Natural Disaster Recovery and Quadrilaterals

I. UNIT OVERVIEW & PURPOSE:

In this unit, students will apply their knowledge of quadrilaterals to solve mathematics problems concerning a tornado that struck a small town in Southwest Virginia. Students will verify the properties of quadrilaterals, incorporate the Pythagorean Theorem, and utilize area and perimeter formulas to solve real-world problems.

II. UNIT AUTHOR:

Jessica Newman, Lee High School, Lee County Public Schools

III. COURSE:

Mathematical Modeling: Capstone Course

IV. CONTENT STRAND:

Geometry

Measurement

V. OBJECTIVES:

Quadrilaterals, Right Triangles, Perimeter, Area. In this unit, students will determine and apply the basic properties of quadrilaterals. Students will apply the perimeter and area formulas of quadrilaterals. The students will verify and prove the area formula for a trapezoid. The students will solve real-world problems using the Pythagorean Theorem and properties of right triangles. The students will determine and apply units of measurement to solve and describe the solutions to math problems.

VI. MATHEMATICS PERFORMANCE EXPECTATION(s):

MPE.4 Verify characteristics of quadrilaterals and use properties of quadrilaterals to solve real-world problems

MPE.5 Solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry

VII. CONTENT:

Other than the mathematical ideas of the unit, content such as natural disasters, carpentry, roofing, and fencing will be utilized to allow students to apply geometry to real-life application problems.

VIII. REFERENCE/RESOURCE MATERIALS:

- http://www.doe.virginia.gov/VDOE/Instruction/ESL/LEPmathResource.pdf
- http://www.pbs.org/teachers/mathline/concepts/outdoors/activity1.shtm
- Fencing Materials Worksheet (Included in Lesson 1)
- Roofing Materials Worksheet (Included in Lesson 2)
- Land Assessment Worksheet (Included in Lesson 3)

IX. PRIMARY ASSESSMENT STRATEGIES:

Students will be given both informal and formal assessments. Informal assessments will be based upon the teacher's observations of student participation in small group work and whole group discussions. Formal assessments will be based upon the completion and correctness of the work that students submit for the Fencing Materials Worksheet, Roofing Materials Worksheet, and Land Assessment Worksheet. Answer keys are

provided at the end of each lesson. Accommodations will be provided as needed.

X. EVALUATION CRITERIA:

Students will be evaluated based on participation in small groups and whole group discussions. Students will also be evaluated based on the completion and correctness of the worksheets involved in the unit.

XI. INSTRUCTIONAL TIME:

150 minutes. Each of the three lesson plans in the unit will take approximately 50 minutes to complete.

Trapezoids and Land Assessments

Strand
Geometry
Measurement

Mathematical Objective(s)
Quadrilaterals, Right Triangles, Perimeter, Area. In this lesson, students will determine and apply the basic properties of quadrilaterals. Students will apply the perimeter and area formulas of quadrilaterals. The students will verify and prove the area formula for a trapezoid. The students will solve real-world problems using the Pythagorean Theorem and properties of right triangles. The students will determine and apply units of measurement to solve and describe the solutions to math problems.

Mathematics Performance Expectation(s)

- MPE.4 Verify characteristics of quadrilaterals and use properties of quadrilaterals to solve real-world problems
- MPE.5 Solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry

Related SOL

- G.7 The student will solve practical problems involving right triangles by using the Pythagorean Theorem, properties of special right triangles, and right triangle trigonometry. Solutions will be expressed in radical form or as decimal approximations.

- G.8 The student will
    - investigate and identify properties of quadrilaterals involving opposite sides and angles, consecutive sides and angles, and diagonals;
    - prove these properties of quadrilaterals, using algebraic and coordinate methods as well as deductive reasoning; and
    - use properties of quadrilaterals to solve practical problems.

NCTM Standards

- Analyze properties and determine attributes of two- and three-dimensional objects
- Use geometric models to gain insights into, and answer questions in, other areas of mathematics

- Understand and use formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders

Materials/Resources

- Classroom set of graphing calculators
- Smart Board (Chalkboard or dry erase board if a Smart Board is not available)
- Land Assessment Worksheet (Attached)

Assumption of Prior Knowledge

- The typical student would have already taken an Algebra I and Geometry course. It is recommended that students should be at or above the abstraction Van Hiele level.
- To succeed in this lesson, students should have an understanding of the basic properties of quadrilaterals, specifically the area formula for particular quadrilaterals.
- The students should have an understanding of the Pythagorean Theorem and properties of triangles.
- The relevant real life contexts in this problem are natural disasters and land assessment.

Introduction: Setting Up the Mathematical Task

A recent tornado in a small town in Southwest Virginia destroyed and damaged multiple plots of land. The insurance company has been surveying these plots of land to write their report on the damage assessment. One plot of land is the shape of a trapezoid, and the insurance company must determine the area of the land. (10 minutes)

- What information will you need to complete this problem? (small groups for 5 minutes; followed by whole class for 5 minutes)
- Some sample responses from students may include the length and the width of the land, a picture of the land, etc.

Student Exploration 1:

The students will be provided with a worksheet to solve the land assessment problem. The students will use their knowledge of the area of trapezoids to complete the problem individually. (10 minutes)

Student/Teacher Actions:

- The students will work individually on the Part 1 of the worksheet. This should take approximately 5 minutes.
- Every student will be asked to show their work and provide explanations in their own words as needed.

- The teacher will provide feedback to the students by walking around to provide assistance as needed.
- The teacher can provide hints to the students as necessary. For example, the teacher may need to assist the students by helping the students label the bases and height of the trapezoid so that students can find the area of the trapezoid.
- The teacher can show the land assessment worksheet on the Smart Board and work the problems as needed. The Smart Board will allow the students to have a larger, visual representation of the roofing problem. If the teacher does not have a Smart Board, the problems can be drawn on the chalkboard or dry erase board.
- The teacher will allow a whole class discussion after the activity for the students to share their responses. Members from each group can come to the Smart Board and complete portions of the problem.

Student Exploration 2:

The students will be provided with a worksheet to solve the land assessment problem. The students will use their knowledge of quadrilaterals and triangles to complete the problem in small groups of 2-3 students. (30 minutes)

Student/Teacher Actions:

- The teacher will work the first example problem given on the Part 2 worksheet so that students understand the directions for the assignment. Student input can be used as the teacher applies area formulas for triangles and rectangles.
- The students will work in small groups of 2-3 students. The students will be provided with Land Assessment worksheet (Part 2) to complete. The attachment contains the problems for their lesson and the answer key.
- All group members are expected to participate and provide input. Every student will be asked to show their work and provide explanations in their own words.
- The teacher will provide feedback to the groups by walking around to provide assistance to the groups as needed.
- The teacher can provide hints to the students as necessary. For example, the teacher may need to assist the students by helping the students divide the trapezoid into triangles or other quadrilaterals.
- The teacher can show the land assessment worksheet on the Smart Board and work the problem as needed. The Smart Board will allow the students to have a larger, visual representation of the land assessment problem. If the teacher does not have a Smart Board, the problems can be drawn on the chalkboard or dry erase board.

- The teacher will allow a whole class discussion after the small group activity for the students to share their responses. Members from each group can come to the Smart Board and complete portions of the problem.

Monitoring Student Responses

- Students make their mathematical thinking and understanding public by responding to the worksheet questions in their small groups.
- Students are expected to show their work on the land assessment worksheet.
- Ensure that each member of the group is working on the project by monitoring the group work.
- Simplify this model for a group of students who are experiencing difficulty. This could be accomplished by clarifying the directions or scenario of the problem, providing hints to assist the group of students, asking students further questions to determine how to solve the problem, displaying the problems on a bigger display for an additional visual representation, or any other method determined by the teacher that will assist the students.
- To summarize the lesson, students will be provided 10 minutes after the completion of the worksheet for whole class discussions concerning their ideas and reasoning on the roofing materials problem.

Assessment

- The students will be assessed by the completion of the land assessment worksheet. The answer key for the land assessment worksheet is attached. Students can be given partial credit if their work has been shown.
- Students will also be assessed by their participation in the small group work and the whole class discussion. The participation grade will be at the teacher's discretion.

Extensions and Connections (for all students)

- To extend this lesson, the students will be asked to write a ½ -1 page journal entry. The students can answer the following questions:
    - How do you think that land is surveyed? Do you think that surveyors use geometry formulas to determine the area of land? Why or why not?
    - Possible extra credit project: Research land surveying or interview a land surveyor. What are the responsibilities of a land surveyor? What do land surveyors do on a daily basis? Describe the tools and math that land surveyors are required to use.

Strategies for Differentiation

The differentiation strategies might include but are not limited to the following list created specifically for ESL students. Feel free to adopt these to your lesson:

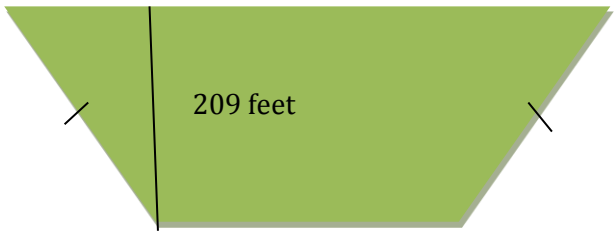
- Make instruction more concrete, visual, collaborative, and hands-on
- Focus on mathematical content rather than on linguistic form (simplify word problems without changing the math meaning)
- Language and content should be presented simultaneously
- Preferential Seating (near teacher or next to a buddy, native language if possible)
- Write legibly and in print
- Step by step instructions (orally and in writing).
- Give EOL students more time for questions and answers.
- Use of Dictionaries
- Simplify the language used rather than the mathematical concepts taught (use known vocabulary and simple sentence constructions).
- Observe and record students' participation in small group activities.

  The accommodations are adopted from the following source.
  http://www.doe.virginia.gov/VDOE/Instruction/ESL/LEPmathResource.pdf

Land Assessment Worksheet

A recent tornado in a small town in Southwest Virginia destroyed and damaged multiple plots of land. The insurance company has been surveying these plots of land to write their report on the damage assessment. One plot of land is the shape of a trapezoid, and the insurance company must determine the area of the land.

374 feet



228 feet

Part 1:

1. What is the area formula for a trapezoid?

2. What is the area of the trapezoid shown above?

3. Based on the picture provided above, determine the other two side lengths of the trapezoid.

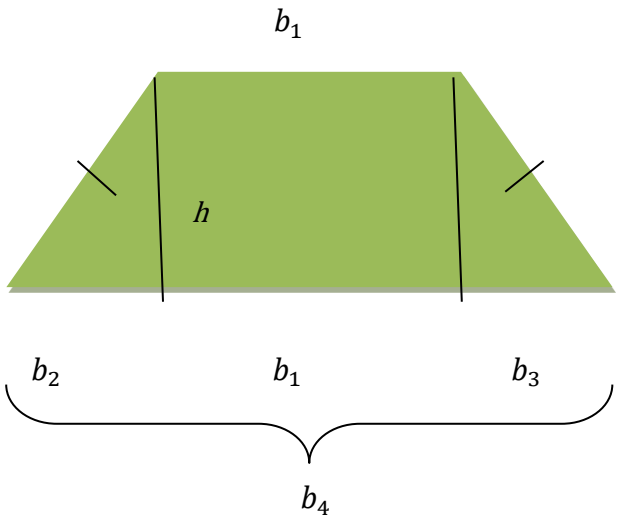
4. What is the perimeter of the trapezoid?

Part 2:

Determine and verify 3 other methods to prove that area formula for a trapezoid. You may draw any sort of trapezoid that you want. Show each step of your work.

Area formula of a trapezoid: $A = \frac{1}{2}h(b_1 + b_2)$

Example:



Divide the trapezoid into 2 triangles and a rectangle. Note that the sum of $b_2 + b_1 + b_3 = b_4$

The area of the rectangle is: $A = b_1h$

The areas of the triangles are: $A = \frac{1}{2}b_2h$ and $A = \frac{1}{2}b_3h$

Then, the area of the trapezoid is the sum of the areas.

$A = b_1h + \frac{1}{2}b_2h + \frac{1}{2}b_3h$

$A = \frac{1}{2}h(2b_1 + b_2 + b_3)$

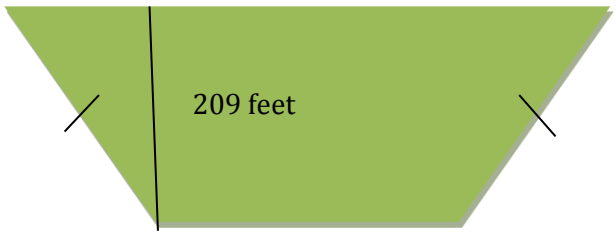
$A = \frac{1}{2}h(b_1 + b_1 + b_2 + b_3)$

- ✓ $A = \frac{1}{2}h(b_1 + b_4)$

Land Assessment Worksheet- Answer Key

A recent tornado in a small town in Southwest Virginia destroyed and damaged multiple plots of land. The insurance company has been surveying these plots of land to write their report on the damage assessment. One plot of land is the shape of a trapezoid, and the insurance company must determine the area of the land.

374 feet



228 feet

Part 1:

1. What is the area formula for a trapezoid?

   $A = \frac{1}{2}h(b_1 + b_2)$

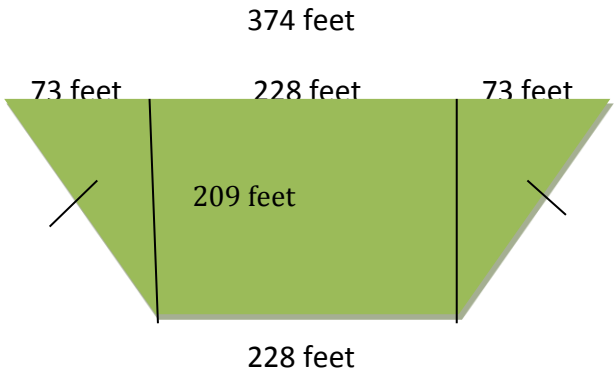
2. What is the area of the trapezoid shown above?

   Given that $h = 209\ ft;\ b_1 = 374\ ft;\ b_2 = 228\ ft$

   $A = \frac{1}{2} * 209(374 + 228)$

   $A = 62909\ ft^2$

3. Based on the picture provided above, determine the other two side lengths of the trapezoid.



We are able to split b_1 into 3 portions by calculating: 374-228 = 146/2 = 73.

Using the Pythagorean Theorem,

$$209^2 + 73^2 = c^2$$

$$49010 = c^2$$

$$c = 221.38$$

The two unknown sides of the quadrilateral both are 221.38 feet since the two lengths are congruent.

4. What is the perimeter of the trapezoid?

$$P = 374 + 228 + 2(221.38) = 1044.76$$

The perimeter of the trapezoid is 1044.76 feet.

Part 2:

Answers can vary. Students could divide the trapezoid into 2 triangles, one rectangle and one triangle, 3 triangles, etc. The teacher can examine the students' work and determine if the verification process is correct.