# Business Ownership

**I. UNIT OVERVIEW & PURPOSE:**

This unit will give students an opportunity to deal with some aspects of owning a business. Students will look at employee salary, maximizing profit, and revenue growth of the business. Students will gain knowledge in this area while using linear, quadratic, and exponential functions. Students will gain experience in both mathematical verbal and content knowledge. Students will have experience using their mathematical skills through application of a real world problem.

**II. UNIT AUTHOR:** Cathryn Stanley

**III. COURSE:**

Mathematical Modeling: Capstone Course (the course title might change)

**IV. CONTENT STRAND:**

Algebra

**V. OBJECTIVES:**

Students will be translating between a verbal expression to an algebraic equation. Students will work with linear, quadratic, and exponential functions to address domain, range, zeros, intercepts, end behavior, finding values for a function, and making a real world connection using linear, quadratic, and exponential functions.

**VI. MATHEMATICS PERFORMANCE EXPECTATION(s):**

2. The student will collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.
12. The student will transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Students will select and use appropriate representations for analysis, interpretation, and predictions.
13. The student will investigate and describe the relationships among solutions of an equation, zeros of a function, x-intercepts of a graph, and factors of a polynomial expression.
16. The student will investigate and analyze functions (linear, quadratic, exponential, and logarithmic families) algebraically and graphically. Key concepts include a) continuity; b) local and absolute maxima and minima; c) domain and range, including limited and discontinuous domains and ranges; d) zeros; e) x- and y-intercepts; f) intervals in which a function is increasing or decreasing; g) asymptotes; h) end behavior; i) inverse of a function; j) composition of multiple functions; k) finding the values of a function for elements in its domain; and l) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic.

**VII. CONTENT:**

This unit shows a real world application problem of business ownership. Students are

able to make mathematical connections outside of the classroom. Students will be applying their understanding of linear functions through a salary activity. Students will be applying their understanding of quadratic functions by looking at how to maximize profit. Students will be applying their mathematical understanding of exponential functions by looking at revenue growth.

**VIII. REFERENCE/RESOURCE MATERIALS:**

Students will be using a graphing calculator, paper, pencil, a salary template, and an introduction problem.

**IX. PRIMARY ASSESSMENT STRATEGIES:**

Students will be given both informal and formal assessments. An informal assessment will occur through observation of small group and whole group discussion. A formal assessment will be given based on student work submitted. Within each lesson a listing of questions to be answered are provided. Accommodations if needed will be provided to students. Listings of those accommodations are provided in each lesson.

**X. EVALUATION CRITERIA:**

Students will be given both informal and formal assessments. An informal assessment will occur through observation of small group and whole group discussion. A formal assessment will be given based on student work submitted. Within each lesson a listing of questions to be answered are provided.

**XI. INSTRUCTIONAL TIME:**

This unit will consist of 3 lessons of approximately 60 minutes each for 3 days. This time may various due to other instructional duties such as warm ups, quizzes, and homework checks. The teacher can appropriately define the timeframe for the lesson.

# Salary – How much will my employees make?

**Strand**
Algebra

**Mathematical Objective(s)**
The mathematical topic addressed in this lesson is Linear Functions. In this lesson students will develop a mathematical model using linear functions given a salary situation. Students will use their knowledge of algebraic functions specifically linear functions to create a mathematical model and solve a problem. In the mathematical model students will list any assumptions and discuss how these assumptions may change the mathematical model. Students will be expected to make any necessary unit conversions needed for the mathematical model so that all units are the same. Students will be investigating linear functions, domain/range, x and y intercepts, end behavior, finding values of a function, and making connections with the real world.

**Mathematics Performance Expectation(s)**
2. The student will collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.
12. The student will transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Students will select and use appropriate representations for analysis, interpretation, and predictions.
13. The student will investigate and describe the relationships among solutions of an equation, zeros of a function, x-intercepts of a graph, and factors of a polynomial expression.
16. The student will investigate and analyze functions (linear, quadratic, exponential, and logarithmic families) algebraically and graphically. Key concepts include a) continuity; b) local and absolute maxima and minima; c) domain and range, including limited and discontinuous domains and ranges; d) zeros; e) x- and y-intercepts; f) intervals in which a function is increasing or decreasing; g) asymptotes; h) end behavior; i) inverse of a function; j) composition of multiple functions; k) finding the values of a function for elements in its domain; and l) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic.

**Related SOLs**

A.4 The student will solve multistep linear and quadratic equations in two variables, including
   a) solving literal equations (formulas) for a given variable;
   b) justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets;

c) solving quadratic equations algebraically and graphically;
d) solving multistep linear equations algebraically and graphically;
e) solving systems of two linear equations in two variables algebraically and graphically; and
f) solving real-world problems involving equations and systems of equations.

Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.

A.7 The student will investigate and analyze function (linear and quadratic) families and their characteristics both algebraically and graphically, including
a) determining whether a relation is a function;
b) domain and range;
c) zeros of a function;
d) *x*- and *y*-intercepts;
e) finding the values of a function for elements in its domain; and
f) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic.

AII.7 The student will investigate and analyze functions algebraically and graphically. Key concepts include
a) domain and range, including limited and discontinuous domains and ranges;
b) zeros;
c) *x*- and *y*-intercepts;
d) intervals in which a function is increasing or decreasing;
e) asymptotes;
f) end behavior;
g) inverse of a function; and
h) composition of multiple functions.

Graphing calculators will be used as a tool to assist in investigation of functions.

AII.8 The student will investigate and describe the relationships among solutions of an equation, zeros of a function, *x*-intercepts of a graph, and factors of a polynomial expression.

AII.9 The student will collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

AFDA.1 The student will investigate and analyze function (linear, quadratic, exponential, and logarithmic) families and their characteristics. Key concepts include
a) continuity;
b) local and absolute maxima and minima;
c) domain and range;
d) zeros;

e) intercepts;
f) intervals in which the function is increasing/decreasing;
g) end behaviors; and
h) asymptotes.

AFDA.3 The student will collect data and generate an equation for the curve (linear, quadratic, exponential, and logarithmic) of best fit to model real-world problems or applications. Students will use the best fit equation to interpolate function values, make decisions, and justify conclusions with algebraic and/or graphical models.

AFDA.4 The student will transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Students will select and use appropriate representations for analysis, interpretation, and prediction.

**NCTM Standards**

- understand relations and functions and select, convert flexibly among, and use various representations for them;
- analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior;
- identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships;
- draw reasonable conclusions about a situation being modeled.
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others

**Additional Objectives for Student Learning**
N/A

**Materials/Resources**
- Graphing Calculator for each student
- Paper
- Pencil
- Salary Table
- Introduction Problem

| T-shirts sold | Employee Salary |
|---|---|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 20 | |
| 100 | |
| 500 | |
| 1000 | |

**Assumption of Prior Knowledge**

- Students should have already taken Algebra I and Algebra II.
- Students should have an understanding of linear functions.
- Students should be able to recognize a linear function.
- Students should have knowledge of using a graphing calculator for linear functions.
- Students should have some basic knowledge of a real life application problem of owning a business.

## Introduction: Setting Up the Mathematical Task

You are a small business owner of a school t-shirt printing company. Because your business is small you have 4 employees who work on commission. Your job will be to model a salary equation based on a defined problem and find the monthly salaries for the employees. The goal of this lesson is for students to recognize a linear function, discuss domain and range, zeros, x- and y-intercepts, end behavior, finding specific values of the function, and make an algebraic connection between the data and graph.

- The timeframe for this activity would be between 45 – 60 minutes.
- You are a small business owner of a school t-shirt printing company. Because your business is small you have 4 employees who work on commission. Your job will be to model a salary equation based on a defined problem and find the monthly salaries for the employees for number of shirts sold being 0, 1, 2, 3, 4, 5, 20, 100, 500, and 1000 (table provided). Students will model the following scenario: Each employee has a monthly base salary of 500 dollars and receives 10% of each t-shirt sold. Each t-shirt will cost $20 and includes any color t-shirt, printing of school mascot, and printing of a name.
- For an extension of the lesson you could have students provide their own values for the needed information. You could use the following. You are a small business owner of a school t-shirt printing company. Because your business is small you have (you choose the

number of employees) employees who work on commission. Your job will be to model a salary equation based on a defined problem and find the monthly salaries for the employees for number of shirts sold being (you choose the number of t-shirts sold). Students will model the following scenario: Each employee has a monthly base salary of (you choose) dollars and receives (you choose the percent) a percent of each t-shirt sold. Each t-shirt will cost (you choose the cost) and includes any color t-shirt, printing of school mascot, and printing of a name.

- Students will be divided into groups of 2 for this activity.
- It would be helpful for students within their groups to make a list of necessary things they think are important in owning a business specifically in paying their employees.
- Students will need to be thinking about the connection between their total salary and selling of t-shirts. How can I model this scenario? How can I make a prediction of future sales? How can I make a prediction of increased salary?
- Students will find the stated salaries from the above scenario of t-shirts sold. Students will translate between the verbal expression to an algebraic equation. Students will model the data graphically.
- Upon completion of the assigned task there will be a whole class discussion of the findings of the activity.

## Student Exploration 1:

- You are a small business owner of a school t-shirt printing company. Because your business is small you have 4 employees who work on commission. Your job will be to model a salary equation based on a defined problem and find the monthly salaries for the employees for number of shirts sold being 0, 1, 2, 3, 4, 5, 20, 100, 500, and 1000. Students will model the following scenario: Each employee has a monthly base salary of 500 dollars and receives 10% of each t-shirt sold. Each t-shirt will cost $20 and includes any color t-shirt, printing of school mascot, and printing of a name.

**Student/Teacher Actions:**

- Students will need to set up their equation for computing salary such as Income = Base Salary + (Commission * Cost of T-shirt * Number of shirts sold).
- Within student groups make sure students are discussing the relationship between salary and number of t-shirts sold.
- Students will need to plot the data found into a graphing calculator.
- Students need to verify the function is y =500 + 2x. There can be a classroom discussion to make sure the equation is correct.
- Students will need to define the domain and range and describe their meaning.
- Students will need to define any x- and y-intercepts and describe their meaning. You will need to make sure students make a connection between x-intercept and zeros.

- Students will need to describe the end behavior. What is the graph doing? What happens if no t-shirts are sold? A few t-shirts? Many t-shirts?
- Students will be finding specific values for the function by completing the table given.
- Students will need to write a description of the activity and their findings.
- As students are working in groups the teacher should be constantly monitoring student work and discussion taking place within each group.
- Students may need to be reminded when using the graphing calculator their window size may need to be changed.

**Monitoring Student Responses**

- Students will make their mathematical thinking and connections public within their small groups and with the whole class discussion.
- Discussing the process and connection will help develop the mathematical content and vocabulary.
- Have students discuss what it means to have a motivated employee. Is it better for the employer to have an employee who is willing to work hard and sell as many t-shirts as possible? What is best for the employee?
- To extend the lesson some students might want to develop their own business plan.
- The whole class discussion of the activity should take about 10 minutes.

## Assessment

- Students will have an informal assessment as the teacher is monitoring the small group and whole class discussion.
- The formal assessment will come from work the students submit at the end of the activity.
- The formal assessment should include the following:
    - a description of the activity
    - the correct function for the employee salary
    - the correct values found for the function
    - a graph of the data
    - a description of the domain and range
    - a description of zeros
    - a description of x- and y-intercepts
    - a description of end behavior

## Extensions and Connections (for all students)

- Some students may have an interest in owning their own business and may want to set up a business plan to present to the class.
- This lesson can be expanded into looking at maximizing profits and overall revenue growth.

## Strategies for Differentiation

- Preferential seating (near teacher or a buddy)
- Mathematical language and content should be presented simultaneously
- Make instruction more concrete, visual, collaborative, and hands-on
- An awareness of differences in communication of the mathematical concept
- Step by step instructions may be necessary
- Simplification of language may be necessary instead of simplification of the mathematical concept
- Let students express themselves without a focus on grammatical skills and accuracy
- Observe students' participation with small group discussion

# Profit – How much can I make?

**Strand**
Algebra

**Mathematical Objective(s)**
The mathematical topic addressed in this lesson is Quadratic Functions. In this lesson students will develop a mathematical model using quadratic functions given a specific profit and maximizing situation. Students will use their knowledge of algebraic functions specifically linear and quadratic functions to create a mathematical model and solve a problem. In the mathematical model students will list any assumptions and discuss how these assumptions may change the mathematical model. Students will be expected to make any necessary unit conversions needed for the mathematical model so that all units are the same. Students will be investigating linear functions, quadratic functions, domain/range, x and y intercepts, end behavior, finding values of a function, and making connections with the real world.

**Mathematics Performance Expectation(s)**
2. The student will collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.
12. The student will transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Students will select and use appropriate representations for analysis, interpretation, and predictions.
13. The student will investigate and describe the relationships among solutions of an equation, zeros of a function, x-intercepts of a graph, and factors of a polynomial expression.
16. The student will investigate and analyze functions (linear, quadratic, exponential, and logarithmic families) algebraically and graphically. Key concepts include a) continuity; b) local and absolute maxima and minima; c) domain and range, including limited and discontinuous domains and ranges; d) zeros; e) x- and y-intercepts; f) intervals in which a function is increasing or decreasing; g) asymptotes; h) end behavior; i) inverse of a function; j) composition of multiple functions; k) finding the values of a function for elements in its domain; and l) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic.

**Related SOLs**

A.1 The student will represent verbal quantitative situations algebraically and evaluate these expressions for given replacement values of the variables.

A.4 The student will solve multistep linear and quadratic equations in two variables, including
a) solving literal equations (formulas) for a given variable;

b) justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets;
c) solving quadratic equations algebraically and graphically;
d) solving multistep linear equations algebraically and graphically;
e) solving systems of two linear equations in two variables algebraically and graphically; and
f) solving real-world problems involving equations and systems of equations.

Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.

A.7 The student will investigate and analyze function (linear and quadratic) families and their characteristics both algebraically and graphically, including
a) determining whether a relation is a function;
b) domain and range;
c) zeros of a function;
d) *x*- and *y*-intercepts;
e) finding the values of a function for elements in its domain; and
f) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic.

AII.5 The student will solve nonlinear systems of equations, including linear-quadratic and quadratic-quadratic, algebraically and graphically. Graphing calculators will be used as a tool to visualize graphs and predict the number of solutions.

AII.7 The student will investigate and analyze functions algebraically and graphically. Key concepts include
a) domain and range, including limited and discontinuous domains and ranges;
b) zeros;
c) *x*- and *y*-intercepts;
d) intervals in which a function is increasing or decreasing;
e) asymptotes;
f) end behavior;
g) inverse of a function; and
h) composition of multiple functions.

Graphing calculators will be used as a tool to assist in investigation of functions.

AII.8 The student will investigate and describe the relationships among solutions of an equation, zeros of a function, *x*-intercepts of a graph, and factors of a polynomial expression.

AII.9 The student will collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

AFDA.1 The student will investigate and analyze function (linear, quadratic, exponential, and logarithmic) families and their characteristics. Key concepts include
a) continuity;
b) local and absolute maxima and minima;
c) domain and range;
d) zeros;
e) intercepts;
f) intervals in which the function is increasing/decreasing;
g) end behaviors; and
h) asymptotes.

AFDA.3 The student will collect data and generate an equation for the curve (linear, quadratic, exponential, and logarithmic) of best fit to model real-world problems or applications. Students will use the best fit equation to interpolate function values, make decisions, and justify conclusions with algebraic and/or graphical models.

AFDA.4 The student will transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Students will select and use appropriate representations for analysis, interpretation, and prediction.

**NCTM Standards**

- understand relations and functions and select, convert flexibly among, and use various representations for them;
- analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior;
- identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships;
- draw reasonable conclusions about a situation being modeled.
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others

**Additional Objectives for Student Learning**

Students will see the importance of mathematics in the real world and how mathematics can be applied to specific situations.

**Materials/Resources**

- Graphing Calculator for each student
- Paper
- Pencil
- Dictionary
- Introduction Problem

**Assumption of Prior Knowledge**

- Students should have already taken Algebra I and Algebra II.
- Students should have an understanding of linear functions and quadratic functions.
- Students should be able to recognize a linear function and a quadratic function.
- Students should have knowledge of using a graphing calculator for linear functions and quadratic functions.
- Students should have some basic knowledge of a real life application problem of owning a business.

## Introduction: Setting Up the Mathematical Task

You are a small business owner of a school t-shirt printing company. Because your business is small you have 4 employees who work for a base salary of 500 and receive a 10% commission of each t-shirt sold. The cost of each t-shirt is $20. The company has several operating costs some of these cost being fixed costs and variable costs. Suppose your business has fixed costs of $1000. Your job will be to model a salary equation, an operating cost function, and a revenue function based on a defined problem. From these functions your job will be to find the break-even point for the company, the maximum revenue, and maximizing profit. The goal of this lesson is for students to recognize a linear function and quadratic function, discuss domain and range, zeros, x- and y-intercepts, end behavior, finding specific values of the function, and make an algebraic connection between the data and graph.

- The timeframe for this activity would be between 60 – 90 minutes.
- Students will be divided into groups of 2 for this activity.
- Students will need to define the following terms: commission, operating costs, fixed costs, variable costs, revenue, break-even, maximum, and profit.
- It would be helpful for students within their groups to make a list of what they think are variable costs and fixed costs.
- Students will model the following scenario: You are a small business owner of a school t-shirt printing company. Because your business is small you have 4 employees who work for a base salary of 500 and receive a 10% commission of each t-shirt sold. The cost of each t-shirt is $20. The company has several operating costs some of these cost being fixed costs and variable costs. Suppose your business has fixed costs of $1000. Your job will be to model a salary equation, an operating cost function, and a revenue function based on a defined problem. From these functions your job will be to find the break-even point for the company, the maximum revenue, and maximizing profit.
- For an extension of the lesson you could have students provide their own values for the needed information. You could use the following. You are a small business owner of a school t-shirt printing company. Because your business is small you have (you choose the number of employees) employees who work for a base salary of (you choose the salary) and

receive a (you choose the percent of commission) commission of each t-shirt sold. The cost of each t-shirt is (you choose the cost of each t-shirt). The company has several operating costs some of these cost being fixed costs and variable costs. Suppose your business has fixed costs of (you choose the amount of fixed costs). Your job will be to model a salary equation, an operating cost function, and a revenue function based on a defined problem. From these functions your job will be to find the break-even point for the company, the maximum revenue, and maximizing profit.

- Students will need to be thinking about the connection between their operating costs and selling of t-shirts. How can I model this scenario? How can I make a prediction of future sales? How can I make a prediction of increased profit?
- Students will translate between the verbal expressions to an algebraic equation. Students will model the data graphically. Students will model a salary equation, an operating cost function, and a revenue function.
- Upon completion of the assigned task there will be a whole class discussion of the findings of the activity.

## Student Exploration 1:

You are a small business owner of a school t-shirt printing company. Because your business is small you have 4 employees who work for a base salary of 500 and receive a 10% commission of each t-shirt sold. The cost of each t-shirt is $20. The company has several operating costs some of these cost being fixed costs and variable costs. Suppose your business has fixed costs of $1000. Your job will be to model a salary equation, an operating cost function, and a revenue function based on a defined problem. From these functions your job will be to find the break-even point for the company, the maximum revenue, and maximizing profit.

**Student/Teacher Actions:**

- Students will need to set up their equation for computing operating cost and revenue of the business.
- Operating cost might be modeled as Cost = Employee Commission * Cost of T-shirt * Number of Shirts Sold + Fixed Costs.
- Profit costs might be modeled as Profit = (t-shirt cost – Company Commission) * number of t-shirts sold.
- Within student groups make sure students are discussing the relationship between operating cost, revenue, and number of t-shirts sold.
- Students will need to enter the equations found into a graphing calculator.
- Students need to verify the functions are: Cost = 2x + 500 + 1000 = y =1500 + 2x and Profit = $y = 20x – 3/5x^2$. This equation comes from the selling price of the t-shirts: (20 – 3/5x) times

the number x of t-shirts sold). There can be a classroom discussion to make sure the equation is correct. Quadratic equations can be used to find the breakeven point.

- Students will need to find the break-even point, the maximum revenue, the maximum profit that can be made based on the scenario, and what is the cost of t-shirt that could maximize the company's profit. When students are finding these answers they will need to describe them using the following vocabulary found below.
- Students will need to define the domain and range and describe their meaning.
- Students will need to define any x- and y-intercepts and describe their meaning. Students will need to make a connection between the terminology x-intercepts and zeros.
- Students will need to describe the end behavior. What is the graph doing?
- Students will need to write a description of the activity and their findings.
- As students are working in groups the teacher should be constantly monitoring student work and discussion taking place within each group.
- Students may need to be reminded when using the graphing calculator their window size made need to be changed.

**Monitoring Student Responses**

- Students will make their mathematical thinking and connections public within their small groups and with the whole class discussion.
- Discussing the process and connection will help develop the mathematical content and vocabulary.
- Have students discuss what it means to have operating costs and revenue. Will the cost of the company's t-shirt need to be increased? Does the company need fewer, the same, or more employees? What would help reduce operating costs but increase revenue?
- To extend the lesson some students might want to develop their own business plan.
- The whole class discussion of the activity should take about 10 – 20 minutes.

## Assessment

- Students will have an informal assessment as the teacher is monitoring the small group and whole class discussion.
- The formal assessment will come from work the students submit at the end of the activity.
- The formal assessment should include the following:
    - a description of the activity
    - the correct functions for cost and profit
    - the correct values found for the break-even point and maximizing profit
    - a graph of the data
    - a description of the domain and range
    - a description of zeros

- a description of x- and y-intercepts
- a description of end behavior

## Extensions and Connections (for all students)

- Some students may have an interest in owning their own business and may want to set up a business plan to present to the class.
- This lesson can be expanded by looking at a company's overall revenue growth.

## Strategies for Differentiation

- Preferential seating (near teacher or a buddy)
- Mathematical language and content should be presented simultaneously
- Make instruction more concrete, visual, collaborative, and hands-on
- An awareness of differences in communication of the mathematical concept
- Step by step instructions may be necessary
- Simplification of language may be necessary instead of simplification of the mathematical concept
- Let students express themselves without a focus on grammatical skills and accuracy
- Observe students' participation with small group discussion
- Provide Dictionaries to students

# Revenue – Will the company's profit continue to grow?

**Strand**
Algebra

**Mathematical Objective(s)**
The mathematical topic addressed in this lesson is Exponential Functions. In this lesson students will develop a mathematical model using exponential functions given a specific revenue growth or decline situation. Students will use their knowledge of algebraic functions specifically exponential functions to create a mathematical model and solve a problem. In the mathematical model students will list any assumptions and discuss how these assumptions may change the mathematical model. Students will be expected to make any necessary unit conversions needed for the mathematical model so that all units are the same. Students will be investigating exponential functions, domain/range, x and y intercepts, end behavior, finding values of a function, and making connections with the real world.

**Mathematics Performance Expectation(s)**
2. The student will collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.
12. The student will transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Students will select and use appropriate representations for analysis, interpretation, and predictions.
13. The student will investigate and describe the relationships among solutions of an equation, zeros of a function, x-intercepts of a graph, and factors of a polynomial expression.
14. The student will recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and will convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.
16. The student will investigate and analyze functions (linear, quadratic, exponential, and logarithmic families) algebraically and graphically. Key concepts include a) continuity; b) local and absolute maxima and minima; c) domain and range, including limited and discontinuous domains and ranges; d) zeros; e) x- and y-intercepts; f) intervals in which a function is increasing or decreasing; g) asymptotes; h) end behavior; i) inverse of a function; j) composition of multiple functions; k) finding the values of a function for elements in its domain; and l) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic.

**Related SOLs**

A.1 The student will represent verbal quantitative situations algebraically and evaluate these expressions for given replacement values of the variables.

A.4 The student will solve multistep linear and quadratic equations in two variables, including

a) solving literal equations (formulas) for a given variable;
b) justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets;
c) solving quadratic equations algebraically and graphically;
d) solving multistep linear equations algebraically and graphically;
e) solving systems of two linear equations in two variables algebraically and graphically; and
f) solving real-world problems involving equations and systems of equations.

Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.

A.7 The student will investigate and analyze function (linear and quadratic) families and their characteristics both algebraically and graphically, including

a) determining whether a relation is a function;
b) domain and range;
c) zeros of a function;
d) *x*- and *y*-intercepts;
e) finding the values of a function for elements in its domain; and
f) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic.

AII.5 The student will solve nonlinear systems of equations, including linear-quadratic and quadratic-quadratic, algebraically and graphically. Graphing calculators will be used as a tool to visualize graphs and predict the number of solutions.

AII.6 The student will recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and will convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.

AII.7 The student will investigate and analyze functions algebraically and graphically. Key concepts include

a) domain and range, including limited and discontinuous domains and ranges;
b) zeros;
c) *x*- and *y*-intercepts;
d) intervals in which a function is increasing or decreasing;
e) asymptotes;
f) end behavior;

g) inverse of a function; and
h) composition of multiple functions.

Graphing calculators will be used as a tool to assist in investigation of functions.

AII.8 The student will investigate and describe the relationships among solutions of an equation, zeros of a function, *x*-intercepts of a graph, and factors of a polynomial expression.

AII.9 The student will collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

AFDA.1 The student will investigate and analyze function (linear, quadratic, exponential, and logarithmic) families and their characteristics. Key concepts include
a) continuity;
b) local and absolute maxima and minima;
c) domain and range;
d) zeros;
e) intercepts;
f) intervals in which the function is increasing/decreasing;
g) end behaviors; and
h) asymptotes.

AFDA.3 The student will collect data and generate an equation for the curve (linear, quadratic, exponential, and logarithmic) of best fit to model real-world problems or applications. Students will use the best fit equation to interpolate function values, make decisions, and justify conclusions with algebraic and/or graphical models.

AFDA.4 The student will transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Students will select and use appropriate representations for analysis, interpretation, and prediction.

**NCTM Standards**

- understand relations and functions and select, convert flexibly among, and use various representations for them;
- analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior;
- understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions;
- identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships;
- draw reasonable conclusions about a situation being modeled.

- communicate their mathematical thinking coherently and clearly to peers, teachers, and others

**Additional Objectives for Student Learning**
Students will see the importance of mathematics in the real world and how mathematics can be applied to specific situations.

**Materials/Resources**
- Graphing Calculator for each student
- Paper
- Pencil
- Dictionary
- Introduction Problem
- Revenue Chart – Let year 1 = 2008

| Year | T-shirts Sold |
|---|---|
| 2008 | 1300 |
| 2009 | 1800 |
| 2010 | 2400 |
| 2011 | 3100 |
| 2012 | ? |
| 2013 | ? |
| 2014 | ? |
| 2015 | ? |
| 2016 | ? |
| 2017 | ? |

**Assumption of Prior Knowledge**
- Students should have already taken Algebra I and Algebra II.
- Students should have an understanding of exponential functions.
- Students should be able to recognize an exponential function.
- Students should have knowledge of using a graphing calculator for exponential functions.
- Students should have some basic knowledge of a real life application problem of owning a business.

## Introduction: Setting Up the Mathematical Task

You became a small business owner in 2008 of a school t-shirt printing company. In the first year your company's sells 1300 t-shirts. Your job will be to model a revenue growth equation based on the information given in the revenue chart. From this function you will be able to

determine if your revenue will grow or decline or find revenue for a particular year. The goal of this lesson is for students to recognize an exponential function, discuss domain and range, zeros, x- and y-intercepts, end behavior, finding specific values of the function, and make an algebraic connection between the data and graph.

- The timeframe for this activity would be between 45-60 minutes. This timeframe involves students having classroom discussion, group discussions, and completing the work. You must take into consideration all students will not work at the same pace and ample time must be provided for all students to complete the work. The timeframe can be adjusted based on teacher discretion.
- Students will be divided into groups of 2 for this activity.
- Students will need to define the following terms: revenue, growth, decline, exponential.
- It would be helpful for students within their groups to make a list of what they think would cause a company to grow in revenue or decline in revenue.
- Students will model the following scenario: You became a small business owner in 2008 of a school t-shirt printing company. In the first year your company's sells 1300 t-shirts. Your job will be to model a revenue growth equation based on this information.
- Students will need to be thinking about the connection between revenue and selling of t-shirts. How can I model this scenario? How can I make a prediction of future sales? How can I make a prediction of increased profit?
- Students will translate between the verbal expressions to an algebraic equation. Students will model the data graphically. Students will model a revenue function.
- Upon completion of the assigned task there will be a whole class discussion of the findings of the activity.

## Student Exploration 1:

You became a small business owner in 2008 of a school t-shirt printing company. In the first year your company's sells 1300 t-shirts. Your job will be to model a revenue growth equation based on this information. From this function you will be able to determine if your revenue will grow or decline or find revenue for a particular year.

**Student/Teacher Actions:**

- Students will need to find their equation for computing revenue of the business using a graphing calculator and exponential form $y = ab^x$.
- Within student groups make sure students are discussing the relationship between revenue, and number of t-shirts sold.
- Students will need a graphing calculator. I have included the steps if using a TI calculator. Students will need to enter the data into a list using L1 and L2. Remind students to enter 1,

2, 3, 4, etc. as the years. Then again if using the TI Calculator use the ExpReg option to find the exponential equation for the data. Using STAT ->CALC->0:ExpReg

- Students will need to enter the equations found into a graphing calculator.
- Students need to verify the function are: Revenue = $990.60(1.34)^x = y = ab^x$
- Students will need to complete the revenue chart provided.
- When students are finding these answers they will need to describe them using the following vocabulary found below.
- Students will need to define the domain and range and describe their meaning. Make sure students are aware the domain would not include negative values or zero. This should be discussed within small group and whole group discussion.

- Students will need to define any x- and y-intercepts and describe their meaning. Make sure students understand the connection between x-intercepts and zeros. If there are no zeros discuss with students why there are none.
- Students will need to describe the end behavior. What is the graph doing?
- Students will need to write a description of the activity and their findings.
- As students are working in groups the teacher should be constantly monitoring student work and discussion taking place within each group.
- Students may need to be reminded when using the graphing calculator their window size made need to be changed.

**Monitoring Student Responses**

- Students will make their mathematical thinking and connections public within their small groups and with the whole class discussion.
- Discussing the process and connection will help develop the mathematical content and vocabulary.
- Have students discuss what it means to have continued revenue growth. Making a connection between lesson 1 and lesson 2 the following questions could be discussed: Will the cost of the company's t-shirt need to be increased? Does the company need fewer, the same, or more employees? What would help reduce operating costs but increase revenue?
- To extend the lesson some students might want to develop their own business plan or instead of using a graphing calculator try using Excel.
- The whole class discussion of the activity should take about 10 – 20 minutes.

## Assessment

- Students will have an informal assessment as the teacher is monitoring the small group and whole class discussion.
- The formal assessment will come from work the students submit at the end of the activity.
- The formal assessment should include the following:
    - a description of the activity
    - the correct functions for revenue
    - the correct values found for the revenue of the missing years
    - a graph of the data
    - a description of the domain and range
    - a description of zeros
    - a description of x- and y-intercepts
    - a description of end behavior

## Extensions and Connections (for all students)

- Some students may have an interest in owning their own business and may want to set up a business plan to present to the class.
- This lesson can be expanded by looking at a company's overall revenue growth using Excel.

## Strategies for Differentiation

- Preferential seating (near teacher or a buddy)
- Mathematical language and content should be presented simultaneously
- Make instruction more concrete, visual, collaborative, and hands-on
- An awareness of differences in communication of the mathematical concept
- Step by step instructions may be necessary
- Simplification of language may be necessary instead of simplification of the mathematical concept
- Let students express themselves without a focus on grammatical skills and accuracy
- Observe students' participation with small group discussion
- Provide Dictionaries to students

Answer Key for Lesson 1 Formal Assessment

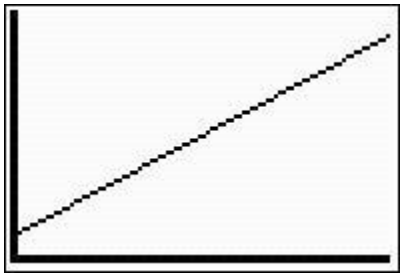
- The formal assessment should include the following:
  - a description of the activity
    (This will be in students own words. Answers will vary)

  - the correct function for the employee salary
    ($y = 500 + 2x$)

  - the correct values found for the function

| T-shirts sold | Employee Salary |
|---|---|
| 0 | 500 |
| 1 | 502 |
| 2 | 504 |
| 3 | 506 |
| 4 | 508 |
| 5 | 510 |
| 20 | 540 |
| 100 | 700 |
| 500 | 1500 |
| 1000 | 2500 |

  - a graph of the data



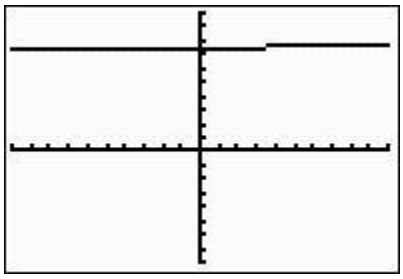
  - a description of the domain and range
    Domain $x \geq 0$ and Range $y \geq 500$

  - a description of x- (zeros) and y-intercepts
    x-intercept is 0 and y-intercept -250

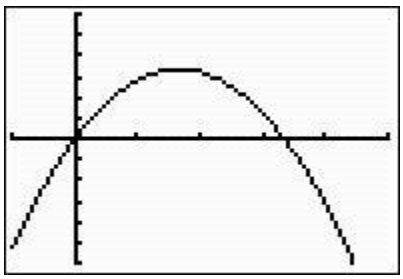
  - a description of end behavior
    As long a t-shirts are sold the graph will continue to increase (answers may vary)

Answer Key for Lesson 2 Formal Assessment

- The formal assessment should include the following:
  - a description of the activity
    Answers may vary by student
  - the correct functions for cost and profit
    Cost = 2x + 500 + 1000 = y =1500 + 2x and Profit = y = 20x – $3/5x^2$
  - the correct values found for the break-even point and maximizing profit
    If zero t-shirts are sold the break-even point is 1500.
    If 16 t-shirts are sold the maximum profit is 166.5.
  - a graph of the data
    Cost



    Profit



  - a description of the domain and range
    Cost – Domain All Reals, Range All Reals
    Profit – Domain $x \geq 16$, Range All Reals
  - a description of x- and y-intercepts
    Cost no x-intercepts, y-intercept 1500
    Profit x-intercept (zero) at 0 and 33.3, y –intercept 0
  - a description of end behavior
    Cost continues, Profit reaches a maximum at (16.7, 166.7)

Answer Key for Lesson 3 Formal Assessment

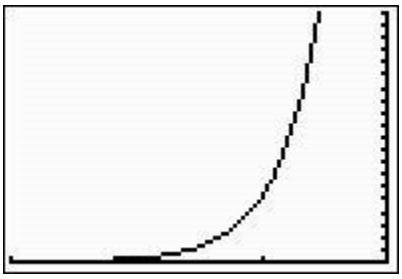
- The formal assessment should include the following:
  - a description of the activity
    Answers may vary

  - the correct functions for revenue
    Revenue = $990.60(1.34)^x$

  - the correct values found for the revenue of the missing years

| Year year 1 is 2008 | T-shirts Sold approximate |
|---|---|
| 2008 | 1300 |
| 2009 | 1800 |
| 2010 | 2400 |
| 2011 | 3100 |
| 2012 | 4000 |
| 2013 | 6000 |
| 2014 | 8000 |
| 2015 | 10000 |
| 2016 | 14000 |
| 2017 | 18000 |

  - a graph of the data



  - a description of the domain and range
    Answers may vary as x increases y increases

  - a description of x- and y-intercepts
    none

  - a description of end behavior
    increases to infinity