

# Surface Area and Volume of 3-D Objects

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## **I. UNIT OVERVIEW & PURPOSE:**

Throughout this unit, students will design and budget a fish tank for production – focusing on the surface area (amount of glass needed) and volume (amount of water it will hold). Students will need to work within a budget and design requirements for the fish tank in order to not waste materials or funds while at the same time maximizing the space the fish have to move and the view the owner will have. The first lesson will review the understanding of surface area and volume.

## **II. UNIT AUTHOR:**

Helen Price, Salem High School, Salem City Schools

## **III. COURSE:**

Mathematical Modeling: Capstone Course (the course title might change)

## **IV. CONTENT STRAND:**

Geometry

## **V. OBJECTIVES:**

Calculating the surface area and volume of 3-D objects; calculating the missing length of a 3-D object; compare ratios of lengths areas, and volumes; determine how the surface area or volume affects one or more of the lengths; how the lengths of a 3-D object affects the surface area or volume; solve real-world problems using 3-D objects

## **VI. MATHEMATICS PERFORMANCE EXPECTATION(s):**

MPE.6 The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

MPE.7 The student will use similar geometric objects in two- or three-dimensions to

- a) compare ratios between side lengths, perimeters, areas, and volumes;
- b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;
- c) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and
- d) solve real-world problems about similar geometric objects.

## **VII. CONTENT:**

In this unit I plan to address the issues of manufacturing, shopping for materials (comparing costs), and budgeting.

## **VIII. REFERENCE/RESOURCE MATERIALS:**

Students will need to have access to the internet – be familiar with searching for items (materials), and have a basic working knowledge of excel (including formulas).

**IX. PRIMARY ASSESSMENT STRATEGIES:**

Students will be required to budget the production of a fish tank. They will need to keep the total cost below a given amount and determine the amount of materials needed for the project. A detailed explanation of their project and findings will be graded using a rubric.

**X. EVALUATION CRITERIA:**

A rubric will be given to the students to aid in knowing what they will be graded on.

**XI. INSTRUCTIONAL TIME:**

There should be at least 20 – 25 minutes of instructional time devoted to the beginning of this unit; during which time you will need to explain the overview of the project, expectations and objectives of the project, and review internet access rules. At the start of each lesson, you should be clear with what the expectations of the day are and what should be completed at a minimum. At least four 50 minute classes will be needed to complete the entire unit.

# Lesson 1: How Surface Area and Volume Change

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## Strand

Geometry

## Mathematical Objective(s)

In this lesson, surface area and volume will be reviewed as well as converting measurements from one unit to another.

## Mathematics Performance Expectation(s)

Problem Solving, Decision Making, and Integration: 6) Use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

**Related SOL** List all applicable SOL for each lesson.

- G.13 The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.
- G.14 The student will use similar geometric objects in two- or three-dimensions to
- compare ratios between side lengths, perimeters, areas, and volumes;
  - determine how changes in one or more dimensions of an object affect area and/or volume of the object;
  - determine how changes in area and/or volume of an object affect one or more dimensions of the object; and
  - solve real-world problems about similar geometric objects.

## NCTM Standards

- Learning about Length, Perimeter, Area, and Volume of Similar Objects
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others
- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other context.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.
- Make and investigate mathematical conjectures.
- Organize and consolidate their mathematical thinking through communication.

## Materials/Resources

Describe the materials and resources (including instructional technology) you plan to use in each lesson.

- Classroom set of calculators (graphing or non-graphing) and computers (excel or a similar program is needed)

## Assumption of Prior Knowledge

- Students should be familiar with the formulas for surface area and volume of 3-D figures.
- Students may find difficulty with converting the volume of a given object in cubic feet/inches into the number of gallons it will hold.

- Student may find difficulty with finding the surface area for the fish tank which does not include the top of the tank.

## Introduction: Setting Up the Mathematical Task

- In this lesson, students will investigate the relationship between lengths and volume of a rectangular solid, as well as convert the volume in cubic feet/inches into gallons.
- Explanation of the overall project should last for about 20 minutes:
  - You are designing a fish tank for production and need to know how much glass (surface area and dimensions) as well as the volume (gallons) the tank will hold. You do not want to spend more than \$50 towards the glass, however you want to maximize the volume (gallons) and view the owner will have while minimizing the cost and use of materials.
  - Ask students why they should maximize the volume (gallons) and view the owner will have while minimizing the cost and use of materials.
- In the first lesson, students will calculate the surface area (with and without the top) and volume of the given tank. Then they will convert the volume into gallons.
- Two similar tanks will be drawn where the lengths, widths, or heights are multiples of each other. In follow up questions, students will relate the volumes.
- Students will work independently on the worksheet provided. Upon completion they will get into pairs to discuss their findings.
- Students should notice in pairs, if not on their own, that there is a relationship between the volume of the object and its related sides. This will aid the students in their design of the fish tank for production.
- Students will be asked to draw upon their prior knowledge during the initial discussion of the project as well as throughout the lesson when they will have to give examples of how to alter the volume with the dimensions of the figure.
- There will be a group discussion of our findings after the paired discussion to ensure that students have all grasped the relationship between the dimensions of a figure and its volume.
- Each pair of students will need to explain in their own way their findings of the relationship between surface area and volume.

## Student Exploration 1:

- What should students be doing?
  - Students should be working on the Surface Area and Volume worksheet. They should be calculating the surface area and volume of each figure. Upon completion they will compare the surface area and volume between two figures that are almost identical – this will aid in their understanding of how one dimension affects both the surface area and volume.
  - Students should then be paired up to further the discussion of the relationship between the two measurements.
- What should teachers be doing to facilitate learning?
  - The teacher will be circulating the classroom to ensure that the students are using the formulas for surface area and volume correctly. If need be, the teacher can ask leading questions to help students discover the relationship between the two measurements.

- After the students have had a chance to discuss within pairs their findings, the class should be brought together for a whole group discussion on their findings.
- An additional discussion should be conducted about the importance of knowing the difference between the surface area of the entire figure and that of a fish tank. Reviewing how to convert a measurement from one unit of measure to another should also be discussed, as well as its importance in this problem.
- Students can then work on completing the chart at the bottom of page two.
- Include possible solutions to the exploration, possible questions to pose to promote student thinking, possible misconceptions or errors, and possible questions to address those misconceptions or errors.
  - Students may think that there is a direct relationship to the surface area or volume and the altered lengths – when it is not as simple as a direct relationship.
  - Students may think that there is no relationship between the lengths and the calculated measurements – you could use the lengths as variables and show them what is occurring during the calculations (or help stronger students to do this and share it with the class).
  - Students may not understand why the width (depth) of the figure never changed from one figure to the next – and that it does not matter with respect to the surface area or volume which measurement is altered.
  - Students may have a hard time finding the relationship between the calculated measurements if more than one measurement was altered at a time.
  - Using the formula aspect of Excel may prove to be advantageous to quickly see the changes in surface area or volume when one length is changed by a factor.

### **Monitoring Student Responses**

- I expect students will use phrases such as: “gets bigger/smaller”, “increases/decreases” instead of “is multiplied by/reduced by a factor” in their explanation of the relationship between the measurements.
- I plan on summarizing the lesson through a classroom discussion of our findings – including input from each pair of students. If needed, include an algebra lesson on how altering one of the lengths affects the calculated measurement – through the use of manipulating the formula.

### **Assessment**

- Describe and attach the assessments for each lesson objective.
  - **Questions**
    - How does the surface area/volume change when only one length of the rectangular solid is altered?
    - What happens when you change the width of the front of the tank? What if you change the width of the side instead? The height?
    - When calculating the volume of a fish tank, why is it important to calculate the number of gallons it will hold?
  - **Journal/writing prompts**
    - Explain in your own words how altering a length on a rectangular solid affects the surface area and volume.
    - Can there be two different sets of dimensions that will yield the same surface area and/or volume? Why or why not? Provide examples to support your answer.

## **Extensions and Connections (for all students)**

- Throughout the rest of the unit, students will be able to add to and apply the knowledge discovered in this unit.
- Students in the sciences – Chemistry and Physics – are required to convert units as incorporated in this lesson.

## **Strategies for Differentiation**

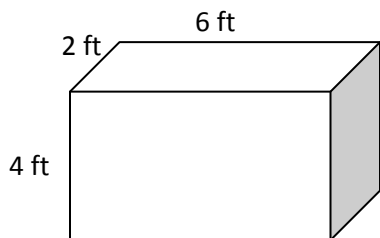
- List ideas for addressing needs of a diverse population of students such as:
  - Providing different sized tissue boxes or cereal boxes may help some students visualize that when a length has changed its surface area/volume has too.
  - Additional problems could be given where the two figures differ in more than just one length (there is one problem like this on the worksheet); or where the change is easier to notice.

Name: \_\_\_\_\_

For the following figures, calculate the surface area and volume of each.

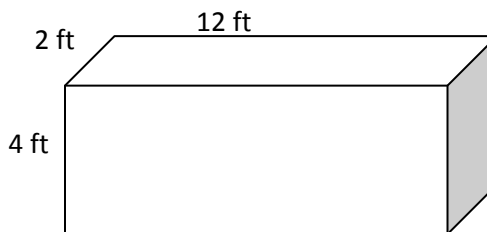
1. Surface Area: \_\_\_\_\_

Volume: \_\_\_\_\_



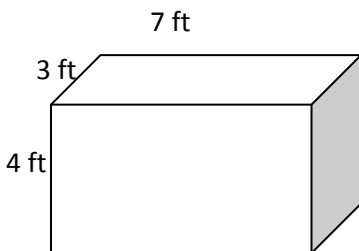
2. Surface Area: \_\_\_\_\_

Volume: \_\_\_\_\_



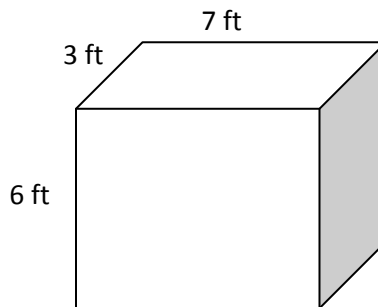
3. Surface Area: \_\_\_\_\_

Volume: \_\_\_\_\_



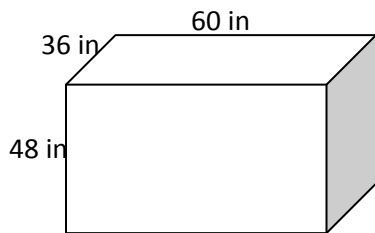
4. Surface Area: \_\_\_\_\_

Volume: \_\_\_\_\_



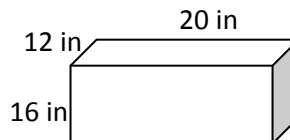
5. Surface Area: \_\_\_\_\_

Volume: \_\_\_\_\_



6. Surface Area: \_\_\_\_\_

Volume: \_\_\_\_\_



How did the shape change from **#1 to #2**?

How did this affect the surface area and volume?

How did the shape change from **#3 to #4**?

How did this affect the surface area and volume?

How did the shape change from **#5 to #6**?

How did this affect the surface area and volume?

*Now pretend that you are manufacturing fish tanks with given dimensions from the reverse of this worksheet. Fill in the chart below. Keep in mind that a fish tank does not have glass on all sides (the top of the fish tank is open, there is no glass). Fish tanks are defined in the industry by the number of gallons they will hold. Convert the volume each tank will hold in cubic feet/inches into gallons (1 cubic foot = 576/77 gallons and 1 gallon = 231 cubic inches).*

Problem Number	Dimensions (l, w, h)	Surface Area of Tank	Volume of Tank (cubic feet/inches)	Volume of Tank (gallons)
1	6, 2, 4			
2	12, 2, 4			
3	7, 3, 4			
4	7, 3, 6			
5	60, 36, 48			
6	20, 12, 16			



# Lesson 2: Minimize Cost Maximize Volume

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## Strand

Geometry

## Mathematical Objective(s)

In this lesson, students will try to minimize the cost by limiting the use of glass while maximizing the volume of the fish tank. Students will continue to calculate the surface area and volume in addition to converting measurements from one unit to another.

## Mathematics Performance Expectation(s)

Problem Solving, Decision Making, and Integration: 6) Use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

**Related SOL** List all applicable SOL for each lesson.

- G.13 The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.
- G.14 The student will use similar geometric objects in two- or three-dimensions to
- compare ratios between side lengths, perimeters, areas, and volumes;
  - determine how changes in one or more dimensions of an object affect area and/or volume of the object;
  - determine how changes in area and/or volume of an object affect one or more dimensions of the object; and
  - solve real-world problems about similar geometric objects.

## NCTM Standards

- Learning about Length, Perimeter, Area, and Volume of Similar Objects
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others
- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other context.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.
- Make and investigate mathematical conjectures.
- Organize and consolidate their mathematical thinking through communication.

## Materials/Resources

- Classroom set of calculators and computers

## Assumption of Prior Knowledge

- Students should be familiar with the formulas for surface area and volume of 3-D figures.
- Students may find difficulty with converting the volume of a give object from cubic feet/inches into the number of gallons it will hold.

- Student may find difficulty with finding the surface area for the fish tank which does not include the top of the tank.
- Students may find difficulty with finding the surface area for the front view of the tank.

## Introduction: Setting Up the Mathematical Task

- In this lesson, students will try to minimize the cost by limiting the use of glass while maximizing the volume of the fish tank. Students will continue to calculate the surface area and volume in addition to converting measurements from one unit to another.
- The teacher will review the class's findings from the previous day's lesson. A brief overview of the project should begin student's focus on today's lesson.
  - You are designing a fish tank for production and need to know how much glass (surface area and dimensions) as well as the volume (gallons) the tank will hold. You do not want to spend more than \$50 towards the glass however you want to maximize the volume (gallons) and view the owner will have while minimizing the cost and use of materials.
  - Ask student why they should maximize the volume (gallons) and view the owner will have while minimizing the cost and use of materials.
  - Ask students what the view the owner will have refers to.
- In the second lesson, students will calculate the maximum amount of glass they can purchase given the cost per square foot; determine what the dimensions of the tank and volume could be.
- Students will be asked to see if there are other dimensions that could be given for the same amount of glass used – along with the volume.
- In follow up questions, students will discuss their findings.
- Students will work independently on the worksheet provided. Upon completion the class will compile a list of the largest volume and smallest amount of glass used (lowest cost).
- A group discussion of our findings will end class.

## Student Exploration 1:

- What should students be doing?
  - Students should be working on the Minimize the Cost/Maximize the Volume worksheet. They should be finding the maximum amount of glass they could purchase, the dimensions of that tank in addition to its volume.
  - Students will then need to draw and label three figures that area possible designs for the proposed fish tank – minimal cost/maximum volume(the use of a spreadsheet may help students see how the dimensions affect the measurements).
- What should teachers be doing to facilitate learning?
  - The teacher will be circulating the classroom to ensure that the students have found the maximum amount of glass needed in the initial problem as well as the dimensions will produce such a surface area. If need be, the teacher can ask leading questions to help students find possible dimensions.

- The teacher will continue to ensure students stay on task and do not have difficulty with finding the surface area of the front view for the fish tank.
- After the students have completed the worksheet, the class should be brought together for a group discussion on their findings which should include a list of the largest volume and smallest amount of glass used (lowest cost).
- An additional discussion should be conducted about the importance of knowing the difference between the surface area of the entire figure and that of the front view of the fish tank.
- Include possible solutions to the exploration, possible questions to pose to promote student thinking, possible misconceptions or errors, and possible questions to address those misconceptions or errors.
  - Students may forget that there is a difference in the surface area of a rectangular solid and the fish tank.
  - Using the formula aspect of Excel may prove to be advantageous to see the changes in surface area or volume when length(s) change.
  - Students may not understand why maximizing the front view is necessary. They may have a difficult time visualizing the orientation of the fish tank.

### **Monitoring Student Responses**

- Towards the end of class, everyone will be brought together for a group discussion on their findings which should include a list of the largest volume and smallest amount of glass used (lowest cost).
- An additional discussion should be conducted about the importance of knowing the difference between the surface area of the entire figure and that of the front view of the fish tank.

## **Assessment**

- Describe and attach the assessments for each lesson objective.
  - **Questions**
    - Why is important to maximize the volume?
    - Why is it important to minimize the cost?
    - What is important about the surface area of the front view?
      - Would you want to consider maximizing/minimizing the front view, why or why not?
  - **Journal/writing prompts**
    - Why is important to maximize the volume?
    - Why is it important to minimize the cost?
    - What is important about the surface area of the front view?
      - Would you want to consider maximizing/minimizing the front view, why or why not?
    - What is the most important aspect of a fish tank in your opinion, with respect to the different criteria we have discussed so far? (Volume, Surface Area, etc.) Explain.

## **Extensions and Connections (for all students)**

- Throughout the rest of the unit, students will be able to add to and apply the knowledge discovered in this unit.

- Students in the sciences – Chemistry and Physics – are required to convert units, in this lesson; they too need to convert units.
- Students in business classes are required to use Excel – including the formula functions – students are given the opportunity and encouraged to use Excel in determining the best design for the fish tank.
- Students could investigate whether certain shapes (tall or skinny tanks) will or will not have strength issues or problems holding water – determine whether the students’ design could actually work.

## Strategies for Differentiation

- List ideas for addressing needs of a diverse population of students such as:
  - Using different sized tissue boxes or cereal boxes may help some process that a length has changed and thus its surface area/volume has too.
  - Using construction paper or card board to physically make a fish tank may help some students with the surface area, volume and front view of the fish tank.
  - Supply students with an excel spread sheet that has the student put in the dimensions of the tank and it will produce the desired information: surface area, volume, etc.
  - Additional requirements could be given to stronger students.

Name: \_\_\_\_\_ *Minimize the Cost/Maximize the Volume (lesson 2)*

*The thickness of glass for fish tanks ranges from 6 mm to 20 mm (depending on the dimensions). On average, the thickness is 9 mm and the cost per square foot of 9 mm glass is \$1.25.*

**You have only \$50 to spend on the glass for the tank.**

- What is the maximum amount of glass you could purchase and at what total price?
- What would the dimensions of that tank be?
  - Are there other dimensions that are also possible?
- What is the volume (both in cubic feet/inches and gallons)?
  - What is the volume for the second set of dimensions?
- Which set of dimensions would you prefer and why?

**Draw and label three figures that are possible designs for your proposed fish tank – try to maximize the volume and minimize the cost. You may want to use a spread sheet/excel to see how the dimensions affect the calculated measurements.**

1.

2.

3.

Drawing	Dimensions (l, w, h)	Surface Area of Front View	Surface Area of Tank	Total Cost of Glass	Volume of Tank (cubic feet/inches)	Volume of Tank (gallons)
1						
2						
3						

Why is it important to maximize the volume?

Why is it important to minimize the cost?

What is important about the surface area of the front view?

Would you want to consider maximizing/minimizing the front view, why or why not?

# Lesson 3: Final Design of Fish Tank

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## Strand

Geometry

## Mathematical Objective(s)

In this lesson, students will create their final design of their fish tank. In doing so they should minimize the cost by limiting the use of glass while maximizing the volume of the fish tank. Students will continue to calculate the surface area and volume in addition to converting measurements from one unit to another.

## Mathematics Performance Expectation(s)

Problem Solving, Decision Making, and Integration: 6) Use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

**Related SOL** List all applicable SOL for each lesson.

- G.13 The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.
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- compare ratios between side lengths, perimeters, areas, and volumes;
  - determine how changes in one or more dimensions of an object affect area and/or volume of the object;
  - determine how changes in area and/or volume of an object affect one or more dimensions of the object; and
  - solve real-world problems about similar geometric objects.

## NCTM Standards

- Learning about Length, Perimeter, Area, and Volume of Similar Objects
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- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.
- Make and investigate mathematical conjectures.
- Organize and consolidate their mathematical thinking through communication.

## Materials/Resources

- Classroom set of calculators and computers

## Assumption of Prior Knowledge

- Students should be familiar with the formulas for surface area and volume of 3-D figures.
- Students may find difficulty with converting the volume of a give object from cubic feet/inches into the number of gallons it will hold.
- Student may find difficulty with finding the surface area for the fish tank which does not include the top of the tank.

- Students may find difficulty with finding the surface area for the front view of the tank.

## Introduction: Setting Up the Mathematical Task

- In this lesson, students will try to minimize the cost by limiting the use of glass while maximizing the volume of the fish tank. Students will continue to calculate the surface area and volume in addition to converting measurements from one unit to another.
- The teacher will review the class's findings from the previous day's lesson. A brief overview of the project should begin student's focus on today's lesson.
  - You are designing a fish tank for production and need to know how much glass (surface area and dimensions) as well as the volume (gallons) the tank will hold. You do not want to spend more than \$50 towards the glass however you want to maximize the volume (gallons) and view the owner will have while minimizing the cost and use of materials.
  - Ask student why they should maximize the volume (gallons) and view the owner will have while minimizing the cost and use of materials.
  - Ask students what the view the owner will have refers to.
- In the third lesson, students will finalize their fish tank design – stating the length, width and height. They will be required to calculate the amount of glass they will need to purchase, the amount of money spent, the surface area of the front of the tank, and the volume (in two different units).
- In follow up questions, students will discuss the designs.
- Students will work independently on the worksheet provided. Upon completion the class will describe their design and explain why they think it is the best.
- A group discussion of our findings will end class.

## Student Exploration 1:

- What should students be doing?
  - Students should be working on the Final Design worksheet. They should be sketching the fish tank, labeling the dimensions and calculating the surface area, volume, etc.
  - Students will then have to describe their design to the class and explain why theirs is the best.
- What should teachers be doing to facilitate learning?
  - The teacher will be circulating the classroom to ensure that the students are completing the assignment. At this point, no help should be needed from the students to calculate any of the desired information. Students can refer back to previous worksheets if need be.
  - It may be helpful to have the students color the front view of the tank to better identify with the dimensions of that particular side.
  - After the students have completed the worksheet, the class should be brought together for a group discussion on their designs. Students will need to describe their design and explain why theirs is the best. Listing the dimensions on the board may be helpful to students to see if there are any similarities – the width/depth in store bought tanks is always 1 foot (or very close to it) .

## Monitoring Student Responses

- Towards the end of class, everyone will be brought together for a group discussion on their designs which should include a description of their design and explanation of why theirs is the best.
- Listing the dimensions on the board may be helpful for students to see if there are any similarities – the width/depth in store bought tanks is always 1 foot (or very close to it).

## Assessment

- Describe and attach the assessments for each lesson objective.
  - **Questions**
    - If you were to go to purchase a fish tank, what criteria would you find important?
    - If two tanks had the same volume, describe how you would choose between them.
  - **Journal/writing prompts**
    - List the different criteria we looked at in designing our fish tank in order of least to most important and explain why you put them in that order of importance: surface area or the entire tank, viewing surface area, volume, etc.

## Extensions and Connections (for all students)

- Students could look online or in a catalog to find the dimensions, surface area, volume, etc. of other fish tanks and see how they compare to their own design.
- Students could build their design for their fish tank out of different materials and create a “store” where other classes could “buy” or vote on the one they would want to purchase. The design could include the information found/calculated through the Final Design worksheet.

## Strategies for Differentiation

- List ideas for addressing needs of a diverse population of students such as:
  - Using construction paper or card board to physically make a fish tank may help some students with the surface area, volume and front view of the fish tank.



Name: \_\_\_\_\_

*Final Design of Fish Tank (lesson 3)*

Using the space below, draw the fish tank you want to manufacture (including the dimensions both in feet and inches). Then list the specifications for the tank.

Length (ft/in): \_\_\_\_\_

Width (ft/in): \_\_\_\_\_

Height (ft/in): \_\_\_\_\_

Surface Area of Front View ( $\text{ft}^2/\text{in}^2$ ): \_\_\_\_\_

Surface Area of Tank ( $\text{ft}^2/\text{in}^2$ ): \_\_\_\_\_

Total Cost of Glass: \_\_\_\_\_

Volume of Tank ( $\text{ft}^3/\text{in}^3$ ): \_\_\_\_\_

Volume of Tank (Gallons): \_\_\_\_\_