENT:

Other than the mathematical ideas of the unit, content such as natural disasters, carpentry, roofing, and fencing will be utilized to allow students to apply geometry to real-life application problems.

VIII. REFERENCE/RESOURCE MATERIALS:

- <http://www.doe.virginia.gov/VDOE/Instruction/ESL/LEPmathResource.pdf>
- <http://www.pbs.org/teachers/mathline/concepts/outdoors/activity1.shtml>
- Fencing Materials Worksheet (Included in Lesson 1)
- Roofing Materials Worksheet (Included in Lesson 2)
- Land Assessment Worksheet (Included in Lesson 3)

IX. PRIMARY ASSESSMENT STRATEGIES:

Students will be given both informal and formal assessments. Informal assessments will be based upon the teacher's observations of student participation in small group work and whole group discussions. Formal assessments will be based upon the completion and correctness of the work that students submit for the Fencing Materials Worksheet, Roofing Materials Worksheet, and Land Assessment Worksheet. Answer keys are

provided at the end of each lesson. Accommodations will be provided as needed.

X. EVALUATION CRITERIA:

Students will be evaluated based on participation in small groups and whole group discussions. Students will also be evaluated based on the completion and correctness of the worksheets involved in the unit.

XI. INSTRUCTIONAL TIME:

150 minutes. Each of the three lesson plans in the unit will take approximately 50 minutes to complete.

Fencing Materials

Strand

Geometry

Measurement

Mathematical Objective(s)

Quadrilaterals, Perimeter, Area. In this lesson, students will determine and apply the basic properties of quadrilaterals. Students will apply the perimeter and area formulas of quadrilaterals. The students will determine and apply units of measurement to solve and describe the solutions to math problems.

Mathematics Performance Expectation(s)

- MPE.4 Verify characteristics of quadrilaterals and use properties of quadrilaterals to solve real-world problems

Related SOL

- G.8 The student will
 - investigate and identify properties of quadrilaterals involving opposite sides and angles, consecutive sides and angles, and diagonals;
 - prove these properties of quadrilaterals, using algebraic and coordinate methods as well as deductive reasoning; and
 - use properties of quadrilaterals to solve practical problems.

NCTM Standards

- Use geometric models to gain insights into, and answer questions in, other areas of mathematics
- Understand and use formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders

Materials/Resources

- Classroom set of graphing calculators
- Smart Board (Chalkboard or dry erase board if a Smart Board is not available)
- Fencing Materials Worksheet (Attached)

Assumption of Prior Knowledge

- The typical student would have already taken an Algebra I and Geometry course. It is recommended that students should be at or above the abstraction Van Hiele level.
- To succeed in this lesson, students should have an understanding of the basic properties of quadrilaterals, specifically the perimeter and area formula of rectangles.

- The relevant real life contexts in this problem are natural disasters and fencing.

Introduction: Setting Up the Mathematical Task

A recent tornado in a small town in Southwest Virginia destroyed and damaged multiple homes and farms. Sam owns a rectangular field where a fence enclosing the field was destroyed. The insurance company assessed that Sam could have a new fence built around his rectangular field. (10 minutes)

- What information does Sam need to know in order to build a fence around his rectangular field? (small groups for 5 minutes; followed by whole class for 5 minutes)
- Some sample responses from students may include the length and width of the field, the perimeter of the field, the amount of materials needed, the cost of the fencing material, etc.

Student Exploration 1:

The students will be provided with a worksheet to solve the fencing material problem. The students will use their knowledge of quadrilaterals to complete the problem in small groups of 2-3 students. (40 minutes)

Student/Teacher Actions:

- The students will work in small groups of 2-3 students. The students will be provided with a fencing materials worksheet to complete. The attachment contains the problems for their lesson and the answer key.
- All group members are expected to participate and provide input. Every student will be asked to show their work and provide explanations in their own words.
- The teacher will provide feedback to the groups by walking around to provide assistance to the groups as needed.
- The teacher can provide hints to the students as necessary. For example, the teacher may need to assist the students by helping the students determine the dimensions of the rectangular field.
- The teacher can show the fencing materials worksheet on the Smart Board and work the problems as needed. The Smart Board will allow the students to have a larger, visual representation of the fencing problem. If the teacher does not have a Smart Board, the problems can be drawn on the chalkboard or dry erase board.
- The teacher will allow a whole class discussion after the small group activity for the students to share their responses. Members from each group can come to the Smart Board and complete portions of the problem.

Monitoring Student Responses

- Students make their mathematical thinking and understanding public by responding to the worksheet questions in their small groups.
- Students are expected to show their work on the fencing materials worksheet.
- Ensure that each member of the group is working on the project by monitoring the group work.
- Simplify this model for a group of students who are experiencing difficulty. This could be accomplished by clarifying the directions or scenario of the problem, providing hints to assist the group of students, asking students further questions to determine how to solve the problem, displaying the problems on a bigger display for an additional visual representation, or any other method determined by the teacher that will assist the students.
- To summarize the lesson, students will be provided 10 minutes after the completion of the worksheet for whole class discussions concerning their ideas and reasoning on the fencing materials problem.

Assessment

- The students will be assessed by the completion of the fencing materials worksheet. The answer key for the fencing materials worksheet is attached. Students can be given partial credit if their work has been shown.
- Students will also be assessed by their participation in the small group work and the whole class discussion. The participation grade will be at the teacher's discretion.

Extensions and Connections (for all students)

- To extend this lesson, ask the students to consider the total amount of money that they determined it would cost to replace the fence around the rectangular field. Is this a true estimate of the amount of money that Sam will need to replace the fence? Why or why not? The students will be asked to write a $\frac{1}{2}$ -1 page journal entry answering the question.

Strategies for Differentiation

The differentiation strategies might include but are not limited to the following list created specifically for ESL students. Feel free to adopt these to your lesson:

- Make instruction more concrete, visual, collaborative, and hands-on
- Focus on mathematical content rather than on linguistic form (simplify word problems without changing the math meaning)

- Language and content should be presented simultaneously
- Preferential Seating (near teacher or next to a buddy, native language if possible)
- Write legibly and in print
- Step by step instructions (orally and in writing).
- Give EOL students more time for questions and answers.
- Use of Dictionaries
- Simplify the language used rather than the mathematical concepts taught (use known vocabulary and simple sentence constructions).
- Observe and record students' participation in small group activities.

The accommodations are adopted from the following source.

<http://www.doe.virginia.gov/VDOE/Instruction/ESL/LEPmathResource.pdf>

Fencing Materials Worksheet

A recent tornado in a small town in Southwest Virginia destroyed and damaged multiple homes and farms. Sam owns a rectangular field where a fence enclosing the field was destroyed. The insurance company assessed that Sam could have a new fence built around his rectangular field.

1. Sam knows that the perimeter of his rectangular field is $P = 1604$ feet. However, he did not know the exact length and width of the field. The width of the field is the sum of 3 times the length and 2 feet. Use this information to determine the length and width of the rectangular field. You may use the following drawing to assist with this problem.



2. The fencing material may be purchased in rolls. Each roll contains 150 feet of fencing material, and the fencing material cost \$82.95 per roll.
 - a. How many rolls of material will Sam need to enclose his field?
 - b. What is the total cost of the fencing material that Sam needs to enclose his field?
 - c. How were you able to solve this problem?

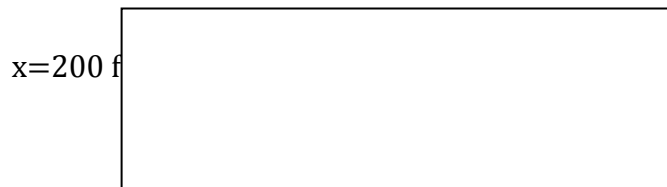
3. Determine the area of the entire rectangular field.

4. Sam wants to buy sod for a portion of his field. He only wants to cover a 145 ft x 145 ft region of the field. If the sod patches costs \$.30 per square foot, how many sod patches will Sam need to cover the 145 ft x 145 ft portion of his field? What will the total cost of the sod be?

5. Using your answers from Problems 3 and 4, what is the total area of the field that will not contain new sod?

Fencing Answer Key:

1. $3x + 2 = 602 \text{ ft}$



To determine the length and widths of the field,

$$P = x + x + (3x + 2) + (3x + 2) = 1604$$

$$8x + 4 = 1604$$

$$8x = 1600$$

$$x = 200$$

Width = 602 feet; Length = 200 feet

2. a) $1604/150 = 10.6933 \cong 11$ rolls of fencing material

b) $11 * \$82.95 = \912.45

c) Sample: Divide the total perimeter by the amount of fencing material (1604/150). Estimate the number of rolls needed. Multiply the number of rolls by the price per roll.

3. $A = l * w$

$$A = 200 \text{ ft} * 602 \text{ ft}$$

$$A = 120400 \text{ ft}^2$$

4. $145 \text{ ft} * 145 \text{ ft} = 21025 \text{ ft}^2$

$$21025 \text{ ft} * \$.30 = \$6307.50$$

Sam will need 21025 sod patches to cover his field and the total cost for sod will be \$6307.50

5. Area of entire field = 120400 ft^2

$$\text{Area of sod portion of field} = 21025 \text{ ft}^2$$

$$\text{Area of the field without sod} = 120400 - 21025 = 99375 \text{ ft}^2$$

Roofing Materials

Strand

Geometry

Measurement

Mathematical Objective(s)

Quadrilaterals, Right Triangles, Perimeter, Area. In this lesson, students will determine and apply the basic properties of quadrilaterals. Students will apply the perimeter and area formulas of quadrilaterals. The students will solve real-world problems using the Pythagorean Theorem and properties of right triangles. The students will determine and apply units of measurement to solve and describe the solutions to math problems.

Mathematics Performance Expectation(s)

- MPE.4 Verify characteristics of quadrilaterals and use properties of quadrilaterals to solve real-world problems
- MPE.5 Solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry

Related SOL

- G.7 The student will solve practical problems involving right triangles by using the Pythagorean Theorem, properties of special right triangles, and right triangle trigonometry. Solutions will be expressed in radical form or as decimal approximations.
- G.8 The student will
 - investigate and identify properties of quadrilaterals involving opposite sides and angles, consecutive sides and angles, and diagonals;
 - prove these properties of quadrilaterals, using algebraic and coordinate methods as well as deductive reasoning; and
 - use properties of quadrilaterals to solve practical problems.

NCTM Standards

- Analyze properties and determine attributes of two- and three-dimensional objects
- Use geometric models to gain insights into, and answer questions in, other areas of mathematics
- Understand and use formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders

Materials/Resources

- Classroom set of graphing calculators
- Smart Board (Chalkboard or dry erase board if a Smart Board is not available)
- Roofing Materials Worksheet (Attached)

Assumption of Prior Knowledge

- The typical student would have already taken an Algebra I and Geometry course. It is recommended that students should be at or above the abstraction Van Hiele level.
- To succeed in this lesson, students should have an understanding of the basic properties of quadrilaterals, specifically the area formula for particular quadrilaterals.
- The students should have an understanding of the Pythagorean Theorem and properties of right triangles.
- The relevant real life contexts in this problem are natural disasters and roofing.

Introduction: Setting Up the Mathematical Task

A recent tornado in a small town in Southwest Virginia destroyed and damaged multiple homes. Although Mr. Turner's home did not get destroyed, his roof suffered major damage by the hail and high winds. The insurance company assessed the damage to Mr. Turner's roof and determined that the roof on his home should be replaced. Determine the amount of roofing material needed to repair the home and the cost of supplies. (10 minutes)

- What information will you need to complete this problem? (small groups for 5 minutes; followed by whole class for 5 minutes)
- Some sample responses from students may include the shape and dimension of the roof, the shape and dimension of the roofing material, the cost of the roofing material, etc.

Student Exploration 1:

The students will be provided with a worksheet to solve the roofing material problem. The students will use their knowledge of quadrilaterals and right triangles to complete the problem in small groups of 2-3 students. (40 minutes)

Student/Teacher Actions:

- The students will work in small groups of 2-3 students. The students will be provided with a roofing materials worksheet to complete. The attachment contains the problems for their lesson and the answer key.

- All group members are expected to participate and provide input. Every student will be asked to show their work and provide explanations in their own words.
- The teacher will provide feedback to the groups by walking around to provide assistance to the groups as needed.
- The teacher can provide hints to the students as necessary. For example, the teacher may need to assist the students by helping the students determine that they will first need to determine the hypotenuse of the right triangle by utilizing the Pythagorean Theorem.
- The teacher can show the roofing materials worksheet on the Smart Board and work the problems as needed. The Smart Board will allow the students to have a larger, visual representation of the roofing problem. If the teacher does not have a Smart Board, the problems can be drawn on the chalkboard or dry erase board.
- The teacher will allow a whole class discussion after the small group activity for the students to share their responses. Members from each group can come to the Smart Board and complete portions of the problem.

Monitoring Student Responses

- Students make their mathematical thinking and understanding public by responding to the worksheet questions in their small groups.
- Students are expected to show their work on the roofing materials worksheet.
- Ensure that each member of the group is working on the project by monitoring the group work.
- Simplify this model for a group of students who are experiencing difficulty. This could be accomplished by clarifying the directions or scenario of the problem, providing hints to assist the group of students, asking students further questions to determine how to solve the problem, displaying the problems on a bigger display for an additional visual representation, or any other method determined by the teacher that will assist the students.
- To summarize the lesson, students will be provided 10 minutes after the completion of the worksheet for whole class discussions concerning their ideas and reasoning on the roofing materials problem.

Assessment

- The students will be assessed by the completion of the roofing materials worksheet. The answer key for the roofing materials worksheet is attached. Students can be given partial credit if their work has been shown.
- Students will also be assessed by their participation in the small group work and the whole class discussion. The participation grade will be at the teacher's discretion.

Extensions and Connections (for all students)

- To extend this lesson, ask the students to consider the picture that was on the roofing materials worksheet. We assumed that the chimney was not present on the roof. The students will be asked to write a ½ -1 page journal entry about their method for solving the mathematics problem if there was a chimney on the roof. The students can answer the following questions:
 - How would the chimney affect the way that you solve this problem? How would you address this situation to determine the amount of shingles and the price of supplies?
 - From the roofing materials worksheet, the roof actually appears to extend beyond the 54 ft x 18 ft dimensions of the house. Why do you think that the roof extends beyond the house? Why would this be important?

Strategies for Differentiation

The differentiation strategies might include but are not limited to the following list created specifically for ESL students. Feel free to adopt these to your lesson:

- Make instruction more concrete, visual, collaborative, and hands-on
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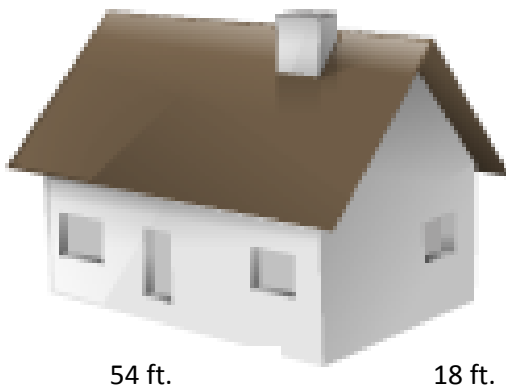
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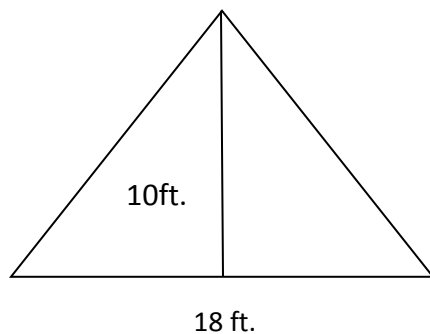
Roofing Material Worksheet

A recent tornado in a small town in Southwest Virginia destroyed and damaged multiple homes. Although Mr. Turner's home did not get destroyed, his roof suffered major damage by the hail and high winds. The insurance company assessed the damage to Mr. Turner's roof and determined that the roof on his home should be replaced. Use the information below to answer these questions.

1. The dimensions for the house (54 feet x 18 feet) are displayed in the figure below.
Note: Although the picture of the house below shows has a chimney, we will ignore the chimney for the purpose of this problem.



The measurements for side of the house at the end of the roof are listed below.

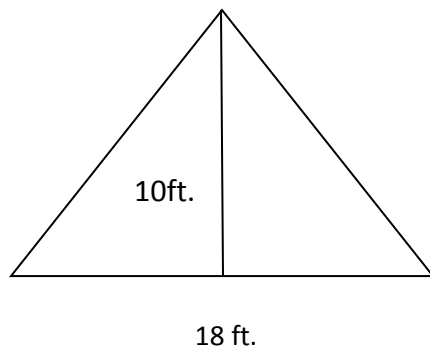


2. The contractor will use packs of shingles to repair the roof. Each pack covers 100 square feet on the roof. Each pack of shingles cost \$120.
3. Answer the following questions and show all of your work:
 - 1) What is the total area of the roof?
 - 2) How many packs of shingles will be needed to cover the roof?
 - 3) What will the total price of shingles be?

Roofing Answer Key:

1. What is the total area of the roof? 1452.6 ft^2
2. How many packs of shingles will be needed to cover the roof? 15 packs of shingles
3. What will the total price of shingles be? \$1800

1. The students will need to determine the hypotenuse of the triangle that is at the end of the roof.



Using the Pythagorean Theorem,

$$10^2 + 9^2 = c^2$$

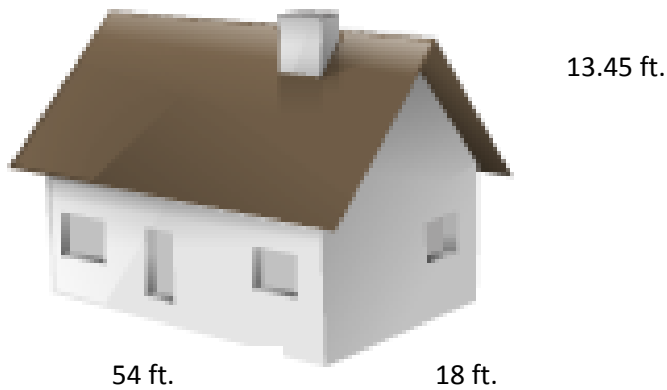
$$100 + 81 = c^2$$

$$181 = c^2$$

$$13.45 = c$$

This means that the hypotenuse of the triangle is approximately 13.45 feet.

Next, the students will need to determine the area of the roof. We know the hypotenuse of the triangle; we now know that the slant of the roof is 13.45 feet.



Since the roof is a parallelogram, the area formula is $A = b * h$

$$A = 54 \text{ ft} * 13.45 \text{ ft} = 726.3 \text{ ft}^2$$

We also have to consider the two sides of the roof, $726.3 * 2 = 1452.6 \text{ ft}^2$

2. To determine the number of packs of shingles needed for the roof, divide the area of the roof by the area of the packs of shingles. That is, $1452.6 \text{ ft}^2 \div 100 \text{ ft}^2 = 14.536$

This means that the contractor will need 15 packs of square shingles to repair the damage on the roof.

3. Since the shingles cost \$120 per pack, the total price for the shingles will be

$$\$120 * 15 = \$1800$$

*Lesson was modified from the following website:

<http://www.pbs.org/teachers/mathline/concepts/outdoors/activity1.shtm>

