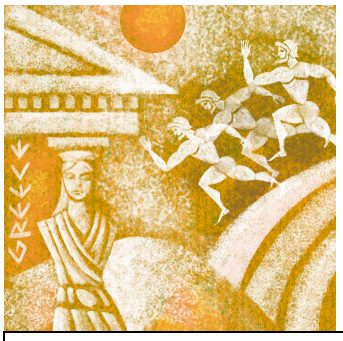

Right Triangle Sports

I. UNIT OVERVIEW & PURPOSE:

This unit will be involving the students in mathematical models relating sport ideas along with algebraic properties to understand the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry.

II. UNIT AUTHOR:

Holly Bautts Kidd; Woodside High School; Newport News, Virginia
Allena M. Poles; King William High School; King William, Virginia
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III. COURSE:

Mathematical Modeling: Capstone Course

IV. CONTENT STRAND:

Geometry & Algebra

V. OBJECTIVES:

- Understand the Pythagorean Theorem and how it is related to the side lengths of a right triangle
- Use the Pythagorean equation to solve for a missing side length in a right triangle given the measures of the other two side lengths.
- Discover the converse of the Pythagorean Theorem
- Use the formulas for special right triangles (45°-45°-90° and 30°-60°-90°) to solve for missing side lengths in these types of triangles
- Solve right triangles using trigonometric ratios of sine, cosine, and tangent

VI. MATHEMATICS PERFORMANCE EXPECTATION(s):

MPE. 1 - The student will solve practical problems involving rational numbers (including numbers in scientific notation), percents, ratios, and proportions.
MPE. 5 - The student will solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry.

VII. CONTENT:

Learning properties of right triangles and trigonometry and applying the properties to designing various sporting complexes.

VIII. REFERENCE/RESOURCE MATERIALS:

Smart Board (preferred; other software could be used)
TI-84 Graphing Calculator/ Projector
Geogebra Software
GeoSketch Pad software

IX. PRIMARY ASSESSMENT STRATEGIES:

Students will be assessed with journal writings, presentations, answering questions, completing/designing spreadsheets (to include formulas) and various projects that will be graded using a rubric.

EVALUATION CRITERIA:

For the final lesson, there is a rubric that is attached.

X. INSTRUCTIONAL TIME:

This unit will be based upon 5-90 minute block classes.

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in collaboration with
Mr. Michael Bolling, Virginia Department of Education

Lesson 1: Pythagorean Sports

Strand
Geometry

Mathematical Objective(s)
The student will:
- Identify the parts of a right triangle (right angle, acute angles, hypotenuse, and legs)
- Find the square root of a value and express this new value as a simplified square root.
- Solve a quadratic equation that involves computations including addition or subtraction and square root.
- Understand the Pythagorean Theorem and how it is related to the side lengths of a right triangle
- Use the Pythagorean equation to solve for a missing side length in a right triangle given the measures of the other two side lengths.

Mathematics Performance Expectation(s)

MPE. 1 - The student will solve practical problems involving rational numbers.
MPE. 5 - The student will solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry.

Related SOL
SOL G.8 - The student will solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry.
(This lesson will focus on the Pythagorean Theorem. Other concepts within this SOL will be addressed at other times in the unit.)

NCTM Standards
- use symbolic algebra to represent and explain mathematical relationships.
- draw reasonable conclusions about a situation being modeled
- analyze properties and determine attributes of two- and three-dimensional objects
- use trigonometric relationships to determine lengths and angle measures.
- use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture.

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Additional Objectives for Student Learning (include if relevant; may not be math-related):

- The student will apply geometry background knowledge and concepts to solve real-world applications.
- The student will use Geogebra software to investigate concepts dynamically.
- The students will apply their knowledge of right triangles to designing different types of sports and sports fields

Materials/Resources

- Classroom Set of TI-84 Graphing Calculators
- Geogebra Software
- Image of Baseball Field
- Scissors
- Paper
- Index Cards

Assumption of Prior Knowledge

- Students should be knowledgeable of the properties of a square (four congruent sides, four right angles, diagonals are congruent and bisect each other and the angles, and that a square forms two right triangles when the square is cut by a diagonal) They also should be knowledgeable of midpoints.
- Students should be knowledgeable of the properties of a diamond (rhombus) and the relationship it has with a square.
- Students should be knowledgeable of vocabulary involving a right triangle including right angle, acute angle, legs, and hypotenuse. The student should also be knowledgeable of the definition of an isosceles triangle, as well as that of a scalene triangle.
- Students should be operating on at least a Level 2, which is the Analysis level on the Van Hiele scale. They should be able to recognize and analyze the properties of triangles, but more importantly they need to recognize, analyze, and synthesize real-world applications using right triangles.
- Students should understand the concept of what it means for a number to be squared and inversely should understand what occurs when the square root of a number is taken.
- Students should be able to recognize and utilize the area formula of a square and be able to find the area of square given the measure of one of its side lengths.
- Students should have some basic knowledge of a baseball field; if not, then pictures of various baseball fields will be mentioned and explained. If it seems more students are not familiar with a baseball field, then the class will walk to the baseball field and actually "walk the field."
- Students should have basic knowledge of how to research various sport fields/courts to find and look up their dimensions and shape.

Introduction: Setting Up the Mathematical Task

- The goal of this lesson is to recognize and utilize the Pythagorean Theorem by connecting it to the dimensions of 1) a baseball field and then later 2) various sport fields/courts.
- To get students involved with the lesson and start gathering background knowledge, the following questions will be posed in order to facilitate general class discussion:
    - How many of you play or like baseball?
    - How many of you have seen a baseball game either on TV or in person?
- Students will be paired up and given the following "Think About It" Questions. This will be a 15 minute activity, which will be student centered and the teacher will act as a facilitator. One member of the pair will record their answers and then as a pair they will share their information with the entire class.
    - Think About It Questions:
        - Why is a baseball field usually called a diamond?
        - What is a better name for the shape that bases make in the main part of the baseball field?
        - Why does the baseball field have the shape you described in the previous questions?
        - What are some more specific properties of the shape of the infield of the baseball diamond?
    - The "Think About It" discussion will lead students to understand that a baseball field is a square and that if a square is cut along one of its diagonals it forms a right triangle. This follow-up discussion should be approximately 10 minutes.

Student/Teacher Actions:

Introduction/ Background

- To demonstrate, the teacher will construct a right triangle with legs of 3 and 4 on Geogebra software. Squares will also be constructed adjacent to the side lengths. [If computers are available to students, they can complete the Geogebra exercise below. If students do not have available computers, the teacher will demonstrate].
- Students will individually, calculate the area of the squares adjacent to side lengths at their seats. Teacher will quickly monitor student responses and then call for student feedback.
- Teacher will then construct the square adjacent to the hypotenuse. Teacher will prompt for student responses as this construction takes place. Students will then add to their calculations what the area of the square adjacent to the hypotenuse equals.
- Teacher will prompt for student responses and then students will recall the relationship of the squares of the legs to the square of the hypotenuse.

Student Exploration:

This part of the lesson will be approximately 55 minutes and will show how the Pythagorean Theorem is formed in order to understand how it can be used to solve for missing side lengths within a right triangle.

To the Baseball Field

- The teacher will post an image of a real baseball field on the Smart Board, or other digital sources (a magazine picture could even work) and if students want a hard copy of a baseball field then they can request one; (teacher may accompany this image by wearing a baseball cap or playing an available sound clip of "Take Me Out to the Ball Game" from YouTube or other forms of sound devices)
- Then students will be prompted with the following questions to discuss as a pair and then share with the entire class:
    - Does anyone know exactly how many feet there are between bases?
    - Does anyone know the distance from home plate to second base?
    - What about the distance from the pitcher's mound to second base?

  During this discussion, call on various students (techniques in choosing students may vary per class) for the answers of the measurements; the answers should be recorded for comparison purposes. There may be different correct answers because of the level of baseball being played.
- Students will work together preferably in groups of two, no more than 3 in a group to facilitate peer interaction to complete the following task:

  1) Given the distance between each base, find the distance the catcher would have to throw the ball from home plate to second base. Student should be simplifying any radical solutions (may obtain a decimal answer to help with visualization).

  2) Given the distance from the pitcher's mound to home plate and your previous answer, is the pitcher's mound the midpoint (this should be background knowledge) of home plate and second base? (If students are having trouble with this question, the teacher may need to revisit the idea of midpoint).

Now, the students will explore various "playing fields" and their dimensions:
Then each group will be given an index card: Each index card will have a different sport listed with the dimensions of the playing field included: (teacher may need to have more than one index card for each sport due to the number of groups)

- Volleyball Court – 60 feet x 30 feet
  Use 10 feet : 1 inch scale
- Gymnastics Spring Floor – 40 feet x 40 feet

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Use 10 feet : 1 inch scale

- Hockey Court – 85 feet x 200 feet
  Use 30 feet : 1 inch scale
- Football Field – 160 feet x 300 feet
  Use 50 feet : 1 inch scale
- Soccer Field – 180 feet x 300 feet
  Use 50 feet : 1 inch scale

• The index cards will:

1. Instruct the students to draw to scale (each scale will be given on the index card) the specified playing field and cut it out.
2. Draw one diagonal across the playing field and cut along the diagonal to form two congruent triangles.
3. Draw and cut out three squares – each square should have side lengths congruent to a side of the triangle cut out above.
4. Calculate the area of each square.
5. Sum the areas of the two smaller squares. (This will be different than the earlier activity because the dimensions of the "playing field" are different than that of the baseball field.)

Monitoring Student Responses

- To understand student thinking and knowledge of the task, the teacher will consistently probe students for responses and discussion. Examples of discussion questions that can be asked are: "How did you use the scale to draw your sports field?", "How does your field compare to the baseball diamond?", "How do the areas of your squares compare to the areas of the squares for the baseball diamond?", etc. Students will be encouraged to engage in positive discussion with each other and the teacher.
- Teacher will monitor student responses and seek any mathematical misunderstandings and/ or errors and probe the students to correct their own errors. The teacher will guide the students to correct responses, but allow the students to seek the correct the answers through their own thought and exploration.

Summary

- Each group will be asked to draw their triangle in a Geogebra file (either on their own computer if available, or the teacher can draw them in her computer if student computers are not available), and one file will be saved on a shared drive for students to access. This file will be accessed for the homework assignment below.

Assessment

The students will be assigned a homework activity to access the Geogebra file (in the library if not available at home) containing the triangles of each group and then create a spreadsheet to calculate the diagonal of a rectangle using the Pythagorean Theorem. When the spreadsheet is created they are to add other random lengths and write a paragraph to compare their findings for each different rectangle and discuss their observations.

Scoring Rubric for Spreadsheet

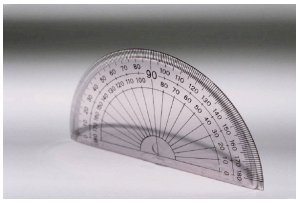
- Dimensions accurately transferred to spreadsheet from Triangle Geogebra File – 10 points
- Spreadsheet Columns are properly labeled – 10 points
- The Hypotenuse Column is properly programmed with the Pythagorean Theorem – 50 points
- Written Paragraph – 30 points

Extensions and Connections (for all students)

- EXTENSION: If students understand this activity and its connections then as an exit question the students will be asked: "If the catcher throws the ball at 75mph, how long would it take for the ball he or she has thrown to reach second base?" This is assuming that the path of the thrown ball is on a straight line.

  (TOOK OUT STUDENTS WRITING THE SIMPLIFIED RADICAL FORM…BECAUSE THEY SHOULD BE DOING THIS ANYWAY)

  - Angles within the field could also be discussed beyond the right angle. i.e. What is the angle made from home plate to second base to first base and why does the angle have that degree measure.
  - To address needs of kinesthetic and visual learners, students could recreate a baseball field within the classroom if time allows or go out to the baseball field; also if the students want/need to create their "playing fields" with the second activity they may do so using paper or constructing a smaller version in class or for homework.



Lesson 2: Discovering the Converse of Pythagorean Theorem

Strand
Geometry, Number, Algebra, and Measurement

Mathematical Objective(s)
Discovering the converse of the Pythagorean Theorem

Mathematics Performance Expectation(s)
MPE. 1 - The student will solve practical problems involving rational numbers (including numbers in scientific notation), percents, ratios, and proportions.

MPE. 5 - The student will solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry.

Related SOL

G.8 The student will solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry.

NCTM Standards

- analyze properties and determine attributes of two- and three-dimensional objects;
- establish the validity of geometric conjectures using deduction, prove theorems, and critique arguments made by others;
- use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture

Additional Objectives for Student Learning (include if relevant; may not be math-related):
The students will use their reading skills and decipher word problems: what is given/known, and what is it asking for/unknown.
The students will use Geogebra or Geosketch Pad software to investigate concepts dynamically**.**

Materials/Resources

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Materials needed for this lesson:

- rulers
- string (two 3 feet strings per group)
- paper clips (six large clips per group)
- protractors
- patty paper
- GeoSketch Pad or Geogebra software

Assumption of Prior Knowledge

- Prior to this lesson the student must understand the Pythagorean Theorem and its importance and usage. The student must also understand the idea of Pythagorean Triples or Pythagorean Triples could be reviewed prior to the investigation.
- Students need to be comfortable reading word problems and deciphering what is given and what is unknown in the problem.
- A typical student must be on level 2 (analysis) of Van Heile, maybe even the next level because the investigation into the converse of the Pythagorean Theorem has the students discover the converse on their own. This will also lead to having the students get started on completing the proof for the converse.
- Some students will require more direction with the investigation because they do not like discovering and seeing what happens; this type of student likes to be told theorems and shown examples and they like to get to work right away. However, a lot of students are going to appreciate the investigation because they will be allowed to do something hands on.
- This lesson will begin with a Van Heile level 1 by offering a concrete reasoning to the converse of the Pythagorean Theorem and then it will move on into a Van Heile level 4 where the students will work on the proof of the Pythagorean Theorem.

Introduction: Setting Up the Mathematical Task

- In this lesson, the student will investigate the converse of the Pythagorean Theorem with a discovery hands-on activity. The student will also be working on reading comprehension, problem-solving skills and cooperative behavior. The student will then work through the proof of the Converse of the Pythagorean Theorem.
- This lesson should take a 90 minute block.
- Have a student write the Pythagorean Theorem on the board and ask students to write the "converse of the conditional statement." (i.e. If $a^2 + b^2 = c^2$, then the triangle is a right triangle.)

- Remind the students if the conditional statement and the converse are both true, then a true biconditional statement could be formed. Ask them to write the biconditional statement and tell them we are going to explore the truth of the converse today.

- Discuss Pythagorean Triples: three positive integers that work in the Pythagorean equation. Example and the most common is 3-4-5. $3^2 + 4^2 = 5^2$. And there are others. List one or two more sets of Pythagorean Triples.
- Get the class into groups (how many depends on size of the class) of 3 to 4 students with at least one student performing at a high level and at least one student performing at a low level.
- "What can you say about triangles in which the side lengths satisfy the Pythagorean equation?" "Can you come up with some Pythagorean Triples that the teacher has not already given?" Circulate and encourage the groups to try various Pythagorean Triples that were listed on the board. Even ask them to try some integers that are not part of the Pythagorean triples because right now we don't know if the converse of the Pythagorean Theorem is true.
- Once the class has discovered several groups of Pythagorean triples, have the students sort them into "families." Ex: one column could be 3-4-5; 6-8-10; 9-12-15; 12-16-20 and then and another could be 5-12-13; 10-24-26; etc. "How are the triples in a column related?" Teacher is trying to get student to discover the triples in a column are multiples of each other.
- "What could be the next triple in the second column?" "Find one more triple for the third column."
- Ask the students to use the discovered side lengths in the Pythagorean Theorem. "What are you noticing when you are making these substitutions into the Pythagorean Theorem?" Then ask the students, "If you were given three side lengths of a triangle, could you tell me if it was a right triangle?"
- While circulating around the room and observing the group work, assist students with the Pythagorean Theorem if and when necessary.
- Groups will share the Pythagorean triples that they discovered.

Student Exploration 1:

Small Group Work (investigation of the converse conjecture)
Whole Class Sharing/Discussion (at the beginning and throughout the lesson)
Student/Teacher Actions:

- The students will:

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- Step 1: As a group select one set of Pythagorean triples from the list. Using your string, mark off 4 points, A, B, C, and D to create three consecutive lengths from your set of triples.
- Step 2: Loop three paper clips on the string and then tie the ends together so that points A and D meet.
- Step 3: Then have three group members pull on a paper clip at the points A, B, or C to stretch the string tight.
- Step 4: With the corner of your paper, check the largest angle. What type of triangle is formed?
- Step 5: Select another set of Pythagorean Triples from the list and repeat the steps 1-4 with the new lengths.
- Step 6: Within your group, discuss the results and state the results as a conjecture.

- During the investigation, the teacher should be walking around and monitoring student responses and concerns.
  - During step 1, teacher may suggest to leave excess string at both ends because it will help tie the ends together easier.
  - During step 4, to verify the right angle, the student can use a corner of a piece of paper or some other object if a protractor is not available.
- Once the groups are done the students can share their work with a variety of lengths. Allow the students to discuss any measurement errors.
- Teacher should continue asking if the converse is always true. And looking at the students' conjectures that were asked for in step 6, agree on a statement of the conjecture. Then ask the students to answer the following question (they may need a few minutes to play with some numbers: "will the conjecture be true if you use different measure units so that the triples change, possibly to non-integers?"
- "Suppose one triangle has sides whose lengths are a Pythagorean triple and another triangle that has sides of a multiple of the Pythagorean triple. How are the triangles related?" (looking for them to answer that triangles with Pythagorean triples that are multiples of each other are similar triangles; this may not be answered until the students have done the next bullet)
- Ask students to draw triangles that are <u>not</u> right triangles. Substitute the side lengths into the Pythagorean Theorem, always using the longest side to substitute for "C". What do you notice about A^2+B^2 and C^2 in these triangles? When is C^2 greater than A^2+B^2, and when is C^2 smaller than A^2+B^2?
- Then the student will try drawing triangles and looking for patterns, but the teacher should try to not answer the question yet. Here the students in their group or individually can get

onto GeoSketch Pad or Geogebra and work on demonstrating this concept. This can help the students reinforce the concept of similar triangles.

Monitoring Student Responses

- To understand student thinking and knowledge of the task, the teacher will consistently probe students for responses and discussion. Students will be encouraged to engage in positive discussion with each other and the teacher.
- Teacher will monitor student responses and seek any mathematical misunderstandings and/ or errors and probe the students to correct their own errors. The teacher will guide the students to correct responses, but allow the students to seek the correct the answers through their own thought and exploration.

Student Exploration 2:

- Step 1: Giving the students the beginning of the proof of the Converse of Pythagorean Theorem; allow them to read and critique the outline of the proof.
  - Conjecture: If the lengths of the three sides of a triangle make the Pythagorean equation true, then the triangle is a right triangle.
  - Given: a, b, c, are the lengths of the sides of Triangle ABC and $a^2+b^2=c^2$
  - Show: Triangle ABC is a right triangle.
  - Plan: Construct a second triangle, right triangle DEF (with angle F being the right angle), with legs of lengths a and b and hypotenuse being x. You need to show that x = c, so that the triangles are congruent. Show angle C and angle F are congruent. If angle C is a right angle, then Triangle ABC is a right triangle. Proof is done.
- As the students are reading this and critiquing the proof, observe their facial expressions and engage in discussions with those who seem to have ideas but maybe don't speak up. Students should be able to go through the outline of the proof and be able to make a geometrical/algebraic understanding of the terminology; if not, then they need to jot down where/how they got confused or off track.
- From step 1, teacher will have two options:
  - Option 1: if students are familiar with proofs, then challenge them to work in pairs on completing the proof.
  - Option 2: if class is commonly weak, then as a class have the discussion to fill in the proof and then working on writing up a great model for the proof.

Assessment

- **Questions**
  - If a triangle has side lengths 7, 15, and 20, will the triangle a right triangle, an obtuse triangle, or an acute triangle? Justify your answer.
  - If the two sides of a triangle have lengths 7 and 15, how long should the third side be to form the hypotenuse of a right triangle? Justify your answer and/or explain the process.
  - If the two sides of a triangle have lengths of 7 and 15, how long should the third side be to form an obtuse triangle? Justify your answer.
  - If the two sides of a atriangle have lengths of 7 and 15, how long should the third side be to form an acute triangle? Justify your answer.
- **Journal/writing prompts**
  - Describe in words how to tell if a triangle is right, acute, or obtuse when the side lengths are given. Will there ever be side lengths given that may not form a triangle at all? Explain your thought process.

Extensions and Connections (for all students)

- As an extension, the students will use Geogebra or GeoSketch Pad software to draw triangles. Make sure students label and measure angles and sides. They need to calculate squares of the lengths of the sides or construct squares on the sides and measure the area. Then they need to drag the vertices until the sum of the squares of the lengths of the two smallest sides equals the square of the length of the largest side. They should find a right angle.
- Also, if this is a more advanced group, at the beginning of the first exploration the teacher may allow the students in groups or individually to come up with several of Pythagorean Triples instead of giving them several groups.

Strategies for Differentiation

- List ideas for addressing needs of a diverse population of students such as:
  - Instruct visual learners to label the sides of the triangles.
  - Be sure to draw pictures on the board and show what sides of the triangle are the legs and the hypotenuse. Talk with them individually to confirm that they understand what the new terms mean.
  - Ask the students to explain how the converse of the Pythagorean Theorem can be used to identify if a triangle is acute or obtuse.

Lesson 3: Special Right Triangles in Sports

Strand
Geometry

Mathematical Objective(s)
TSW:
- Express values of square roots as simplified square roots
- Use the formulas for special right triangles (45°-45°-90° and 30°-60°-90°) to solve for missing side lengths in these types of triangles

Mathematics Performance Expectation(s)

MPE. 1 - The student will solve practical problems involving rational numbers (including numbers in scientific notation), percents, ratios, and proportions.
MPE. 5 - The student will solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry.
MPE. 7 - The student will use similar geometric objects in two- or three-dimensions to
a) compare ratios between side lengths, perimeters, areas, and volumes;
d) solve real-world problems about similar geometric objects.

Related SOL
SOL G.8 - The student will solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry. (This lesson will focus on Special Right Triangles.)

NCTM Standards
- analyze properties and determine attributes of two- and three-dimensional objects;
- establish the validity of geometric conjectures using deduction, prove theorems, and critique arguments made by others;
- use geometric ideas to solve problems in, and gain insights into, other disciplines and other

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areas of interest such as art and architecture

Additional Objectives for Student Learning (include if relevant; may not be math-related):
- The student will use Geogebra software to investigate concepts dynamically.

Materials/Resources
- Classroom Set of TI-84 Graphing Calculators
- Geogebra Software
- Paper
- Markers
- Scissors
- Index Cards

Assumption of Prior Knowledge
- Students should be knowledgeable of the properties of a square (four congruent, four right angles, diagonals are congruent and bisect each other and the angles, and that a square forms two right triangles.) They also should be knowledgeable of midpoints.
- Students should be knowledgeable of the properties of a rectangle, as well.
- Students should be knowledgeable of vocabulary involving a right triangle including right angle, acute angle, legs, and hypotenuse. The student should also be knowledgeable of the definition of an isosceles triangle, as well as that of a scalene triangle and equilateral triangle.
- Students should be operating on at least a Level 2, which is the Analysis level on the Van Hiele scale. They should be able to recognize and analyze the properties of triangles, but more importantly they need to recognize, analyze, and synthesize real-world applications using right triangles.
- Students should understand the concept of what it means for a number to be squared and inversely should understand what occurs when the square root of a number is taken.
- Students should have some basic knowledge of a various sports fields.

Introduction: Setting Up the Mathematical Task

- The goal of this lesson is to recognize and utilize the properties and formulas for Special Right Triangles and relate these properties to that of playing fields of various sports.
- Students will be paired up and given the following "Think About It" Questions. This will be a 15 minute activity, which will be student centered and the teacher will act as a facilitator. One member of the pair will record their answers and then as a group they will share their information with the entire class.

- Think About It Questions:
  - What do you recall about the measurements as it relates to the lengths between bases of the baseball field and the distance from second base to home base? How does this relate to other triangles we worked with that were similar to this one? Also, apply this to what occurred in the rectangular sports fields you explored previously.
  - What type of triangle was made by cutting the baseball diamond along the imaginary line from second base to home plate? Also, think of the type of triangle that was made by cutting a diagonal in the rectangular sports fields?
  - Can you connect your responses to the first question and make a general statement about what occurs in triangles of the type that you described in the second question?

  - The goal of this discussion will be to lead students to the understanding that an isosceles right triangle has two legs that are always congruent and the hypotenuse is the length of the leg times the square root of two. Also, this should lead students to understanding that when triangles are made from the diagonals of rectangles and form 30-60-90 triangles, the hypotenuse is always twice the shorter leg and the longer leg is equal to the shorter leg times the square root of three. This follow-up discussion should be approximately 15 minutes.

Student/Teacher Actions:

Introduction/ Background

- The teacher will construct a square on Geogebra (or similar shape drawing software) and draw one diagonal of the square.
- Teachers will facilitate students and lead conversation about the angles of the square and what occurs when a diagonal is cut in a square. This conversation will lead students to understand that since the diagonals of a square bisects the angles; the square has been cut into two isosceles right triangles with base angles of 45°. Explain to the students that this is identified as a "45-45-90 triangle."
- The teacher will then construct an equilateral triangle on Geogebra (or similar shape drawing software) and repeat the discussion that followed. This conversation will lead students to understand that the bisector of an equilateral triangle forms two congruent scalene right triangles with angle measurements of 30° and 60°. Explain to the students that this is identified as a "30-60-90 triangle."

Student Exploration:

This part of the lesson will be approximately 60 minutes and will first involve the students deriving the formulas for special right triangles and then using them to compare their solutions of measurements found from Pythagorean Theorem in particular playing fields to those found using formulas for Special Right Triangles.

Deriving Formulas for Special Right Triangles (35 minutes)

Students will derive the formulas for special right triangles using the Pythagorean Theorem using the given steps.

Part 1: 45° - 45° - 90° Triangle

1. Cut a square from the paper by folding one corner diagonal to the opposite side of the paper.
2. Draw the diagonal along the fold you made.
3. Label the diagonal "d" and the side lengths of the square "x"
4. Using the variables of "d" and "x", solve for d in terms of x using the Pythagorean Theorem based upon one of the triangles formed on notebook paper.
5. Make a connection between what you have just found and what you discovered in the "Think about It" questions earlier.

Part 2: 30° - 60° - 90° Triangle

1. Construct and cut out an equilateral triangle from your paper.
2. Fold the triangle in half; then trace the fold.
3. Label the fold, "a" as this is the altitude of the triangle. Consider each side of the equilateral triangle to be "x" as in the previous derivation.
4. Consider one of the triangles formed from your fold. Now consider that if the hypotenuse is "x", how would we describe the shorter leg of the triangle formed in terms of x. Label this on your new triangles. (Students should discover that the shorter leg is ½ x).
5. Using the "a", "x", and "½ x". Solve the Pythagorean Theorem using these variables and expressions to solve for a.
6. Make a connection between what you have just found and what you discovered in the "Think about It" questions earlier.

To the Sports Field (25 minutes)

Students will revisit the index cards they created from last class. (Note the addition of the baseball that will have to be made from an additional index card)

- Volleyball Court – 60 feet x 30 feet
- Gymnastics Spring Floor – 40 feet x 40 feet
- Hockey Court – 85 feet x 200 feet
- Football Field – 160 feet x 300 feet
- Soccer Field – 180 feet x 300 feet
- Baseball Field – 90 feet x 90 feet

- Students will access the Geogebra files that were created including all of the triangles.
- Students will be asked if their sport field has the appropriate dimensions to form one of the special right triangles (45-45-90 or 30-60-90).
    - If the answer is no, ask the students to adjust the length of their field so that it will be an appropriate measure to form a 30-60-90 or 45-45-90 triangle.
- Students will then be asked to calculate the diagonals of the rectangles using their new formulas and record them. They will then compare these results to the results that they previously received using the Pythagorean Theorem.

Monitoring Student Responses

- To understand student thinking and knowledge of the task, the teacher will consistently probe students for responses and discussion. Students will be encouraged to engage in positive discussion with each other and the teacher.
- Teacher will monitor student responses and seek any mathematical misunderstandings and/ or errors and probe the students to correct their own errors. The teacher will guide the students to correct responses, but allow the students to seek the correct the answers through their own thought and exploration.

Summary

- Students will be given for homework along with the assignment described in the assessment to write two to three sentences that discuss why the Special Right Triangles formulas worked in the various sports fields.

Assessment

The students will create two of their own "sports". One of the sports must have a square field and the other rectangular. They must create a presentation using poster board for each one of their new sports where one of the side of the board describes how their sport is played, as well as other details (number of players, type of ball, other utilities used) and the other side of the board shows a scale replica of the field used in their sport. Extra points will be given to students who can apply previously learned Geometry topics to other areas of their sport. The student will then be asked to develop the dimensions (side lengths and diagonals) of the playing field and show how these dimensions reflect the formulas for special right triangles. This must also be presented on the explanation side of the poster board.

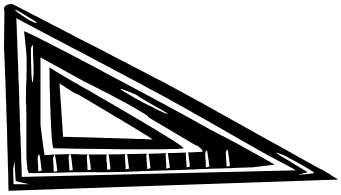
Extensions and Connections (for all students)

- Extension: Students will figure out what happens to the remaining measurements of special right triangles if one measurement is doubled, halved, tripled, etc. by using the formulas for special right triangles derived in this lesson.

Strategies for Differentiation

- Make sure visual learners and ELL learners are always drawing and labeling the triangles. They should label the short leg, long leg, and hypotenuse of the 30-60-90 triangle and label the legs and hypotenuse of the 45-45-90 triangle.

- For high-ability students, give the coordinates of one vertex of a special right triangle. Have them determine two other coordinates for a 45-45-90 triangle, and also the other two coordinates for a 30-60-90 triangle.



Lesson 4: Right Triangle Trigonometry in Sports

Strand
Geometry

Mathematical Objective(s)
TSW:
- Solve right triangles using trigonometric ratios of sine, cosine, and tangent

Mathematics Performance Expectation(s)

MPE. 1 - The student will solve practical problems involving rational numbers (including numbers in scientific notation), percents, ratios, and proportions.
MPE. 5 - The student will solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry.

Related SOL
SOL G.8 - The student will solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry. (This lesson will focus on Right Triangle Trigonometry.)

NCTM Standards
- analyze properties and determine attributes of two- and three-dimensional objects;
- establish the validity of geometric conjectures using deduction, prove theorems, and critique arguments made by others;
- use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture
- use trigonometric relationships to determine lengths and angle measures.

Additional Objectives for Student Learning (include if relevant; may not be math-related):
- The student will use Geogebra software to investigate concepts dynamically.

Materials/Resources
- Classroom Set of Graphing Calculators; TI-84 recommended
- Geogebra Software or other shape building software
- Smart Board and other classroom interactive boards and/or displays
- Paper
- Trig Spreadsheet (Attached)

Developed by Dr. Agida Manizade & Dr. Laura Jacobsen, Radford University MSP project
in collaboration with
Mr. Michael Bolling, Virginia Department of Education

- Index Cards from Previous Lessons

Assumption of Prior Knowledge

- Students should be knowledgeable of vocabulary involving a right triangle including right angle, acute angle, legs, and hypotenuse. The student should also be knowledgeable of the definition of an isosceles triangle, as well as that of a scalene triangle and equilateral triangle.
- Students should be operating on at least a Level 2, which is the Analysis level on the Van Hiele scale. They should be able to recognize and analyze the properties of triangles, but more importantly they need to recognize, analyze, and synthesize real-world applications using right triangles.
- Students should understand the concept of what it means for a number to be squared and inversely should understand what occurs when the square root of a number is taken.
- Students should understand the concept of rounding and what it means to round a number to a certain decimal place.

Introduction: Setting Up the Mathematical Task

- The goal of this lesson is to recognize and utilize the trigonometric ratios involved with right triangles and relate these ratios to solving triangles created in real world sports activities.
- Students will be paired up and given the following "Think About It" Questions with the attached spreadsheet. This will be a 30 minute activity, which will be student centered and the teacher will act as a facilitator. One member of the pair will record their answers and then as a group they will share their information with the entire class.
- The teacher will draw three different right triangles in Geogebra. The lengths of all three sides of the triangle should be labeled, and one of the angles should be labeled. The students should complete the first seven columns of the spreadsheet. (Spreadsheet is attached) When the students are finished, the teacher will ask the students to press the "sin" button in their calculator and input the first angle measure. The students will be asked to compare the answer to the answers in their chart. Ask them if they can explain how the calculator calculated the answer. Have the students label the eighth column "Sin (angle)" and ask them to complete the column. Then have them predict what will happen with cosine and tangent. After they have made their predictions, have the students complete the last two columns of their table with "Cos (angle)" and "Tan (angle)."
- The teacher will create a right triangle on Geogebra with angle measures of 40°, 50°, and 90°, but will only display the 40° and denote the right angle. The teacher will also give the measurement of the hypotenuse and note the measurement of one of the legs as "x" leaving the other leg unlabeled. The following questions will then be given to the students.
    - Think About It Questions:

- What measurements are you given in the triangle? Use proper vocabulary.
- Why are you not able to use the Pythagorean Theorem to find x?
- Why are you not able to use Special Right Triangles to find x?
- Do you think there is a way to solve for x? If so, what do you think it would involve?
- This discussion would hopefully lead to students to understand that because they cannot use the Pythagorean Theorem or properties of Special Right Triangles, that there is some other method that has to be used. Discussion will also include the methods the students devised that could possibly be used. Hopefully some students will see that you have to use the measurements you have in some format along with the fact that it is a right triangle to solve for the desired measurement.

Student/Teacher Actions:

Introduction/ Background

- The teacher will display a right triangle on the Smart Board and facilitate in helping students understand the vocabulary needed to use trigonometric ratios including what the opposite and adjacent sides are in relationship to the acute angles in a right triangle. Students should have prior knowledge that the side opposite of the right angle of a right triangle is the hypotenuse.

Student Exploration:

This part of the lesson will be approximately 60 minutes and will first involve the students solving right triangles.

Solving right triangles for a missing side

Students will solve right triangles using the given steps. The teacher should give the students 5 more right triangles to solve on their own using the steps below.

1. List given information using appropriate vocabulary.
2. Label all parts of the right triangle (hypotenuse, opposite leg, adjacent leg, angle)
3. Decide which trigonometric function is appropriate (sine, cosine, or tangent).

4. Write the necessary equation to solve the right triangle.
5. Solve the right triangle.

Sports

Students will revisit the index cards they created from last class, lesson 3. (Note the addition of the baseball that will have to be made from an additional index card)

- Volleyball Court – 60 feet x 30 feet
- Gymnastics Spring Floor – 40 feet x 40 feet
- Hockey Court – 85 feet x 200 feet
- Football Field – 160 feet x 300 feet
- Soccer Field – 180 feet x 300 feet
- Baseball Field – 90 feet x 90 feet

Teacher will now ask the students, "If you knew the sides of the triangle, how would you find the missing angles?"

The students will use the sports cards and complete all missing sides and angles (when the diagonals are drawn) for the fields.

Monitoring Student Responses

- To understand student thinking and knowledge of the task, the teacher will consistently probe students for responses and discussion. Students will be encouraged to engage in positive discussion with each other and the teacher.
- Teacher will monitor student responses and seek any mathematical misunderstandings and/ or errors and probe the students to correct their own errors. The teacher will guide the students to correct responses, but allow the students to seek the correct the answers through their own thought and exploration.

Summary

- Students will be asked to summarize the following on a note card:
  - When will you use the Pythagorean Theorem to solve a right triangle?
  - When will you use special triangle formulas to solve right triangles?
  - When will you use trigonometric functions to solve right triangles?

Assessment

Students will create a culminating power point presentation on how they were able to use Pythagorean Theorem, Special Right Triangles, and Trigonometric Ratios to find measurements in a triangle, and ultimately solve for a right triangle. They will also connect these methods to their various sports explorations. Their presentation should discuss the connection between these methods and when it is appropriate to use each method in working with right triangles.

Extensions and Connections (for all students)

- Extension: Have students create a unit circle and show the connection between special right triangles and trigonometric ratios; show how the trigonometric ratios for basic increments of angles can be found without using a calculator.

Strategies for Differentiation

- Make sure visual learners and ELL learners are always drawing and labeling the triangles. They should label the opposite leg, adjacent leg, and hypotenuse of all triangles.

- For high-ability students, give the coordinates of three vertices of a right triangle. Ask them to solve the right triangle; that is find the length of the three sides and the measures of the acute angles.

Right Triangle Trigonometry Exploration Worksheet

Meaure of Angle	Length of Hypotenuse	Length of Opposite Leg	Length of Adjacent Leg	Opposite Leg / Hypotenuse (4 decimal places)	Adjacent Leg / Hypotenuse (4 decimal places)	Oposite Leg / Adjacent Leg (4 decimal places)

Developed by Dr. Agida Manizade & Dr. Laura Jacobsen, Radford University MSP project
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Mr. Michael Bolling, Virginia Department of Education

Lesson 5: Right Triangle Trigonometry in Sports

Strand
Geometry

Mathematical Objective(s)
The student will:
- Solve right triangles by choosing trigonometry and the Pythagorean Theorem

Mathematics Performance Expectation(s)

MPE. 1 - The student will solve practical problems involving rational numbers (including numbers in scientific notation), percents, ratios, and proportions.
MPE. 5 - The student will solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry.

Related SOL
SOL G.8 - The student will solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry. **(This lesson will focus on Right Triangle Trigonometry.)**

NCTM Standards
- analyze properties and determine attributes of two- and three-dimensional objects;
- establish the validity of geometric conjectures using deduction, prove theorems, and critique arguments made by others;
- use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture
- use trigonometric relationships to determine lengths and angle measures.

Additional Objectives for Student Learning (include if relevant; may not be math-related):
- The student will use Geogebra software to investigate concepts dynamically.

Materials/Resources

- Classroom Set of TI-84 Graphing Calculators
- Geogebra Software
- Computers with Microsoft Office word and Excel for students
- Internet access for students

Assumption of Prior Knowledge

- Students should be knowledgeable of the properties of right triangles.
- Students should be knowledgeable of vocabulary involving a right triangle including right angle, acute angle, legs, and hypotenuse. The student should also be knowledgeable of the definition of an isosceles triangle, as well as that of a scalene triangle and equilateral triangle.
- Students should be able to recognize when to use the Pythagorean Theorem and when to use trigonometric ratios to solve right triangles.
- Students should be operating on at least a Level 2, which is the Analysis level on the Van Hiele scale. They should be able to recognize and analyze the properties of triangles, but more importantly they need to recognize, analyze, and synthesize real-world applications using right triangles.
- Students should understand the concept of what it means for a number to be squared and inversely should understand what occurs when the square root of a number is taken.
- Students should understand the concept of rounding and what it means to round a number to a certain decimal place.
- Students should have some basic knowledge of a various sports fields.

Introduction: Setting Up the Mathematical Task

- The goal of this lesson is to make use of all right triangle lessons that have been taught in this unit.
- Students will be paired up and given the following "Think About It" Questions. This will be a 15 minute activity, which will be student centered and the teacher will act as a facilitator. One member of the pair will record their answers and then as a group they will share their information with the entire class. The teacher will access stairway designs on the internet.
    - Think About It Questions:
        - Are all stairways designed at the same angle of elevation?
        - Are all steps the same height and depth?
        - What would be the overall height of a 15-step stairway if each step was 6" high and 8" deep?

- Show how right triangles can be used to find the height and angle of elevation of the overall stairway.
- How would this compare to a 15-step stairway if the height of each step was 6", but the depth of each step was 18"?

Student/Teacher Actions:

Introduction/ Background

- The teacher will display a right triangle using a Smart Board or other classroom resource and summarize the use of right triangles and how to solve them using trigonometric ratios, the Pythagorean Theorem, and Special Right Triangles. A review will also assist students to know when to use each technique.

Student Exploration:

This part of the lesson will be approximately 60 minutes. It will be the beginning of a real world design project.

Look at stadium design projects and begin team design (60 min)

Have the students look up various stadium designs on the internet. They should discover that like the stairways discussed, some stadium seating is at a steeper angle of elevation than others. Discuss when it may be better to have a larger angle of elevation and when it may be better to have a smaller angle of elevation.

The students are to design 4 different stadium seating areas. One for 200 fans, one for 2000 fans, one for 20,000, and one for 80,000 fans. They are to show how triangles are to be used in their design. Each individual seating area has to meet minimum space requirements:

Width requires minimum of 18"
Depth requires minimum of 26" (8" for leg space and 18" for seat depth)
Height requires minimum of 36" (18" for leg height and 18" for seat back height)

Designs need to keep the overall height of the stadium to a minimum while still reaching the minimum requirements given. Fans should be high enough to see the stadium over the person in front of them. Sketches of photos would be helpful.
Grading will be based on creativity, proper calculations and use of triangles, neatness of presented project.

Monitoring Student Responses

- To understand student thinking and knowledge of the task, the teacher will consistently probe students for responses and discussion. Students will be encouraged to engage in positive discussion with each other and the teacher.
- Teacher will monitor student responses and seek any mathematical misunderstandings and/ or errors and probe the students to correct their own errors. The teacher will guide the students to correct responses, but allow the students to seek the correct the answers through their own thought and exploration.

Summary

- For homework, the students will be asked to finish their designs.

Assessment: Attached rubric (at the end)

Extensions and Connections (for all students)

- Extension: Students should design alternate seating arrangements for each of the 4 designs they have already completed. Ask them to critique one another's designs and decide which designs are the best and most realistic.

Strategies for Differentiation

- Make sure visual learners and ELL learners are always drawing and labeling the triangles. They should label the short leg, long leg, and hypotenuse of all right triangles.

- For high-ability students, ask them to discuss what factors should limit the height of the stadium seating and what factors should limit the depth of the stadium seating. What should the minimum and maximum limits be?

Rubric for lesson 5

	Excellent	Average	Poor
Triangles (Total 50 Points)	Triangles were properly shown in the design. All right triangles were properly solved. Various sizes of triangles were used, and various methods were used to solve the triangles.	Triangles were correct most of the time. There were some errors in calculations. There was not a variety in the sizes of triangles. The same triangle size was used repeatedly and no variety in calculations was used.	Triangles were not used in the design or the triangles were solved incorrectly.
Presentation (Total 30 Points)	The calculations are all shown. Sketches or photos of the design are included. The presentation is neat and easy to decipher.	Calculations are shown but are difficult to follow. Sketches and/or photos are included, but are lacking in neatness.	Calculations are not shown or are not neat. No sketches or photos of the final design are included.
Completed all 4 designs (Total 10 Points)	All 4 designs were completed.	Only 3 of the 4 designs were completed.	2 or less of the designs were completed.
Creativity (Total 10 Points)	The design is realistic. The design allowed high enough incline for the fans to view but not so high that the overall height of the seating arrangement was unrealistic. A design was given that is unique.	The design is realistic but one that is common. The angle of elevation is reasonable in all or most of the designs.	The design is not realistic and would not be possible to complete in real life.

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